the period from year 0~1 to year T-1~T ($\sum_{\tau=1}^{T} B_\tau$). We call $TPAP_T^\alpha$ in Eq.[4] as "cumulated first marriage effect". It is a period measure which increases/decreases only in response to the change in the 1st marriage hazard. It corresponds with time series of the total average number of births that women in hypothetical cohorts would have, if no change in childbirth hazards and shapes of the age schedule from year 0 to year T. Similarly, $TPAP_T^\beta$ in Eq.[5], "cumulated marital fertility effects", reveals time series of period total average parities with a fixed naptiality. It reflects a cumulative effect of changes in childbirth hazards of the ever-married from year 0 to year T, interpreted as the number of births of women in hypothetical cohorts under a constant 1st- marriage hazard at the level of year 0 with an invariant shape of age pattern. Eq.[2] decomposes an annual average change of TPAP from year 0 to T into contributions of the cumulated nuptiality and marital childbirths.

Defining $TPAP_T^\alpha$ in Eq.[4] and $TPAP_T^\beta$ in Eq.[5] is attractive, because the decomposition result can be graphically summarized and demonstrated in one single figure. Notice from the equations [3], [4] and [5] that the difference between the cumulated marital fertility (first marriage) effect and the TPAP calculated by the multistate lifetable for year t equals to the total first marriage (marital fertility) effects cumulated from year 0~1 up to year t-1~t. Figure 4 depicts Eq. [3], [4] and [5] for Chinese in Singapore, and illustrates that the area between the dotted line and the solid line corresponds to the total decline of TPAP due to the decrease in the marital fertility from 1980 upto each year ($-\sum_{\tau=1}^{t} B_\tau$).

Summary of the results

Figure 5 depicts the decomposition results for Chinese and figure 6 corresponds with the result for Malay's TPAP. Table 1 summarized the calculation of percent distribution of both effects for overall change in 1980-2010.

By the comparison between figure 5 and 6, it is evident that the changes in marital fertility affected TPAP severer for the Chinese than for the Malay. Among the Chinese, table 1 shows that both the 1st marriage- and marital fertility- effects account halves of the decline in TPAP for 1980-2010. Moreover, figure 5 shows that TPAP decreased mainly due to marital fertility effects for 1980-1984 and 1988-1999, while decreases of the 1st marriage increasingly affected TPAP after 2000. Among the Malays, table 1 confirms that marital childbirth hazards had the positive net effects on TPAP for

1980-2010 overall. Also, figure 6 shows positive marital fertility effects increased from the mid-1980s to around 1990 and decreased from around 2000 to the mid-2000s; marital fertility effects stayed almost unchanged in other periods and the 1st marriage effects were attributable to TPAP falls after the early 2000s. Prolonged decline in Malay's TPAP since the 1990s with the stability of the cumulated marital fertility effect imply a role of nuptiality as a primary determinant of Malay's fertility decline, especially after the early 2000s.

Table 2 summarizes the ethnic differentials and similarities in the 1st marriage and marital fertility effect on fertility changes by specific periods. Contrary to impressions from the figures for 1980-2010 overall, ethnic differentials are found only in one period for each of effects. First, in 1990s Malay's marital fertility was almost constant, while Chinese marital fertility decreased by 12%. Second, after 2000 Chinese 1st marriage effect decreased by 13%, while Malay's 1st marriage effect fallen by 30%. For other periods, although the magnitude of the fertility varies by ethnic group, patterns of the fertility coincide among two factors.

Concluding remarks

In this paper, we have discussed about how to construct multistate lifetable when state distributions could not be obtained in parts of years, and proposed a numerical model. Even when statistical tables for population at risk for a specific event are not available, we could still construct multistate lifetables, if the size of the total population and the number of demographic events until the year were known. The reason is that the number of demographic events has strong correlation to hazard rates, and information from vital statistics scaled by total population is enough to construct transition probability matrix.

Moreover, if state distributions are observed in more than years and we have the number of demographic events during interim years, we could improve state distribution estimates. The reason is simply using information both from the beginning and the end of the period is better than using only one of them.

