First, looking at the monthly changes of the overall TFR by birth order, we see that the TFR began to drop suddenly in December 2004, and remained low for six months until May 2005. Then, after bottoming out at this point, the TFR shows a subsequent sharp rise. This rising trend accelerates from around December 2005, exhibiting the largest increase in March 2005. Although the rate of increase significantly drops starting around June 2005, the rise itself continues steadily until October 2008 and exhibits a local maximum in November. The TFR then declines slightly or levels off for seven months afterward.

The period where these changes occurred is divided into the following detailed phases:

(1) December 2004 to May 2005 (6 months): Sudden drop

(2-1) June 2005 to November 2005 (6 months): Sharp rise

(2-2) December 2005 to May 2006 (6 months): Sharpest rise

(3-1) June 2006 to February 2007 (9 months): Level off for 1st and 2nd children

(3-2) March 2007 to November 2008 (21 months): Slow increase

(4) From December 2008 and onward: Level off or decline

Each of the four phases—(1) sudden drop, (2) sharp rise, (3) slow increase, and (4) level off or decline—shows significant change. The phases where the TFR rises, (2) and (3), can further be divided into two sub-periods each, according to the difference in pace. The most remarkable change in this period is the change from phase (1) to phase (2), where the TFR bottoms out in May 2005 and shifts from dropping sharply to rising sharply. The time period and pattern of this change are common to the TFR for all birth orders (except that the fertility rate of the first child does not bottom out until June 2005), and it looks as if a sudden restraint and release of childbearing occurred simultaneously among women of all parities.

The second remarkable change is the change from phase (2) to phase (3), where the TFR of the first and second children sharply increases until December 2006 and then shifts to a slow rise, which continues until December 2008. Note that this pattern is not observed in the TFR of the third and further children; in this particular case, the trend continues to rise at a consistent pace until phase (4).

Since these children were conceived approximately nine months before each phase, it is necessary to retrace the timing of pregnancy for each phase in order to investigate the triggers of phase shifts. However, no obvious factors have been found so far (one significant event that occurred in August 2004, i.e., nine months before May 2005, where the greatest change was observed, is the 28th Summer Olympic Games held in Athens, Greece, from August 13 to 29; however, the influence of this event on pregnancy is unknown).

The leveling-off trend observed among all birth orders at the same time in December 2008 and onward in phase (4) may quite possibly signal the end of the rising trend and should be observed closely. Some care must be taken when computing seasonal adjustment according to X-11, as the method tends to generate instability at the terminal parts of time-series data (values may change due to addition of new data), but there can be no doubt that a new trend is beginning in this phase. This phase corresponds to the time period where the influence of the global financial crisis started to spread. However, the period of conception of those births is nine months earlier, where there are no obvious events that might have influenced childbearing to be found.

3.2 Examination of the Tempo Effect – Is it Due to a Catch-up Effect?

The fact that the monthly changes in the fertility rates show the same patterns among all birth orders suggests that the driving force behind these changes is a period effect. That is, if each cohort goes through different changes, there must be some time lag in terms of the changes occurring among higher birth orders. The term period effect here refers to a change in fertility rates caused by certain temporary factors (usually meaning social economic events, such as times of war and economic crisis). In order to examine such changes in the following, it is necessary to define them more precisely.

One of the important aspects of a period effect is that it leaves little influence on the completed fertility of any cohorts involved, although it may bring about significant changes in annual fertility rates. Here, we will use this characteristic as the definition of a period effect for the time being. That is, a period effect is a fertility rate change observed in a certain period, which does not influence the cohort completed fertility (cohort TFR).

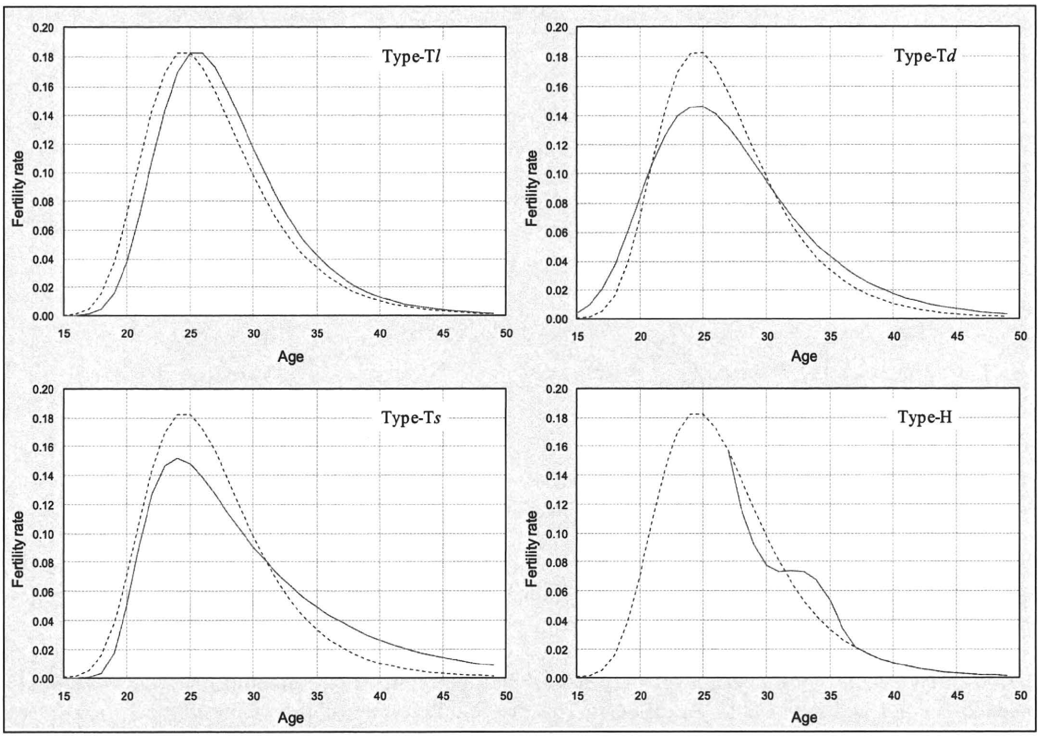
According to this definition, a period effect can be said to be a change in timing occurring in the childbearing schedule in terms of cohort fertility rates. A cohort is considered to have a unique childbearing schedule with a certain potential regularity, and a period effect is a change that causes the actual fertility rates to deviate temporarily from the original schedule without affecting the long-term balance. Not affecting the long-term balance of cohorts means that the change is redeemed by other periods.

It is possible to consider several different types of such changes in cohort childbearing schedules. The first group of changes is the case where the childbearing timing of a cohort as a whole shifts. In this case, a well-known tempo effect acts on the fertility rate for a period. That is, if the mean age at birth (MAB) of a cohort is rising, for example, a tempo effect causes the period TFR to go down. On the other hand, if the rise of the MAB stops or the MAB drops, a tempo effect that pushes up the period TFR comes into play. In this paper, these effects are called type-T period effects (see figure 3 for illustration).

As shown in the figure, there are three different types of effect identified as type-T period effects, i.e. shift in location of fertility schedule on age axis (type-T*l*), shift in dispersion (type-T*d*), and shift in shape (type-T*s*).

Another type of change encompasses disturbances occurring only for parts of a cohort childbearing schedule (the last graph in Figure 3). That is, this type encompasses fertility rate changes caused by a cohort reacting to certain events occurring in the environment and hastening or postponing its childbearing time period.

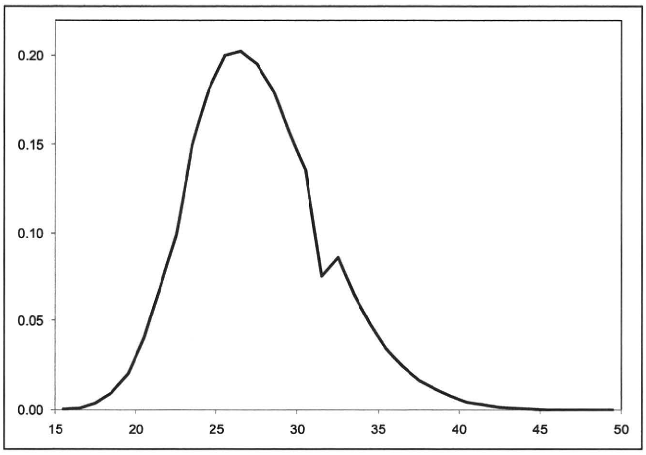
Figure 3. Types of Period Effect in Terms of Cohort Fertility Schedule.



