$$P_E(x)=\frac{\gamma_1\gamma_2}{\gamma_2-\gamma_1}\left\{\frac{1}{\gamma_1}h_1(z)-\frac{1}{\gamma_2}h_2(z)\right\}$$

$$r_E(x)=\frac{h_1(x)-h_2(x)}{h_1(x)/\gamma_1-h_2(x)/\gamma_2}$$

To compute the convolution of the normal and two identical exponential distributions, observe that by the associativity of convolution the exponentials convolve to a gamma distribution. Thus I can view the total convolution as the convolution of the normal and gamma distributions. Its PDF and CDF are given by

$$g(z)=\sigma_e\gamma^2\left\{z^*h(z)+\phi(z^*)\right\} \tag{12}$$

$$G(z)=\Phi(\frac{z-\mu_e}{\sigma_e})-\left\{h(z)+\frac{1}{\gamma}g(z)\right\} \tag{13}$$

where $h(z)=\exp\left\{-\gamma(z-\mu_e)+\frac{1}{2}\sigma_e^2\gamma^2\right\}\Phi(z^*)$.

The proportion who is in the marriage market at age x, and the hazard rate of marrying for those in the market are given by:

$$P_E(x)=h(x)+\frac{1}{\gamma}g(x)$$

$$r_E(x)=\frac{h(x)}{h(x)+g(x)/\gamma}$$

Non-independent "Convolution" Model

All those formulae discussed above depend critically on the assumption of independence between the distributions to be convolved. If this assumption does not hold for the actual target process, their utility as behavioral models is limited. This issue has not been discussed extensively in previous work due to a lack of empirical evidence.

Kaneko (1991a) pointed out the heavy dependence of waiting time (T2+T3) on age at first meeting (X1) based on survey data collected in Japan[9]. He reported, in particular, a strong negative correlation between X1 (age at first encounter with eventual spouse) and T2 (waiting time from encounter to engagement) as -0.449 for Japanese women with sample size N=4682. These correlations make variance of age at first marriage much smaller than that of age at first meeting, which should not happen in the convolution framework.

[9] The Ninth National Fertility Survey (NFS 9) conducted in 1987 in Japan by Institute of Population Problem (currently National Institute of Population and Social Security Research).

Hence the validity of convolution models of first marriage process as behavioral explanatory tools is questionable at least for some populations.

Therefore, in order to develop the behavioral multistage models I should consider models without convolution or with non-independent "convolution", by which I mean a procedure generating the distribution of a sum of random variables assuming dependencies among them. The general formula for the PDF, $g_z(z)$, of sum of two random variables X and T (>0) is given by:

$$g_z(z) = \int_{-\infty}^{z} g_x(x) w_t(z-x|x)dx \tag{14}$$

where $g_x(x)$ is PDF of X and $w_t(t|x)$ is conditional PDF of T given X=x.

With this integral formula, I would obtain $g_z(z)$ by providing functional form of $g_x(x)$ and $w_t(t|x)$.

One possible such model is one whose parameters of the waiting time distributions are functions of age at initiation of waiting, i.e. X. Here I attempt to develop a model of this type to conduct a numerical examination, in a search for appropriate models to describe the observed of multistage process.

I employ the generalized gamma (GG) distribution to represent the mathematical models for waiting time because it is one of the most flexible parametric models for survival times. Its PDF is given by:

$$w(t;\theta,c,v) = \frac{|\theta|}{c\Gamma(\theta^{-2})}(\theta^{-2})^{\theta^{-2}}\left(e^{v}t\right)^{\frac{1}{c\theta}-1}\exp\left[-v-\theta^{-2}\left(e^{v}t\right)^{\frac{\theta}{c}}\right] \tag{15}$$

where $\theta, c,$ and v are three parameters.

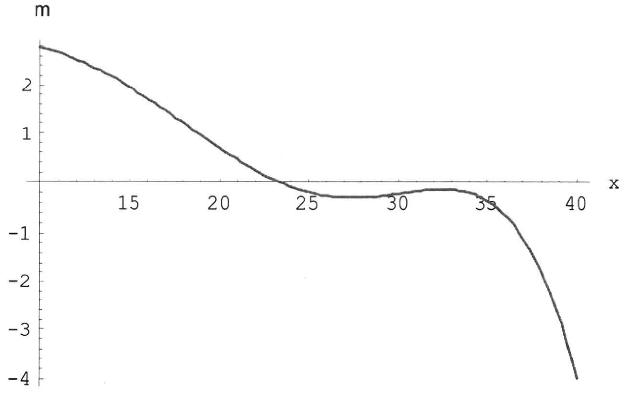
Let the scale parameter v be dependent on age at onset x, and expressed as a polynomial function of x, such as $v(x) = v_0 + v_1x + v_2x^2\cdots$, where $v_0, v_1, v_2\cdots$ are polynomial coefficients. Then PDF of the GG waiting time distribution is given by