Finally, we found ethnic differentials and similarities in the 1st marriage and marital fertility effects. We should be cautious to derive policy implication from the results, because fertility changes as a consequence of many factors that may be endogenous and it is arduous to isolate the effects of policy without an access to more detailed data. However, the fact that Malay's rapid fertility decline after 2000 was a consequence of the 1st marriage effect may call for new research directions to further discuss policy implications, because the government introduced intensive pro-natal policy after 2000. Although Malay's fertility is higher than Chinese in 2010, this fact would suggest that Malay's marriage and fertility behavior be getting resembling Chinese behavior.

Appendix 1. The method for solving the adjustment problem

Let $S=\{1,2,3,4,5,6\}$ denotes the state space. States from 1 to 6 correspond with the marriage and childbirths states as in the following order: nevermarried, evermarried and no child; evermarried and parity 1; evermarried and parity 2; evermarried and parity 3; evermarried and parity 4 and over. Let ${}_5L^{c,S_i,t}_{x\sim x+4}$ be a rate of female population of age x~x+4 who stays in the i[th] state at the time of census in year t. Let ${}^{i\to j}M^{t\to t+1}_{x=x+4\to x+1\sim x+5}$ be the state transition of female of age x~x+4 in year t moving from state i to other states j until age x+1~x+5 by the year t+1. Let ${}_5K^{S_i,t+1}_{x+1\sim x+5}$ be a estimate for the rate of female population of age x+1~x+5 who stays the i[th] state in year t+1. Let ${}_5L^{p,S_i,t+1}_{x+1\sim x+5}$ denote a estimate for the rate of female population of age x+1~x+5 who stays the i[th] state in year t+1. Then $\{{}_5K^{S_i,\tau}_a, {}_5L^{p,S_i,\tau}_a\}$ for $\tau=t+1,\cdots,t+5$ and $a=x+1\sim x+5,\cdots,x+5\sim x+9$ may be solved recursively starting from year-t census distribution until the year t+5 when the next census distribution is available as in Eq.[A1]~[A2].

$$
\begin{aligned}
{}_5K^{S_i,t+1}_{x+1\sim x+5} &= {}_5L^{c,S_i,t}_{x\sim x+4}\cdot\left(1-{}^{i\to i+1}M^{t\to t+1}_{x=x+4\to x+1\sim x+5}\right) \quad \textit{if t is census year}\\
&= {}_5L^{p,S_i,t}_{x\sim x+4}\cdot\left(1-{}^{i\to i+1}M^{t\to t+1}_{x=x+4\to x+1\sim x+5}\right) \quad \textit{otherwise}
\end{aligned}
\qquad \text{Eq.[A1]}
$$

$$
\begin{aligned}
{}_5L^{p,S_i,t+1}_{x+5\sim x+9} &= \delta^{S_i,t\to t+5}_{x\sim x+4\to x+5\sim x+9}+\left({}_5L^{c,S_i,t+1}_{x-4\sim x}+4\cdot{}_5L^{c,S_i,t+1}_{x+1\sim x+5}\right)/5 \quad \textit{if t is census year}\\
&= \delta^{S_i,t\to t+5}_{x\sim x+4\to x+5\sim x+9}+\left({}_5K^{S_i,t+1}_{x-4\sim x}+4\cdot{}_5K^{S_i,t+1}_{x+1\sim x+5}\right)/5 \quad \textit{otherwise}
\end{aligned}
\qquad \text{Eq.[A2]}
$$

where $\{{}_5K^{S_1,t+1}_{16\sim 20}, {}_5K^{S_2,t+1}_{16\sim 20},\cdots,{}_5K^{S_6,t+1}_{16\sim 20}\}=\{1,0,\cdots,0\}$ is given by a radix for the model lifetable, Eq.[A2] defines $\{{}_5L^{p,S_i,t+1}_{x+5\sim x+9}\}$, and we call $\delta^{S_i,t\to t+5}_{x\sim x+4\to x+5\sim x+9}$ as the average error in the estimation for the rate of female population of age x+5~x+9, who stays in i[th] state in year t+5, based on the state distribution in year-t census for the same cohort whose age was x~x+4 in year t. The average error spreads the total error, $\left({}_5L^{p,S_i,t+5}_{x+5\sim x+9} - {}_5L^{c,S_i,t+5}_{x+5\sim x+9}\right)$,

over each predicted values of $\left\{{}_5L^{p,S_i,t}_{x+5\sim x+9}\right\}$ for interim years between two censuses.