Note: The period fertility exhibits similar changes due to different type of changes in the cohort fertility level and schedule. The period effect of type-T is caused by the shift of the cohort fertility schedule. The period effect of type-H is caused by the temporary fluctuation that is redeemed in another period, while the type-H' effect is a temporary fluctuation that continues to change the completed level of cohort fertility. Thus the type-H' effect is not a genuine period effect by our definition.

In fact, a case example that clearly shows the second type of change exists in recent Japanese history: the so-called Hinoe-uma (Fiery Horse) phenomenon, which occurred in 1966. The Hinoe-uma is a calendar event based on Chinese astrology occurring once every 60 years. Due to the superstition that girls born in that year would cause bad luck for their husbands, many couples avoided having children in that year, and the fertility temporarily dropped by one fourth from the average level (the TFR in 1966 was 1.58 or 75 percent of the average level over 1963 through 1969 except 1966, see changes of TFR in Figure 1). However, all the main cohorts involved in childbearing in this year (the cohorts born from 1923 to 44, who were 22 to 49 years of age at that time) compensated for this loss in the following years and no cohorts exhibited TFR values lower than 2.0. In other words, the Hinoe-uma phenomenon had little effect on the cohort TFR, making it an example of a pure period effect (Figure 4). This type of fertility rate change is called type-H period effects here.

Figure 4. Age-specific Fertility Rate of Japanese Female Cohort Born in 1935.

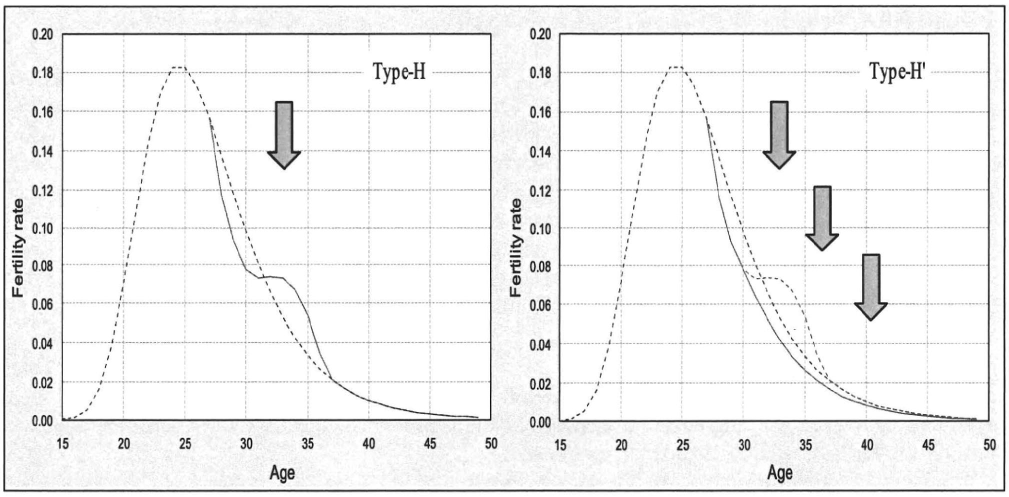


Note: Female cohort born in1935 experienced the Hinoe-uma in1966 at age 31.

It should be noted, however, that some fertility rate changes that occur in reaction to changes in social economy do have lasting influence on the cohort completed fertility. Because they are changes in individual cohorts, it is appropriate to call them period-cohort effects, considering them as a type of cohort effect induced by a certain period. However, whether this kind of period changes is limited to pure period effects (i.e., type-H period effects) or is a period-cohort effect affecting the cohort's long-term balance, cannot be known until the affected cohorts complete their childbearing process. Moreover, in terms of the occurrence mechanism, it is irrelevant whether or not it affects a cohort's long-term balance. For this reason, there should be no problems in handling such period changes as type-H period effects from the viewpoint of investigating causes of the occurrence. Period-cohort effects may simply be considered to be the results of prolonged type-H period effects (hereinafter written as type-H'), as illustrated in Figure 5.

Now, in recent fertility changes in Japan, it is speculated that type-H period effects are important because the same changes are seen among all the fertility rates by birth order, as explained above. Moreover, if the changes that occurred in this period are the results of type-T period effects, it would mean that low fertility rates before reaching the point of reversal were caused by tempo effects due to postponement transitions for each cohort and that upturns of fertility rates would signify regression to cohort fertility level due to the shift in childbearing timing ending. However, this hypothesis can be ruled out by observing the monthly MAB development simultaneously. The MAB has been increasing without leveling off throughout the entire period of drop and upturn in the fertility rate since 2002 for the first and second children, who are the main force of fertility. Thus, it is unlikely that the reversal trend is a sign of reverting to the cohort level due to tempo effects dying out, i.e., "tempo transition."

Figure 5. Types of Period Effect in Terms of Cohort Fertility Schedule.



Bongaarts and Feeney (1998) proposed an index that eliminates tempo effects from period TFR. Here, we will use this index to check the development of the effects acting on the period TFR in Japan and whether or not they are tempo effects[3]. The index proposed by Bongaarts and Feeney is referred to as ATFRp in the following. Figure 6 illustrates the development of ATFRp along with the normal TFR. Tempo effects are represented as the differences between ATFRp and TFR. It is seen that relatively large tempo effects have been in action even after the start of the upturn in 2006, reflecting the continuous rise of the MAB mentioned above. It can furthermore be seen that the tempo effects in 2006 and 2007 amount to 0.17 and 0.14, respectively, which are substantially larger than the value of 0.12 in 2005 when the TFR bottomed out. The value for 2008, 0.09, is only tentative, but at least it does not appear as if the rise of TFR since 2006 is caused by tempo effects dying out (here fertility rates are calculated only with births to Japanese women).

Now, the ATFRp approach estimates tempo effects under certain assumptions. That is, the age-specific period fertility rate is composed of age-specific fertility rates of a large number of cohorts, but the ATFRp index proposed by Bongaarts and Feeney assumes that the age-specific period fertility rate is composed of age-specific fertility rates of all cohorts who are experiencing the timing shift at the same speed, and then eliminates the tempo effects (or tempo distortion) caused by this shift (Bongaarts and Feeney 1988). The uniform timing shift speed $r(t)$ in year t is given as the change in the average age of childbearing in a given period compared to the previous year (in this paper, the average value of change from the previous year and the change to the next year is used).

This view implicitly allows the timing change speed of fertility rate $f(t_c+a, a)$ experienced at a certain age (a) in a certain year ($t= t_c+a$) to fluctuate by age a (that is, for each year t) when focus is placed on a

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[3] Since the fertility rate (i.e., including children with Japanese nationality born to non-Japanese women) and the total fertility rate (see the formula below) defined in the same way as in the Vital Statistics corresponding to the aforementioned fertility rate composition all depend on the demographic compositions of Japanese and non- Japanese women, they can be calculated as a result of population projection. Handling such individually defined fertility rates in the overall fertility rate assumptions of the future population projection makes the projection methodology considerably more complicated, though it is an indispensable mechanism for accurate reproduction of the future population status where international population exchanges have advanced.

Definition of the total fertility rate of the Vital Statistics;

$$\text{(Total fertility rate)} = \sum_{\substack{\text{Sum for ages} \\ (15\text{-}49)}} \frac{\begin{array}{c}\text{(Number of births} \\ \text{by Japanese females)}\end{array} + \begin{array}{c}\text{(Number of births with Japanese} \\ \text{nationality born from} \\ \text{non-Japanese females*)}\end{array}}{\text{(Population of Japanese females)}}$$

*A child with Japanese nationality born from a non-Japanese female is a child whose father is Japanese.

single cohort (birth year t_c) and, instead, assumes that all cohorts involved have a common timing change speed within a given year (period-shift framework). That is: $\mathrm{ATFRp}(t) = \Sigma f(t,a) / (1 - r(t))$, where Σ is the sum for age a (note that this calculation is performed for each birth order and the value is obtained by summing up the results).