$$w(t,x;\theta,c,v_0,v_1,v_2\text{L}\,) = \frac{|\theta|}{c\Gamma(\theta^{-2})}(\theta^{-2})^{\theta^{-2}}\left(e^{v(x)}t\right)^{\frac{1}{c\theta}-1}\exp\left[-v(x)-\theta^{-2}\left(e^{v(x)}t\right)^{\frac{\theta}{c}}\right]. \tag{16}$$

It is possible to make the other parameters, θ and c dependent on X in the same way, though I do not attempt it here.

Parameter estimation of $w(t,x;\theta,c,v_0,v_1,v_2\cdots)$ is conducted by applying it to survey data of delay from age at first meeting to marriage in Japanese women through the maximum likelihood method[10]. The result indicates that the polynomial functions up to five produces statistically significant improvements in fitness, but little improvement is seen thereafter. Function $v(x)$, the scale parameter value of the GG waiting time for those whose age at first meeting is x, is shown in Figure-1 in the form of a fifth degree polynomial.

Figure 1 Scale Parameter of the GG Distribution for Waiting Time as a Function of Age at First Meeting with Eventual Husband: Japanese Women

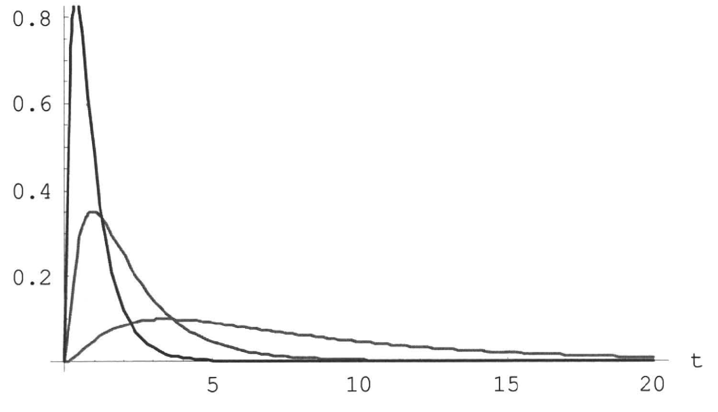


As age at meeting increases, the value of v goes down until mid twenties. Then it stabilizes at slightly below zero until the mid thirties followed by a steep fall. Note that the rapid decline after the late thirties is not reliable due to the small sample size.

Corresponding waiting time distributions for ages 15, 20, and 25 at first meeting are shown in Figure-2. The figure indicates that there are remarkable differences in waiting time distribution depending on age at first meeting, which implies again that convolution model with the independence assumption is simply not realistic.

[10] The Ninth through Eleventh National Fertility Survey conducted in 1987, 1992, and 1997 in Japan by National Institute of Population and Social Security Research.

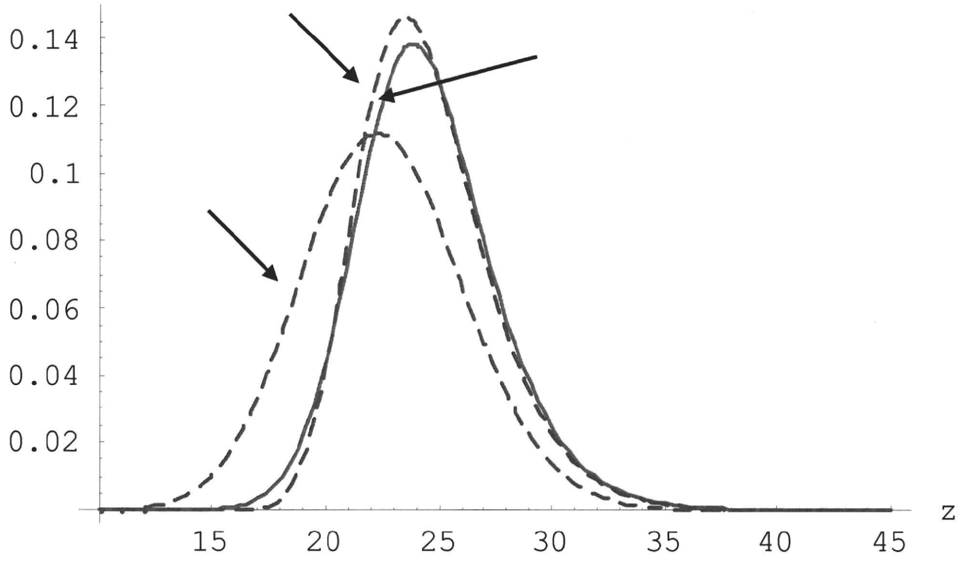
Figure 2 PDF of Estimated GG Distribution for Waiting Time by Age at First Meeting with Eventual Husband: Japanese Women



With this waiting time distribution combined with the estimated GLG distribution of age at first meeting on the same sample, age distribution of first marriage is reconstructed by numerical integration of the non-independent convolution formula (14) and is compared with directly estimated GLG distribution. The result is shown in Figure-3.