We solve the average errors $\left\{\delta^{S_i,t\to t+5}_{x\sim x+4\to x+5\sim x+9}\right\}$ for each state $\{S_i\}$ *for* $i = 2,\cdots,6$ and age $\{x \sim x+4\}$ *for* $x = 20,25,\cdots,45$ by interim period between censuses $\{t \to t+10\}$ *for* $t = 1980,1990,2000$ and $\{t \to t+5\}$ *for* $t = 2005,2010$ by means of minimizing sum of squared errors $\sum_{i=1}^{6}\left(\varepsilon^{i,t\to t+5}_{x\sim x+4\to x+5\sim x+9}\right)^2$, where each of squared errors is calculated by a system of 6 highly nonlinear equations as in Eq.[A3]~[A4]. For the optimization, we rely on the modified newton's method with initial values of $\mathbf{0} = \left\{\delta^{S_i,t\to t+5}_{x\sim x+4\to x+5\sim x+9}\right\}_{i=2,\cdots,6}$. Then, state distributions of all year during the interim period, ${}_5L^{p,S_i,\tau}_{x\sim x+4}$, will be recovered by Eq.[A2].

$$\varepsilon^{i,t\to t+5}_{x\sim x+4\to x+5\sim x+9} = \log\left({}_5L^{p,i,t+5}_{x\sim x+4}/\left(1-{}_5L^{p,i,t+5}_{x\sim x+4}\right)\right) - \log\left({}_5L^{c,i,t+5}_{x\sim x+4}/\left(1-{}_5L^{c,i,t+5}_{x\sim x+4}\right)\right) \textit{ for } i = 2,\cdots,6 \qquad \text{Eq.[A3]}$$

$$\varepsilon^{1,t\to t+5}_{x\sim x+4\to x+5\sim x+9} = \log\left(\left(1-\sum_{i=2}^{6}{}_5L^{p,i,t+5}_{x\sim x+4}\right)\Big/\left(\sum_{i=2}^{6}{}_5L^{p,i,t+5}_{x\sim x+4}\right)\right) - \log\left({}_5L^{c,1,t+5}_{x\sim x+4}/\left(1-{}_5L^{c,1,t+5}_{x\sim x+4}\right)\right) \qquad \text{Eq.[A4]}$$

$${}_5L^{p,S_i,t}_{x\sim x+4} \in [0,1] \quad \forall i,t,x \qquad \text{Eq.[A5]}$$

$$\sum_{i=1}^{6}{}_5L^{p,S_i,t}_{x\sim x+4} = 1 \quad \forall t,x \qquad \text{Eq.[A6]}$$

Note that state distributions are probability distributions so that they must satisfy two conditions specified in Eq.[A4]~[A5]. We apply log-odds transformation as in Eq.[A3] for the first restriction Eq.[A4] and impose the second restriction Eq.[A6] on state 1 as shown in Eq.[A4].

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[ii] For 1980-2010, the number of live births by the order, mother's age and ethnic group can be taken from *Report on the Registration of Births and Deaths* (Registry of Births and Deaths, Immigration and Checkpoints). No data are available for the 1st marriages by women's age and ethnic group, but the 1st marriages by women's age and registration system are available in *Statistics on Marriages and Divorces* (Department of Statistics, Singapore). In 2010, 83% of total marriages was registered under the Women's Charter in which 76% of wives was Chinese. Muslim marriages accounted 13% of total marriages and 73% of muslim wives was Malay. Thus, we expect that the fraction of the 1st marriage among all marriages under Women's Charter given an age should be strongly correlated with Chinese fraction of the 1st marriage among marriages given an age. Then, we indirectly estimate Chinese age-specific first marriages by sum of two parts; Chinese marriage under Women's Charter multiplied by the fraction of 1st marriage under Women's Charter, and the inter-ethnic Muslim marriage of Chinese multiplied by an age distribution of Muslim marriages of other than Malays or Indians and the fraction of 1st marriage under Muslim Law Act.