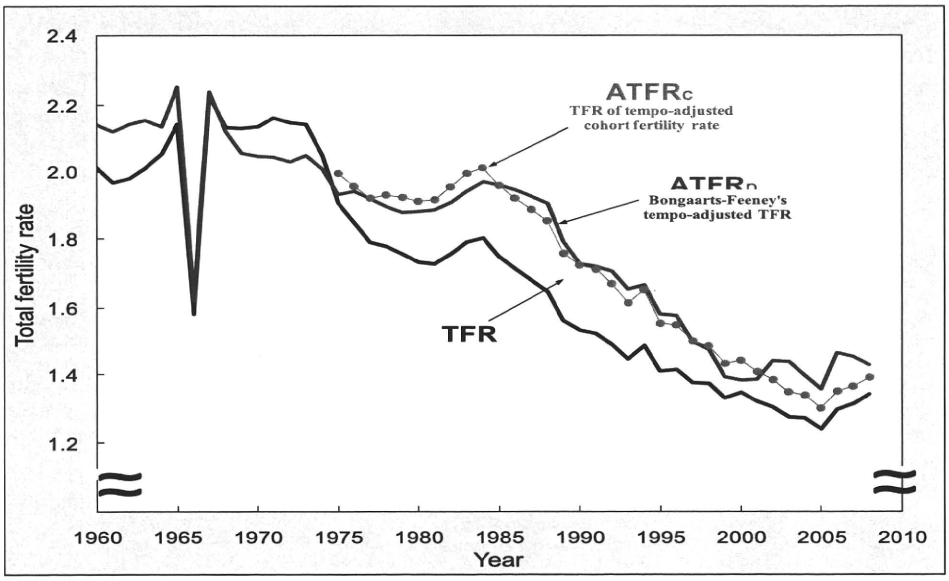
This view prioritizes harmonization among age-specific period fertility rates. However, in some cases it might be more appropriate to give precedence to harmonization of age-specific cohort fertility rates; that is, a framework in which r, the timing change speed of fertility rate, would be seen as a characteristic of a cohort and its tempo effect on the period total fertility is also a characteristic unique to the cohort (We refer to this as a cohort-shift framework). This can be achieved by expressing the timing change speed of a cohort as a function of the cohort born in year t_c, and the timing effect on the periods from this cohort as $\tau(t_c) = 1/(1 + r(t_c))$ (van Imhoff 2001), and the period TFR with adjusted timing effects as $\mathrm{ATFRc}(t) = \Sigma f(t,a) / \tau(t-a)$, where Σ is the sum for age a (note that this calculation is also performed for each birth order and the value is obtained by summing up the results).

Note that in this calculation, in addition to the measured age-specific fertility rates, the timing change for related cohorts is required, and has to be obtained from cohort fertility rate assumptions in the Population Projections. However, we emphasize that it is only the timing changes in future fertility rates that are required - the fertility rates themselves are not used.

Figure 6 shows the result of calculating the ATFRc index from the fertility trend data recorded in Japan. In the period leading up to 2000, both ATFRp and ATFRc follow very similar paths. From 2000, however, they show slightly different behaviors. In particular, in 2000 and onward, ATFRc continues dropping alongside the TFR trend and also indicates a rapid increase at the upturn in the same way as for TFR. Assuming that the cohort-based timing change is essentially continuous, the period effects are leveled out and show smooth development, but persistent tempo effects still appear clearly, suggesting that the true cohort TFR is actually higher than the values observed in each period.

The two adjusted TFR indices show an upturn in the same way as for the measured TFR, suggesting the increase in this period is not the recovery brought about by the elimination of type-T tempo effects, but rather a substantive rise of type-H effects.

Figure 6. Trends of the Total Fertility Rates with/without Tempo-adjustment



Note: The fertility rates are calculated based on births by Japanese women only.

Now, the discussion above suggested that the main cause of the recent upturn in the fertility rate is a type-H period effect. However, it has also been confirmed that the MAB for the first and second children continuously rises over this period, which means that tempo effects that push down the period TFR exist. These tempo effects can also be seen from the development of the ATFRp and ATFRc indices for this period. The question now becomes, how can the scale of type-H period effects be measured while such tempo effects exist.

We propose to apply a model based on the population projections. In the "Population Projections for Japan," a cohort model is used for formulating fertility rate assumptions[4]. In particular, the childbearing schedule in the entire reproductive life course is projected for individual single year cohorts of women, and this schedule is then reorganized in order to project age-specific fertility rates on a yearly basis from the past into the future (Kaneko et al. 2008).

The projection model is particularly good at describing the age-specific cohort fertility rate, and we believe it is fully capable of describing and expressing regularities latent at the base of the cohort childbearing schedule (see figure 7). Of course, there are cases where the achieved values deviate from the regularities for some ages. In fact, these deviations are precisely period effects of type-H. For this reason, the period effect can be obtained as the difference between the fertility rate achieved in a given year/age and the corresponding model value. On the other hand, type-T period effects caused by the shift of cohort childbearing schedule are included in the projected fertility rate and are thus excluded from the period effect obtained as the difference between the projected value and achieved value; only type-H period effects are captured in this way.

Under normal circumstances, the model fertility rates used in the Population Projections for Japan are future predictions that have not yet been achieved. In contrast, the method proposed here uses model values of years and ages in the past. The accuracy of the measurement result achieved by this method depends on the accuracy of the cohort model. For cohorts who have completed their childbearing process up to reasonably high ages, the applicability and accuracy of the cohort model has been established, as shown in the graphs above. For young cohorts with little experience in the childbearing process, however, there are various speculative factors involved in their remaining childbearing schedule and the accuracy of the model is less well understood. Therefore, the measurement values should be treated as provisional for the most recent years, where such cohorts contribute more.

Figure 8 illustrates the result of measuring type-H period effects using the method proposed above. Figure 8-a and Figure 8-b shows projected values of type-H period effects by age group and by birth order as bar graphs, respectively (left scale). Both figures show the total period effect, i.e., period effects on TFR, as line plots (right scale) as well. Note that the right scale is twice as large as the left scale.