In the figure the reconstructed PDF of age at first marriage with the non-independent "convolution" of age at first meeting and dependent waiting time is matched up to the estimated PDF of the CM distribution of age at first marriage. The estimated PDF of age at first meeting with the eventual husband, which is used in the reconstruction, is also shown in the figure. The reconstructed distribution with non-independent convolution is fairly close to the directly estimated distribution, although differences are noticeable. Since the discrepancies remain even after introducing the dependence into the other parameters, it is due to inadequacy of the GG distribution for representing the waiting times, despite of the fact that the GG is one of the most flexible parametric models.

Figure 3 Observed and Estimated PDF of Age Distributions of First Marriage: Jap anese Women



Note: Estimated PDF of age at first marriage with non-independent "convolution" model is compared with estimated PDF of the GLG model. Estimated PDF of age at first meeting with eventual husband is also shown. Source data is from The Ninth through E leventh National Fer tility Survey conducted in 1987, 1992, and 1997 in Japan by National Institute of Population and Social Security Research.

However, the examination indicates the possibility of employing the non-independent "convolution" model as a behavioral explanatory tool. It is shown that the model is able to reproduce distribution of age at first marriage fairly accurately, if the dependency of waiting time on age at meeting is properly represented.

Estimation of waiting time distributions with survey data

First, I estimate waiting time distributions. Second, using the model of waiting time, I estimate preceding event distribution from that of the resulting event. For example, I estimate age distribution of first marriage from distribution of age at first birth. Then I proceed to application to the vital statistics data which has no information on waiting time distributions. The following is the data source and variables.

Data (survey data)

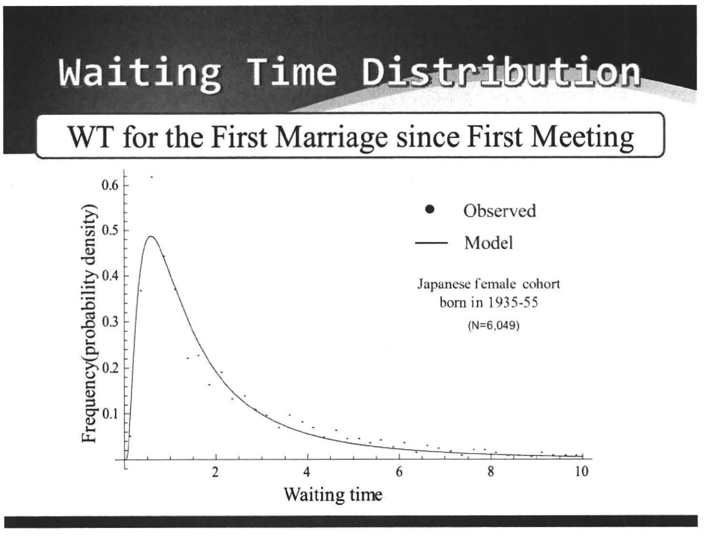
Japanese National Fertility Survey (NFS): 9th-13th (1987-2005) --- nationally representative samples of married Japanese female cohort born in 1940-62

Variables

Age at first meeting with present spouse (N=6,049)
Age at first marriage (N=5,200)
Age at first and second birth (N=4,699)
and waiting times between life events above