[ii] Khoo, Chian Kim(1981) *Singapore: Census of Population 1980, Release No.2*

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Figures and table

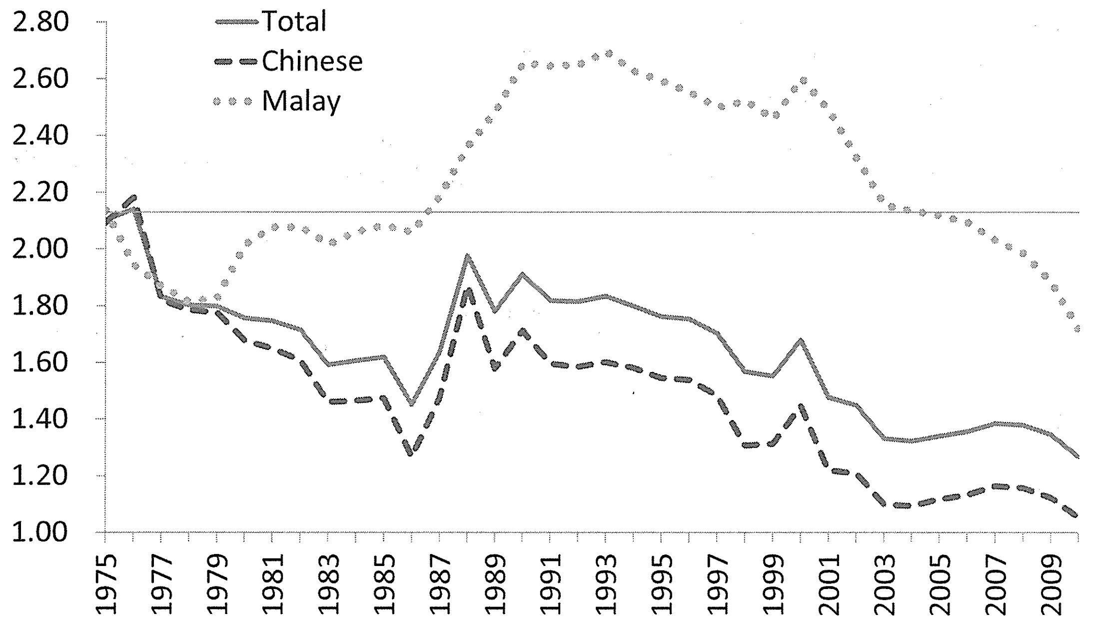
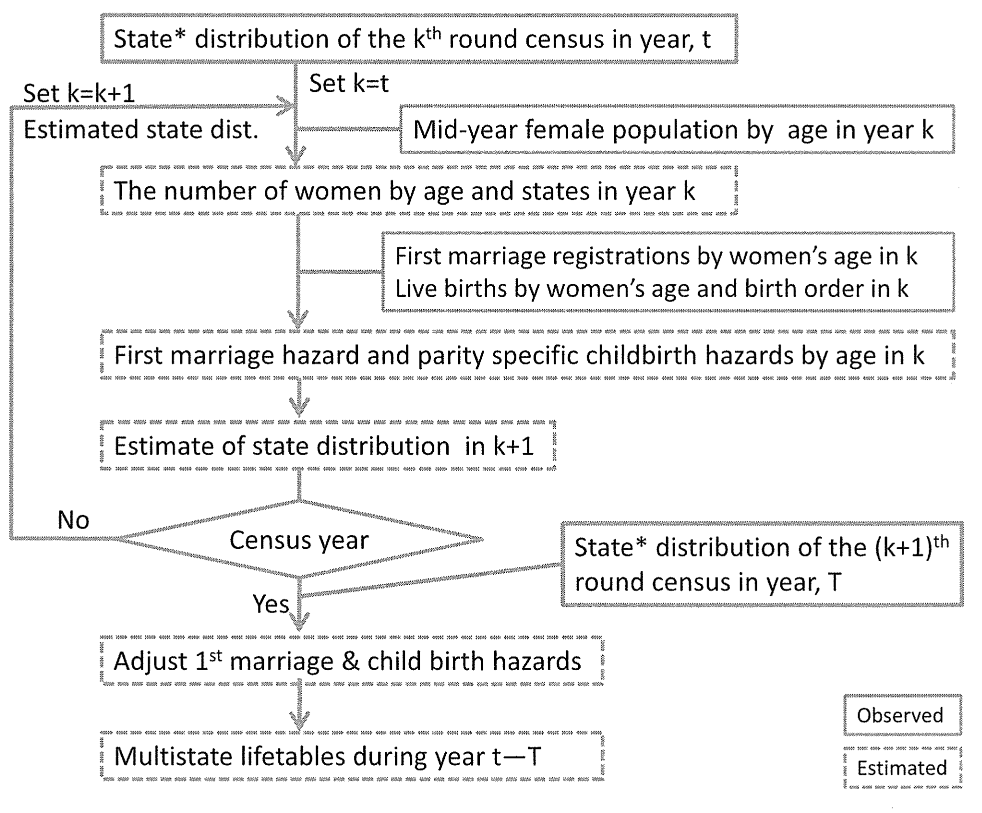


Figure 1. Period TFR by ethnic group in Singapore: 1975-2010.