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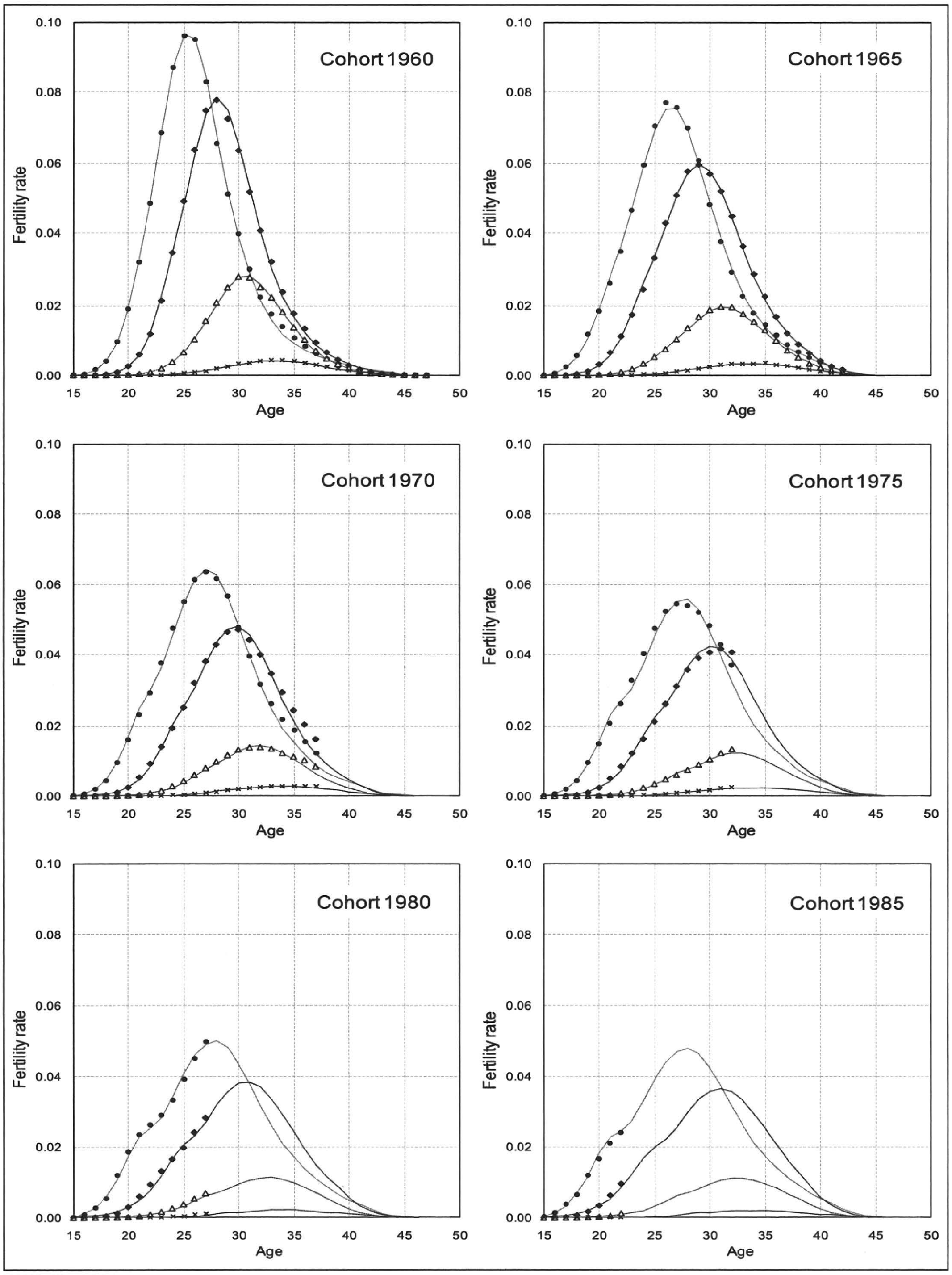
4) The model is based on the probability density function of the generalized log-gamma distribution, which is one of standard distributions in statistics. The fertility rate at age *x* for *n*-th birth is $f_n(x): f_n(x) = C_n \cdot \gamma(x; u_n, b_n, \lambda_n)$

where, $$\gamma(x; u_n, b_n, \lambda_n) = \frac{|\lambda_n|}{b_n \Gamma(\lambda_n^{-2})} (\lambda_n^{-2})^{\lambda_n^{-2}} \exp\left[\lambda_n^{-1}\left(\frac{x-u_n}{b_n}\right) - \lambda_n^{-2} \exp\left\{\lambda_n\left(\frac{x-u_n}{b_n}\right)\right\}\right].$$

Here, γ and exp are the gamma and exponential functions, respectively. C_n, u_n, b_n, and λ_n are parameters of the fertility rate function of birth order *n*; this is an extension of the Coale-McNeil Model. The further adjustment is made so that the distribution will reproduce the characteristics of Japanese age-specific fertility rate precisely. A standard pattern of errors (ε_n) was identified by comparison with the actual fertility rates and the modeled rates and used to adjust the model schedule. As a result, the function of cohort fertility rate by age *x*, *f*(*x*) is given as follows. See Kaneko (2003) for the details.

$$f(x) = \sum_{n=1}^{4+} C_n \cdot \left\{\gamma\,(x; u_n, b_n, \lambda_n) + \varepsilon_n\left(\frac{x-u_n}{b_n}\right)\right\}.$$

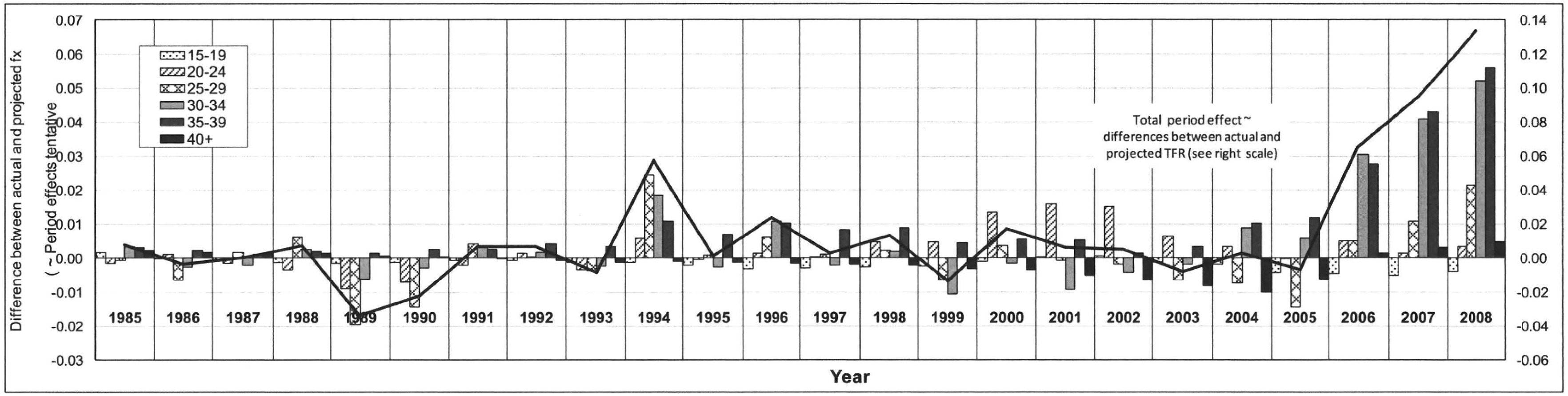
Figure 7. Actual and Modelled Fertility Rates of Japanese Female Cohorts by Birth Order



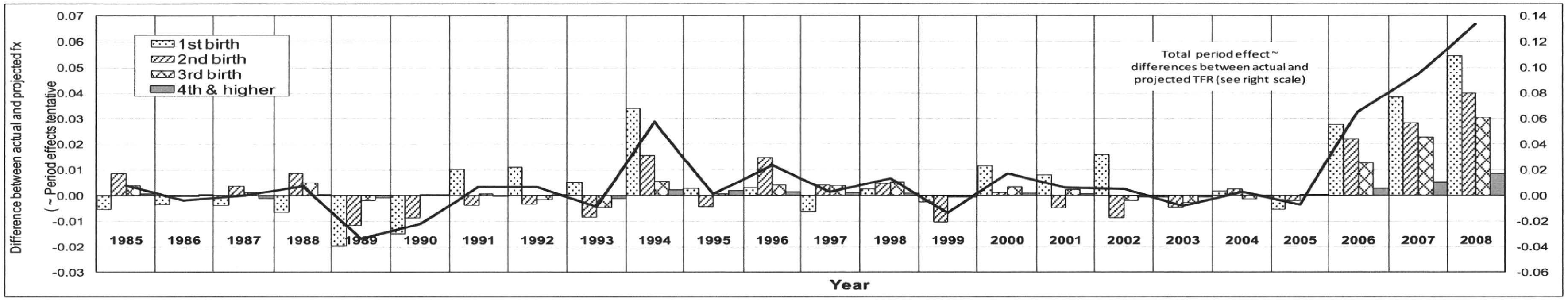
Note: Actual age specific fertility rates by birth order for female cohorts are plotted by dots, while modeled rates are plotted by lines. The actual rates are calculated only for female with Japanese nationality. The model rates are those employed in the official population projection conducted in 2006 as the medium assumption.

Figure 8. Estimates of Period Effects as Differences between Actual and Projected Fertility Rates by Five Year Age Groups: 1985-2008.

a. Age groups



b. Birth orders



***Note*:** The total period effects (solid line – right scale) is drawn in half the scale of the effects by age group (bar graph – left scale). Fertility rates are calculated based on births by Japanese women only here as well.