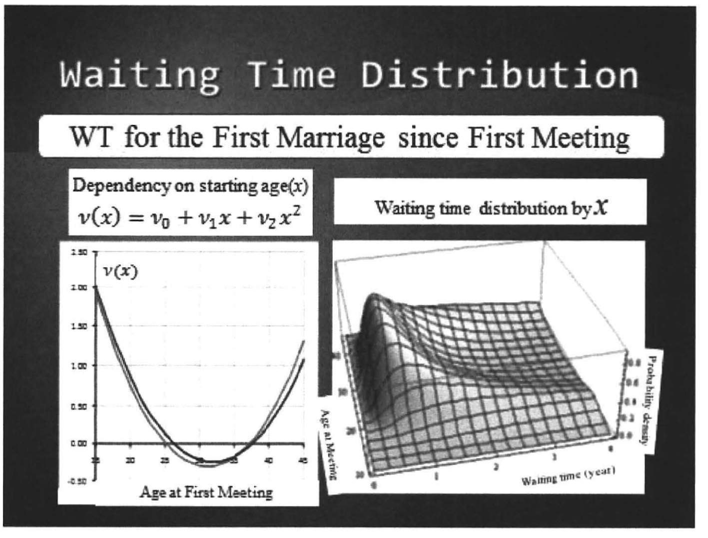
Figure 4 shows the waiting time distribution from the time at first meeting with present spouse to the time at first marriage for Japanese female cohort in our data set. The actual observed distribution is indicated by dots, while the thin line denotes a model of the waiting time, i.e. the Generalized Gamma distribution with the estimated parameters. The model indicates moderately good fit to the data.

Figure 4 Observed and Modeled PDF of Waiting Time from First Meeting to Marriage



The waiting time is dependent on age at first meeting. This dependency is expressed by parameter form which is dependent on starting age. In this case, the model fits the data when the dependency is expressed as a quadratic form. This implies that when you are young, the later you meet your spouse, sooner you get married, while when you get older, waiting times tend to prolong. The resulting waiting time model is shown right in 3D graph in Figure 5.

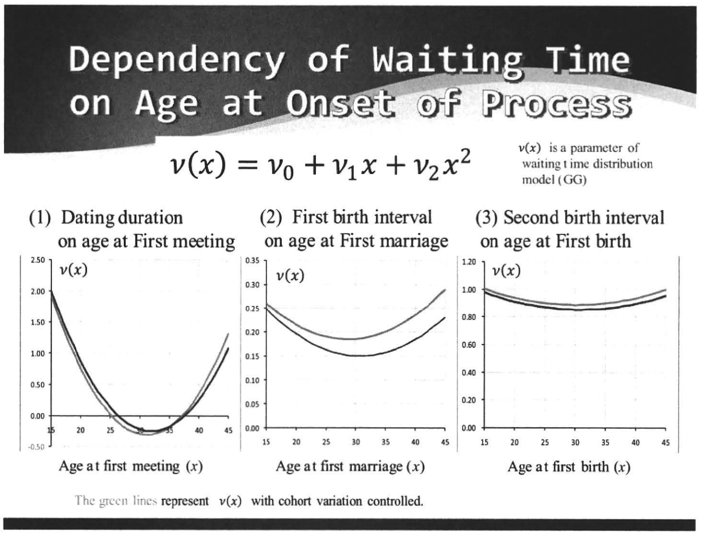
Figure 5 Dependency of the Waiting Time on Age at First Meeting



Note: The green lines represent $v(x)$ with cohort variation controlled.

In the similar way, I examined two other life events with this survey data, i.e. the first and second birth. While the dependency of waiting time for first marriage on age at first meeting is heavy, this is not true for waiting time for first birth, and for second birth as shown in Figure 6 (2) and (3).

Figure 6 Dependencies of Three Waiting Times of Life Events on Age at Onset

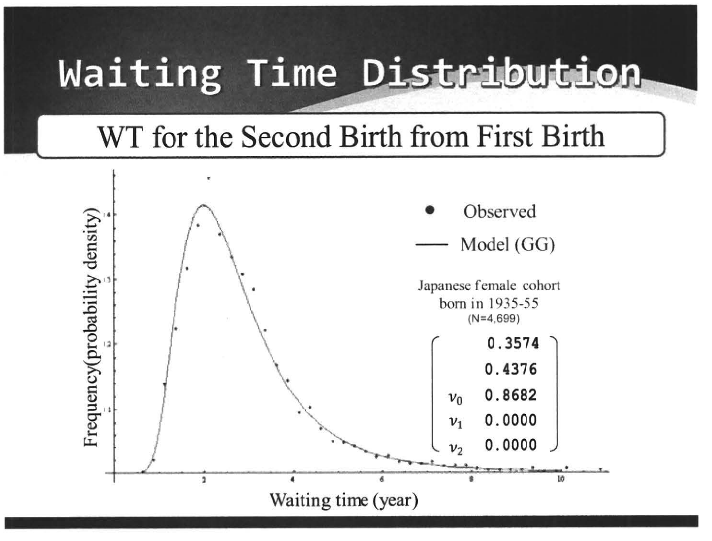


Note: The green lines represent $v(x)$ with cohort variation controlled.

According to these observations, the original independent model could be applied to the first birth and second birth with reasonable goodness of fit. Avoidance of the complexities introduced by the dependency scheme has great advantage in practical applications.