*States={Nevermarried, Married&[No child, parity 1, parity2 , parity 3, parity 4+]}

Figure 2. Multistate lifetable construction with limited data.

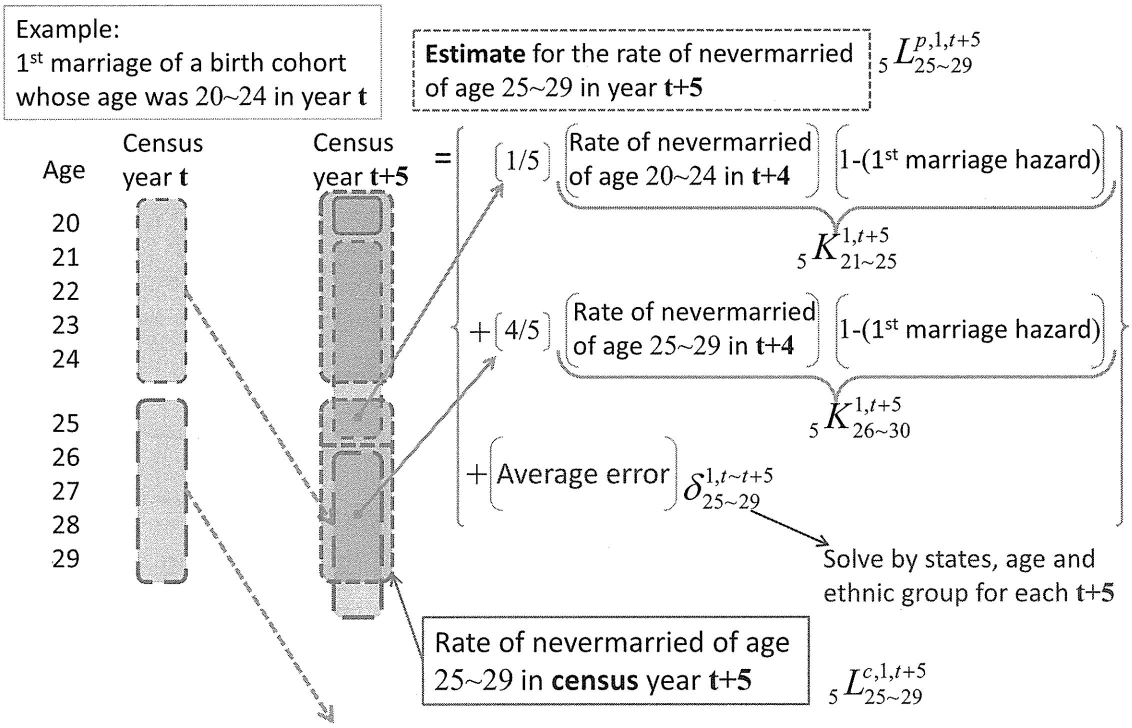


Figure 3. Adjustment strategy for the fist marriage and marital childbirth hazards.