In the period up to 2005, the absolute value of period effects on the TFR (the scale on the right axis) exceeds 0.03 only in 1989 and 1994. In other years, the period effects, in general, amount to very little. In 1989 and 1994, some changes can be recognized in the figures showing the annual TFR development (figures 1). 1989 is the year the TFR dropped below the value in the year of Hinoe-uma and achieved the lowest value in recorded history and is also the year of "Merkmal" that triggered widespread societal awareness in Japan of the low fertility rates. Note that the period effect value is -0.034; the absolute value is not very large. On the other hand, in 1994, the period effect value is 0.058, which is quite prominent in the period up to 2005. Although the cause of this effect is not certain, one possible cause that has been suggested is the marriage between Crown Prince Naruhito to Princess Masako Owada in June of the previous year, which attracted the attention of many citizens.

In other years, positive period effects are observed in the three-year period from 2000 to 2002, among 20 to 24-year old women and for the first children only. Millennium effects were anticipated in this period, but the TFR itself did not show any significant rise. At closer inspection, it is noted that the fertility rates of the age group of 22 to 25 years show actual values higher than expected from the cohort model for first children.

In the sections above, the relatively prominent changes that occurred up to 2005 were discussed. Compared to those changes, the recent three-year period from 2006 to 2008 shows a very strong rise in terms of period effects. The projected period effects are high, 0.065, 0.095, and 0.134, respectively, and indicate a yearly upward trend. Looking at the values by age group (figure 8-a), the upward effects in the 30s age group are notable in each year. In 2007 and 2008, the value shows a dramatic rise for the age group in the latter half of the 20s as well. Looking at the values by birth order (figure 8-b), the period effect seems to contribute to all orders roughly equally.

Table 1 shows the contributions of age-specific and birth-order-specific subgroups to the entire period effects for both 1989 and 1994, for the purpose of comparison. In 1994, which shows relatively large positive period effects, the relative contribution of the age group of 25 to 29 years of age is large, while in 2006 to 2008, the contributions of the age groups of 30 to 34 years of age and especially 35 to 39 years of age are dramatically high. Moreover, in terms of birth order, while the contribution of the first children is large in 1994, the contribution of the third and further children is large in 2006 to 2008. Taking these characteristics into account, the period effect patterns in these recent three years are clearly different from the past.

Table 1. Contribution of Subgroups to Period Effects of Type H in Selected Years

a. Age groups (%)

Age group	Years whose "period effect" exceeds 0.03				
	1989	1994	2006	2007	2008
15-19	4.2	- 2.0	- 6.8	- 5.2	- 2.9
20-24	26.1	10.3	7.8	1.7	2.5
25-29	58.0	42.6	7.9	11.5	16.1
30-34	17.9	32.0	46.5	43.1	39.0
35-39	- 4.0	18.8	42.5	45.5	41.8
40+	- 2.2	- 1.7	2.2	3.4	3.5
Total (values)	100.0 (-0.034)	100.0 (0.058)	100.0 (0.065)	100.0 (0.095)	100.0 (0.134)

b. Birth orders (%)

Birth order	Years whose "period effect" exceeds 0.03				
	1989	1994	2006	2007	2008
1st birth	57.9	58.9	42.5	40.4	40.8
2nd birth	34.3	27.3	33.6	29.8	30.0
3rd birth	5.6	9.7	19.4	24.0	22.8
4th & higher	2.1	4.1	4.5	5.7	6.4
Total (values)	100.0 (-0.034)	100.0 (0.058)	100.0 (0.065)	100.0 (0.095)	100.0 (0.134)

Note: Comparatively outstanding values for the age groups and birth order are underlined.

In general, the rise of fertility in this period is known as "last-minute birth" and similar terms. These descriptions generally imply that women who delayed having children are now having more children while they are still able to. The age patterns of period effects show an upward movement in age groups from the middle of the 30s to the early 40s, which also supports this view. This generation includes the second baby boomers that were born in the period from 1971 to 1974. They tend to be promoters of lower fertility rates, who significantly postponed family formation and/or childbearing. For this reason, if they wish to have a fixed number of children in their lives, this period is their last chance. If only period effect patterns are examined, however, women in this age range not only tended to give birth to the first and second children they felt compelled to have in order to avoid childlessness and having an only-child, but also exhibited an increasing number of births to third and further children in a rather prominent manner. This suggests that the people who shifted towards more reproductive behaviors were not limited to those who had delayed family formation specifically, but encompassed a wider range of people as well. The significance of this interpretation will be examined in the subsequent discussion.

6. Discussion

In this paper, I utilized fertility projection prepared for the official population projection to analyze the period effects that are latent within the past and current fertility trends. Before applying the framework, I operationally defined a period effect as a fertility rate change observed in a certain period of time, which does not influence the cohort completed fertility (cohort TFR). Then I sorted out several types of period effect according to its effect on cohort fertility schedule, i.e. three different types of type-T period effect (T*l*, T*d*, and T*s*), and type-H. Type-T period effect is equivalent to a so called tempo effect.

Using this decomposition, period fertility rates synthesized from projected cohort fertility schedules are compared with observed rates. The former includes cohort changes and type-T period effects, but is free from the type-H period effects. Therefore difference between the projected and observed period fertility rate identifies the type-H (or type-H') period effects which should be induced by some period specific events.

Three temporal aspects of driving factors, i.e. period, cohort and age effects, are recognized in trends of demographic measures in general. In our framework, the age effects are expressed with a function of age as the regularity approximated by a mathematical function, while the cohort effects (variation by cohort) is represented by the different parameter values of the function. The period effects are disturbances to the age schedules shaped by the function with the certain parameter values affecting simultaneously many cohorts at different ages.

The fertility rates dropped continuously until 2005 in Japan, and the so-called lowest low fertility was attained for a three-year period from 2003 to 2005. However, from 2006 to 2008, an upward trend has been observed in the fertility rates, and the breadth of this upsurge is quite extraordinary as compared to past fluctuations seen since the fertility decline below replacement level started in 1974. Considering how important the fertility trends are for a society already in a phase of depopulation and rapid aging, it is extremely interesting to consider whether or not the recent rise in fertility rates is likely to affect the long term outlook. For this reason, this paper investigated the nature of the upturn, by closely examining the monthly development of fertility rates in this period, attempting to estimate tempo effects caused by adjusted TFRs such as the index proposed by Bongaarts and Feeney, and by estimating period effects (type-H period effects) that exclude tempo effects.

As a result, it was estimated that the recent upturn of fertility rates could generally be explained by type-H period effects. That is, we found that the upturn is an emergent change that cannot be reproduced by continuous changes in each cohort and which occurred in a manner deviating from the regularity of childbearing schedules of each cohort. For some cohorts in higher ages concluding their reproductive processes, however, it is likely the completed fertilities become slightly larger than previously estimated, from a windfall type effect of type-H'.

In the US and Europe, upturns of fertility rates have been observed since the 1990s in one country after another, and the majority of the countries experiencing lowest low fertility rates have already broken away from that status at the time of this writing. Thus, it is crucial to identify whether the upturn in Japan is qualitatively similar to those observed in the US and Europe.

However, whereas the upturns in the US and Europe are generally considered to have occurred because the period TFR returned to the long-term cohort TFR as tempo effects die out due to completion of postponement transition (Goldstein et al. 2009), it is suggested that the upturn being observed in Japan is of a different nature and is caused by other, peculiar causes.

From a long-term perspective, if the rise in fertility currently being observed is purely caused by type-H period effects, the period fertility rate should decline again within the next several years and will ultimately not significantly change the long-term outlook for the fertility rate. In fact, according to observation of the monthly development, fertility rates have already been turning around to a downward trend again for at least eleven months since December 2008.

On the other hand, this downturn itself might have been caused by transient disturbances related to the financial crisis starting roughly in September 2008 (Goldstein et al. 2009) and may actually be indicative of a complicated situation where different period effects overlap in some way.

Meanwhile, if the circumstances that brought about the recent upturn also serve to keep the fertility rates at higher levels than in the past for an extended period of time, overcoming this transient period of decline, the end result will be a positive influence on the cohort fertility, where the long-term outlook will involve a higher fertility than in the past. In this case, current period effects are modified from type-H to type-H'.

Therefore, it is important to attempt to understand the causes of the upturn. According to the analysis of observed age-specific fertility rates, the main players in the recent reversal are the so-called second baby boomers, i.e., the generation born in the period from 1971 to 1974. The second baby boomers were expected to give birth to the third baby boomers in the latter half of the 1990s and onward. However, this event was never realized due to significant postponement of family formation and/or childbearing.

On the contrary, their fertility rates kept on falling in 2000 and onward as well and reached the lowest low level in 2003. However, eventually it likely became clear to members of this group that if they wished to have a certain number of children in their lives, they were approaching the age limit to realize this desire. This childbearing urge should have reached super-saturation in 2003 and onward. Assuming these conditions hold, it appears that pregnancy and childbirth were further suppressed from 2004 to the first half of 2005 for some reason. One can thus imagine that excessive energy recoil had been accumulated.

Continuing this line of thought, it is possible that the changes in the social economic mood, which among other things involved a generally improved employment environment, triggered the sudden release of this pent-up desire among the second baby boomer generations.. In other words, although the trigger itself was a very ordinary change, there is a distinct possibility that it led to abrupt, significant changes in the trend due to the interaction with the circumstances of the players.

If that is the case, the decline of fertility from around 2003, particularly the drop in 2005, was caused by type-H period effects in the negative direction and the rise afterward occurred as a rebound to the decline, caused by type-H period effects in the positive direction. Note that the recent rise is actually accelerating and is exceeding the level of the rebound in 2004 to 2005, suggesting the possibility that additional factors are involved.

For example, the large-scale second baby boomer generation's growing desire to get married and/or have children itself forms a market demographic and is likely to gather momentum through mass media and similar channels. Magazines targeting women aged 30 years and up began to feature many positive articles featuring marriage, pregnancy, childbirth, and childrearing, and fashionable new words related to such subjects are becoming commonplace[1]. Furthermore, the national government and local governments are advertising their measures to promote childbearing as well. Spearheaded by the mass media, such measures seem to form a positive feedback relationship with the increasing fertility. Namely the initial rise from the rebound caused an increase in media coverage about marriage and family, which in turn promoted further marriages and caused additional births to occur.

What will happen to this reversal trend of fertility in the future? First, if the rise in most cohorts ends up as a simple type-H period effect from rebound and temporary boom, the fertility rates will regress on the line of the previous prospect. In this scenario the recent boom comes to an end soon and it becomes difficult to maintain the current level when the fertility starts to stagnate and the feedback cycle with popular culture is cut. Signs hereof may be already beginning to show in the monthly development.

On the contrary, if the boom continues for long enough to make the increases type-H' and so raise the levels of completed fertility, and if those age patterns are continually succeeded by the following cohorts, then it means that the shrinking trend of cohort fertility reverses and the long term prospects of fertility should be revised to be as high as improved level of cohort completed fertility. In this case, the feedback relationship would be maintained. It is possible the group of single people and families of under-parity has ballooned to a huge size by now, because it contains the second baby boomers.

There are several other factors affecting the future course of fertility, among which the new child allowance is particularly notable. The new government has promised the adoption of this policy, and it amounts to 26,000 yen (about 290 US dollar) a month per every child through junior high. The current plan is to enact half the amount of allowance in April 2010, and the full amount in April 2011. Though it may have a certain impact on fertility, it seems necessary that it be perpetual and publicly viewed as reliable to alter the long term trends beyond just period fertility in the short term.

7. Conclusion

In this paper, we pursued three objectives: (1) to show the usefulness of population projection models in analyzing demographic processes in the past and present as well as in the future, (2) to measure and understand the period effects in terms of modifiers of the cohort fertility schedule, and (3) to identify factors and mechanisms of the recent peculiar fertility development in Japan, using the proposed framework.

Fertility projection is utilized to analyze the period effects that are latent within the past and current fertility trends. It seems that the framework is useful to separate out the type-H and H' period effects from the type-T period effects (the tempo effects) and cohort effects so that causes of changes in fertility trend may be identified.

The fertility rates in Japan have dropped continuously below the replacement level from the mid 1970s until 2005, and Japan experienced the so-called lowest low fertility for a three-year period from 2003 to 2005. However, in the recent three years from 2006 to 2008, an upturn trend has been observed in the fertility rates, and the TFR rose to 1.37 in 2008.

Using the period effect analysis composed with the fertility projection system, it is estimated that the recent upturn can mainly be explained by the period effect, which does not change cohort completed fertility, and particularly the effects that cause temporal shifts and are redeemed in other periods (termed the period effect of type-H). For some cohorts in higher ages, however, it is likely the completed fertilities become slightly larger than previously estimated as a result of the type-H' effect. These are different in causes from the upturns seen in the US and Europe, where the period fertility rates have been reversed mostly by "the tempo transition" (Goldstein et al 2009) which is the completing phase of the

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[1] "Kon-katsu" (activities to look for marriage partners) - there is affirmative nuance for the activities. "Ara-for"(Around Forty) - a somewhat positive title for single women around age 40, who are typically active in work and romance. "Sosyoku-danshi" (herbivorous boy) – a label for young men who are passive to romance and marriage, suggesting that women should be active to make those come into existence.

"postponement transition"(Kohler et al 2002). This corresponds to the period effect of type-T in our terminology.

The upturn in Japan seems to be caused by a rebound of the short term too-low fertility in the lowest low period, or in 2003-2005, followed by a boom induced mainly by the media targeting the single's and family of under parity's market whose size is unprecedented in these years, partly because it includes the second baby boomers.

It is possible that the long term prospects of fertility formed in the latest population projection, which are based on the data corrected by the year 2005, might be underestimated in light of the present situation. It depends on whether the rise in fertility schedules of cohorts in their mid-thirties and beyond in this period is continually succeeded by the following cohorts ending up with rises in their cohort completed fertilities.

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APPLICATION OF AGE-TRANSFORMATION APPROACHES TO MORTALITY PROJECTION FOR JAPAN

Futoshi ISHII[1]

Introduction

For projecting future mortality in "Population Projection for Japan: 2006-2055" (NIPSSR 2007), a new "age-shifting model", which incorporates age-shifting as well as age-scaling of mortality, has been developed and used (Ishii 2008). These kinds of operations could be incorporated into a more general framework, i.e. an age-transformation approach.

This paper serves to examine and propose a novel method for the mortality projection of Japan that is an application of the age-transformation approach.

1. Two Representations of the Log Mortality Surface

In this section, we discuss two representations of the log mortality surface and define certain functions to describe the log mortality and its inverse functions.

Let $X = [0,+\infty)$ be the space of age and $T = (-\infty,+\infty)$ be the space of time. In the following discussion for modeling mortality, we will use $\mu_{x,t}$ the hazard function for exact age $x \in X$ at time $t \in T$. In this paper, we express the log hazard function of mortality as $y = \lambda_{x,t} = \log \mu_{x,t}$, where $y \in Y = (-\infty,+\infty)$ is the value of the function. Then, the set $S = \{(x,t,y) | y = \lambda_{x,t}\}$ determines a surface in $\Re^3$, called the *log mortality surface*. This is a conventional representation of the log mortality surface. In this representation, $y = \lambda_{x,t}$ would be considered as the height from the $X-T$ plane in $\Re^3$.

Here, we consider another representation of the log mortality surface under a set of assumptions.

We assume that $\lambda_{x,t}$ is a smooth continuous function with respect to x and t defined on $X_0 \times T_0 = [0,\omega] \times [t_0, t_1] \subset X \times T$, where $\omega < +\infty$ is the finite maximum age for mortality models.