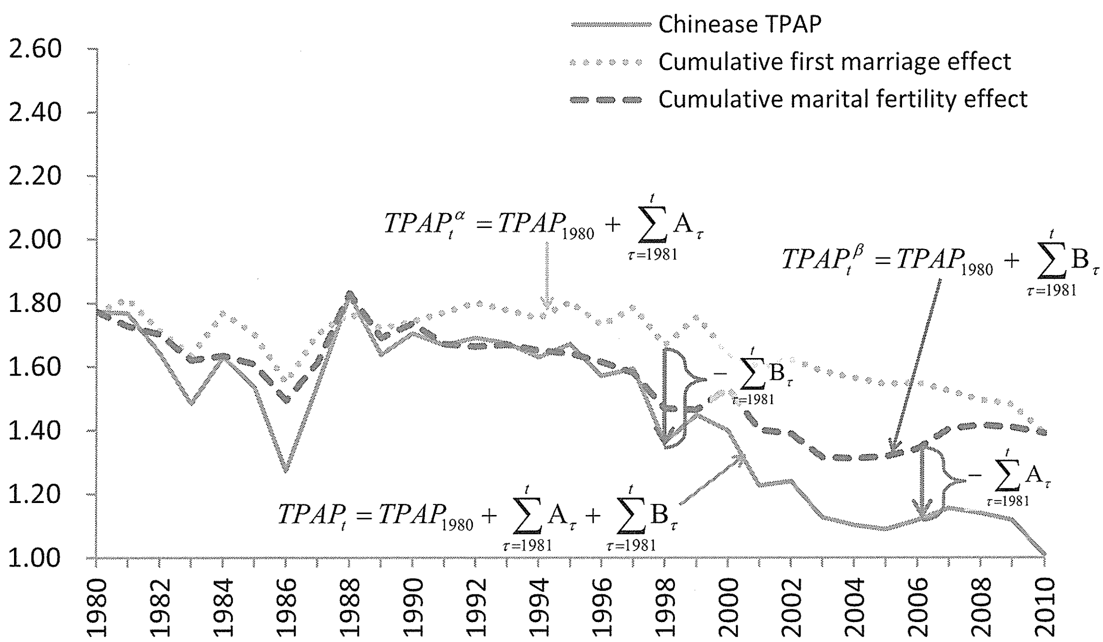


Figure 4. Decomposition of period TPAP into the first marriage component and the marital fertility component.

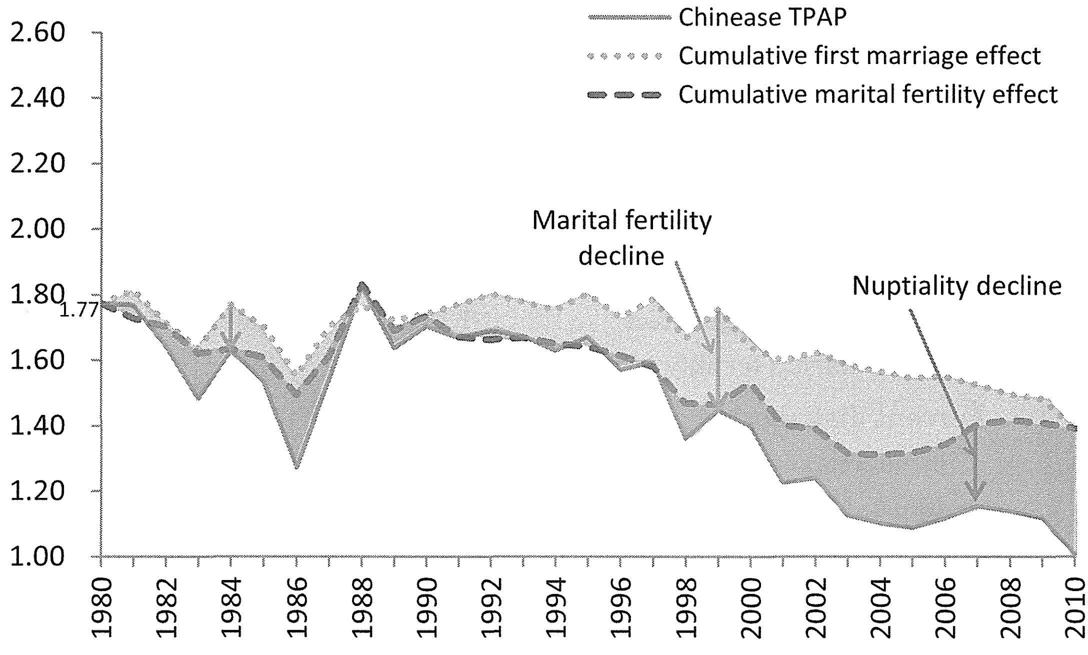


Figure 5. Decomposition result of TPAP into effects of the first marriage and marital fertility: Chinese in Singapore, 1980-2010.