For the purpose of modeling *adult* mortality, we can further assume that $\lambda_{x,t}$ exhibits a strictly monotonic increase with respect to x for each t and $x > x_0(t)$. Here, $x_0(t)$ represents the lower

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bound of x above which $\lambda_{x,t}$ exhibits a strictly monotonic increase for each t. Then, for each t, the function $\lambda_t(x)$ defined by

$$\lambda_t : \tilde{X}_t \to Y, \quad \lambda_t(x) \underset{\text{def}}{=} \lambda_{x,t}$$

is the injective (one to one) function of x, where $\tilde{X}_t = [x_0(t), \omega]$. Let $\tilde{Y}_t = \lambda_t(\tilde{X}_t)$, then $\lambda_t(x) : \tilde{X}_t \to \tilde{Y}_t$ has an inverse function $v_t(y) : \tilde{Y}_t \to \tilde{X}_t$ defined on $\tilde{Y}_t$ for each t.

Let us define Y_0 as follows:

$$Y_0 \underset{\text{def}}{=} [y_0, y_1] \text{ where } y_0 = \sup_{t \in T_0} \min \tilde{Y}_t, \qquad y_1 = \inf_{t \in T_0} \max \tilde{Y}_t$$

Then, we can define $v_{y,t} : Y_0 \times T_0 \to X_0$ by $v_{y,t} \underset{\text{def}}{=} v_t(y)$

$v_{y,t}$ gives the *age* x at which the value of the log hazard function is equivalent to a value y at time t. Moreover, we define the following two differential functions by time t: (1) $\rho_{y,t}$: the mortality improvement rate and (2) $\tau_{y,t}$: the force of age increase.

$$\rho_{x,t} \underset{\text{def}}{=} -\frac{\partial \lambda_{x,t}}{\partial t} = -\frac{\partial \log \mu_{x,t}}{\partial t}$$

$$\tau_{y,t} \underset{\text{def}}{=} \frac{\partial v_{y,t}}{\partial t}$$

2. Age-transformation

Next, we introduce an age-transformation in mortality analysis. In this paper, we define the age-transformation as follows.

Def 1. Let $x, z \in [0, \infty)$ be coordinates for age. If we have a transformation $f_t : z \to x$, which is continuous and monotonically increasing, we call f_t as an age-transformation from x to z at time t.

Let us consider graphical representations of the age-transformation. We use the following two representations, the graph of $x = f_t(z)$ and an "iso transformed-age map".

Here, we look at these graphs with an example of shifting age-transformation, which is defined by the following equation.

$$x = f_t(z) \underset{\text{def}}{=} \max(5t + z, 0) \quad (t = -2, -1, 0, 1, 2)$$

The relationship among x, z and t is expressed in three-dimensional space as shown in Figure 1.

One way to project this relationship onto two-dimensional space is by plotting the graph of $x = f_t(z)$ for each *t* on the X-Z plane. Figure 2 illustrates this graph. From this, we are able to read which age in the original coordinate (*x*) corresponds to the transformed one (*z*).

Another way to project onto two-dimensional space is to consider which ages in the original coordinate are identified by this transformation. We can express this by showing a plot $f_t(y)$ for $y = 0, 1, \cdots, 110$. We call it "iso transformed-age map". Figure 3 is the iso transformed-age map for this shifting age-transformation. The red lines shows the age 0, 10, ..., 110.

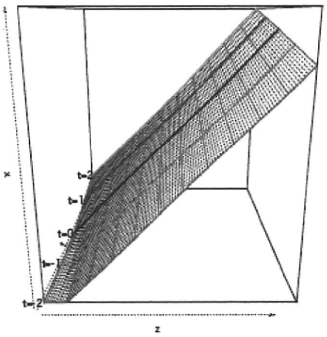


Figure 1: Relationship between x, z *and t* (shifting)

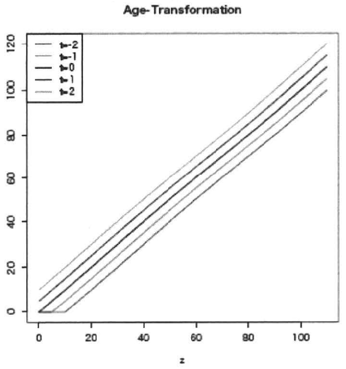


Figure 2. Age-transformation Function (shifting)

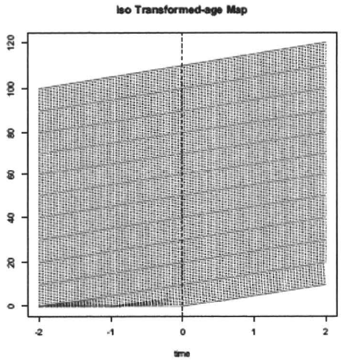


Figure 3. Iso Transformed-age Map (shifting)

3. Lee-Carter model and Age-transformation Approach

In Section 2, we introduced an age-transformation approach for mortality analysis. In this section, we review our preceding work for Japanese mortality projection that combined the Lee-Carter model with age-transformation (Ishii 2008).

The Lee-Carter model (abbreviated as LC) is expressed by the following formula (Lee and Carter 1992).

$$\lambda_{x,t} = \log \mu_{x,t} = a_x + k_t b_x$$

where a_x is a standard age pattern of mortality.

Taking a partial derivative by time *t*, we obtain the following relationship.

$$\rho_{x,t} \underset{\text{def}}{=} -\frac{dk_t}{dt} b_x = -k_t' b_x$$

This equation shows that the age distribution of $\rho_{x,t}$ is constant in the LC model. If we further assume that k_t is linear over time, $\rho_{x,t}$ is constant over time. Therefore, the LC model works well when the age-specific rate of mortality improvement is considered to be constant over time, that is, the mortality improvement is considered as *decline*.

Then, when does the LC model fail to express mortality improvement? To observe this point, we examine the following stylized examples.

Here, we consider two piecewise linear log mortality functions. At $t = 0$, both functions are identical: $\lambda_{x,t} = -2$ for age 0, -8 for age 25, -6 for age 50, -3 for age 75 and -1 for age 100. In Example 1, age-specific rates of improvement are constant over time. The annual rate of decline is 0.12 for age 0, 0.06 for age 25, 0.06 for age 50, 0.07 for age 75 and 0.04 for age 100.

In Example 2, age-specific rates of improvement for ages under 25 are constant and the same as in Example 1. However, for ages above 50, the mortality curve shifts to the right 3/5 years annually.

Figure 4 shows $\lambda_{x,t}$ (top figure) and $\rho_{x,t}$ (bottom figure) for Example 1. From the bottom figure, we can observe that the rates of mortality improvement are constant over time.

Figure 5 shows the same figures for Example 2. From the bottom figure, we can observe that the peak of the rates of mortality improvement is shifting to the right over time. Such mortality improvement could not be expressed by the LC model. The black line shows the rate of mortality improvement, which is equal to the b_x function under the LC model. We can observe that this line exhibits an average rates of mortality improvement for the entire period, even though no actual $\rho_{x,t}$ shows such rates of mortality improvement.

Figure 4. $\lambda_{x,t}$ and $\rho_{x,t}$ Example 1

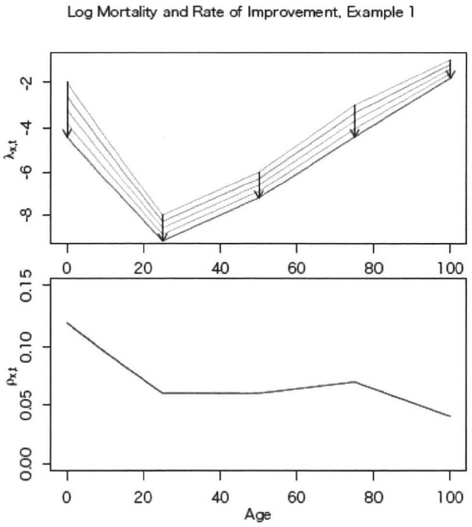
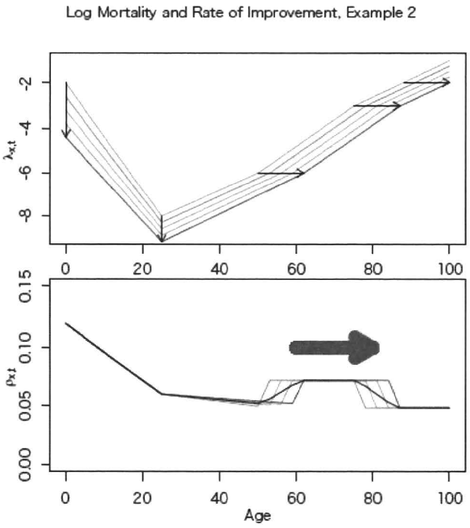


Figure 5. $\lambda_{x,t}$ and $\rho_{x,t}$ Example 2



Following these observations, we could say that use of the LC model may not be considered appropriate if the mortality improvement is considered as *shifting*. We proposed age-transformation approaches for projecting Japanese mortality rates since we observed the recent mortality improvement in Japan could be considered as *shifting*, though this point is reconsidered later.

The age-transformation approach works as follows. Let us denote the LC modeling and projecting procedure as *L*; then the modeled and projected mortality $\hat{\mu}_{x,t}$ by the LC procedure would be obtained as $L(\mu_{x,t})$. We proposed performing the Lee-Carter procedure after some age-transformation, and modelling and projecting the rates by inverse age-transformation, i.e., $A^{-1}L\,A(\mu_{x,t})$.

[Lee-Carter model]

$$\begin{tikzcd}
\mu_{x,t} \arrow[d, "\mathcal{L}"] \\
\hat{\mu}_{x,t}
\end{tikzcd}$$

[Lee-Carter model with Age-transformation]

$$\begin{tikzcd}
\mu_{x,t} \arrow[r, "A"] & \mu_{z,t} \arrow[d, "\mathcal{L}"] \\
\hat{\mu}_{x,t} & \hat{\mu}_{z,t} \arrow[l, "A^{-1}"']
\end{tikzcd}$$

Here, we illustrate how the age-transformation approach will work in Example 2. Let us consider the following age-transformation: shifting mortality curves to the left 3/5t years for the group aged 50 and over as in the top figure in Figure 6. Then the transformed mortality rates are in the bottom figure.

Figure 7 shows the age-transformed $\lambda_{x,t}$ and the rates of mortality improvement $\rho_{x,t}$. We can see that the $\rho_{x,t}$ function for the age-transformed mortality is constant over time, and thus the LC model provides a perfect fit for the age-transformed mortality rates. Therefore, we can model Example 2 using the LC model with age-transformation. This is a core structure of this approach.

Figure 6. Age-transformation for Example 2

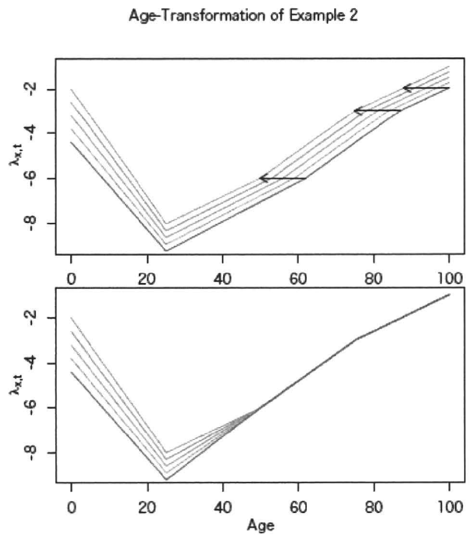
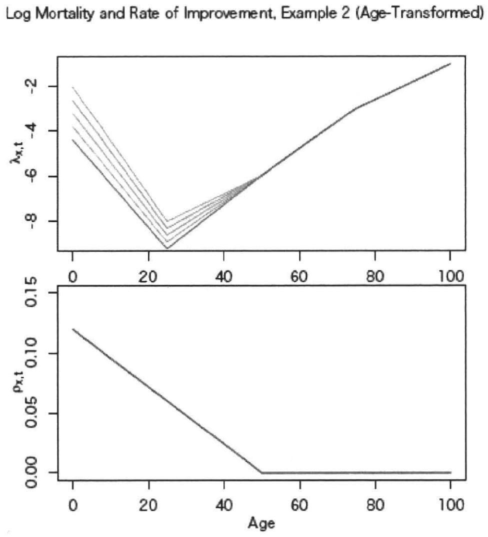


Figure 7. $\lambda_{x,t}$ and $\rho_{x,t}$ Example 2 (with Age-transformation)



In Ishii (2008), we proposed the following age-transformation A for entire age to express the mortality improvement as a *decline* in younger age and a *shift* in older age in order to apply the Lee-Carter procedure.

First, we fit the three parameter logistic curve

$$\mu_{x,t} = \frac{\alpha_t \exp(\beta_t x)}{1 + \alpha_t \exp(\beta_t x)} + \gamma_t$$

to the actual mortality rates. Then, we obtain the parameter $S_t = -\frac{\ln(\alpha_t)}{\beta_t}$, which is used to express the shift amount in the shifting logistic model (Bongaarts 2005), and another parameter β_t which expresses the slope of the curve.

Next, let x be the original age and z be the transformed one, and define the relation $x = f_t(z)$ as follows.

$$f_t(z) \underset{\text{def}}{=} \begin{cases} z & (z \le B_1) \\ \left\{ \dfrac{\beta_{t_0}}{\beta_t}(B_2 - S_{t_0}) + S_t - B_1 \right\} \dfrac{z - B_1}{B_2 - B_1} + B_1 & (B_1 \le z \le B_2) \\ \dfrac{\beta_{t_0}}{\beta_t}(z - S_{t_0}) + S_t & (B_2 \le z) \end{cases}$$

Then set $\hat{\mu}_{z,t} \underset{\text{def}}{=} \mu_{f_t(z),t}$

Figure 8. Age-transformation Function

Age-Transformation

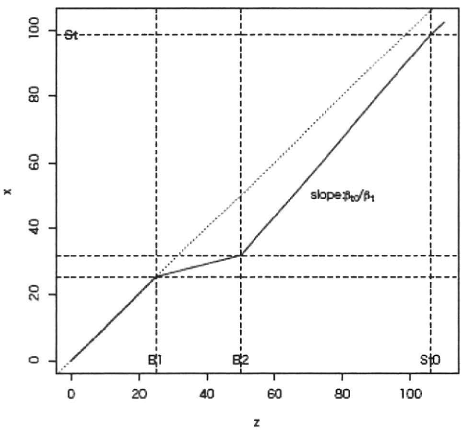


Figure 9. Iso Transformed-age Map

Iso transformed-age Map

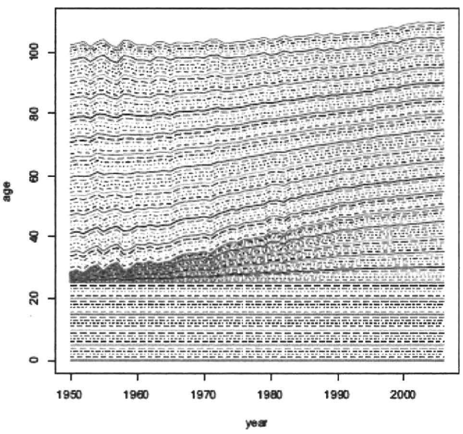


Figure 8 shows an example of age-transformation function, and Figure 9 shows the iso transformed-age map. Using the age-transformation A, modeling and projecting mortality rates are performed as $A^{-1} L A(\mu_{x,t})$.

4. Mortality Improvement: Decline or Shift?

In Section 3, we reviewed the age-transformation approach developed in Ishii (2008). For the modelling of adult mortality, the projection is based on the assumption that the mortality improvement is considered as shifting. It is suggested from the trends in $\mu_{x,t}$ and $l_{x,t}$ that the recent improvement in adult mortality in Japan could be better understood when considering it as shifting. In this section, we reconsider whether it is more plausible to understand mortality improvement in Japan as declining or shifting. First, we describe the definitions of the proportional hazard model and the Lee-Carter model, which are decline-type models. Then, we introduce the horizontal shifting model and the horizontal Lee-Carter model, which are shift-type models corresponding to the two decline-type ones. Through this consideration, we propose a new type of adult mortality model and discuss another way to define age-transformation.