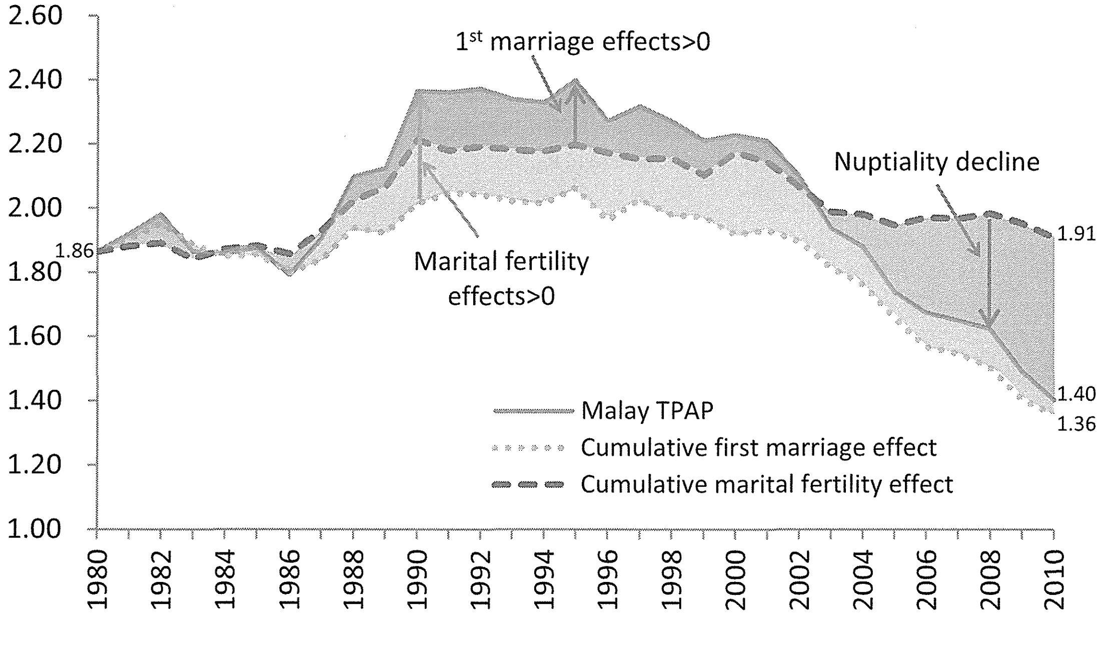


Figure 6. Decomposition result of TPAP into effects of the first marriage and marital fertility: Malay in Singapore, 1980-2010.

Table 1. Decomposition of TPAP into contributions of the 1st marriage and marital fertility effects: Chinese and Malay in Singapore, 1980-2010 overall.

	Years 1980	Years 2010	Δ(2010-1980)
I. Chinese			
Change of period measures			
Total period average parity 1)	1.773	1.008	-0.765
Cum. 1st marriage effect 1)	1.773	1.390	-0.383
Cum. marital fertility effect 1)	1.773	1.391	-0.382
Percent distribution of effects			
1st marriage effect 2)			-50.1
Marital fertility effect 3)			-49.9
II. Malay			
Change of period measures			
Total period average parity 1)	1.863	1.403	-0.461
Cum. 1st marriage effect 1)	1.863	1.358	-0.506
Cum. marital fertitlity effect 1)	1.863	1.908	0.045
Percent distribution of effects			
1st marriage effect 2)			-109.7
Marital fertility effect 3)			9.7

Note: 1) $[TPAP_{2010}-TPAP_{1980}]*B/T$ where B stands for the length of the reproductive years(i.e. age 20-49) and T stands for the length of the period. TPAP should be read as X^a for cumulative 1st marriage effect and X^b for cumulative marital fertility effect, defined in Eq.[6] and Eq.[7], respectively. 2) % ratio of $[X^a_{2010}-X^a_{1980}]$ / $[TPAP_{2010}-TPAP_{1980}]$ where X^a denotes cumulative 1st marriage effect defined in Eq.[6]. 3) % ratio of $[X^b_{2010}-X^b_{1980}]$ / $[TPAP_{2010}-TPAP_{1980}]$ where X^b denotes cumulative marital fertility effect defined in Eq.[7].

Table 2. Ethnic differentials and similarities in the 1st marriage and marital fertility effect on fertility changes.

A. Cumulative marital fertility effect

Period	Simirarity	Dissimilarity	
		Chinese	Malay
1986~1990	Increase*		
1990~2000		Decrease 1.74→1.53(-11.6%)	Almost constant 2.21→2.17(-1.8%)
1999~2000	Increase		
2000~2004	Decrease		
2004~2008	Not Decrease*		

*Patterns differ.

B. Cumulative 1st marriage effect

Period	Simirarity	Dissimilarity	
		Chinese	Malay
1986~1990	Increase		
1990~2000	Little dicrease		
2001~2010		Decrease 1.59→1.39(-13.0%)	Rapid decline 1.93→1.36(-29.6%)