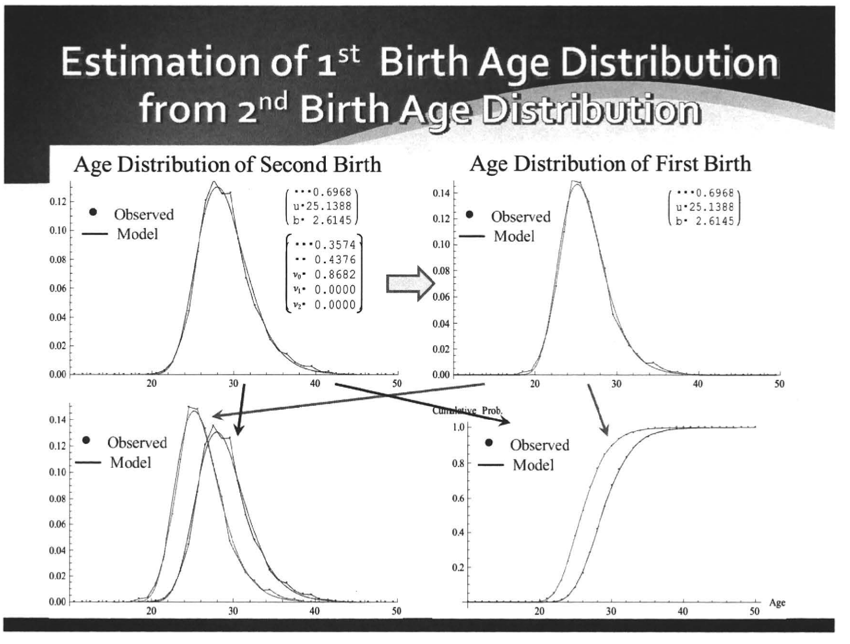
Figure 7 shows the observed and modeled distribution of waiting time from the first birth to the second.

Figure 7 The Observed and Modeled Distribution of Waiting Time from the First Birth to the Second



The model (the Generalized Gamma distribution) well replicates the observed. The age distribution of the first birth is to be estimated by applying the model with this waiting time model to data of age at second birth. The estimated first birth distribution from the second birth data is compared with the actual observed distribution in Figure 8.

Figure 8 Estimation of Age Distribution of the First Birth from the Distribution of the Second Birth: Application to the Survey Data



The estimation of age distribution of age at first birth replicates the actual distribution quite well. In other words, the estimation of age distribution of the first birth is conducted by applying the multistage model to the second birth. In a sense this is an inevitable outcome, because the model utilized the well fitted waiting time distribution to the actual data. Then I attempted estimation of age distribution of the first birth of the vital statistics.

Figure 9 Estimation of Age Distribution of the First Birth from the Distribution of the Second Birth: Application to the Vital Statistics Data

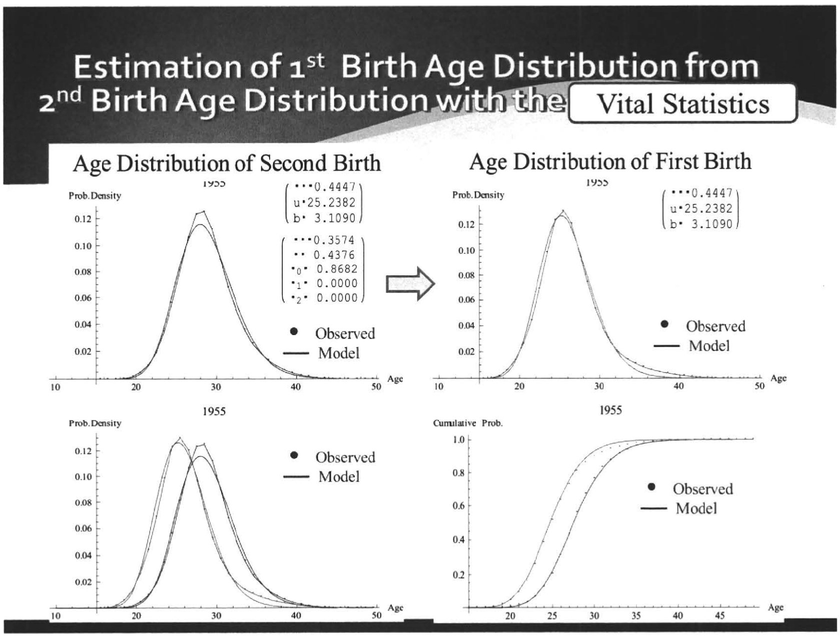
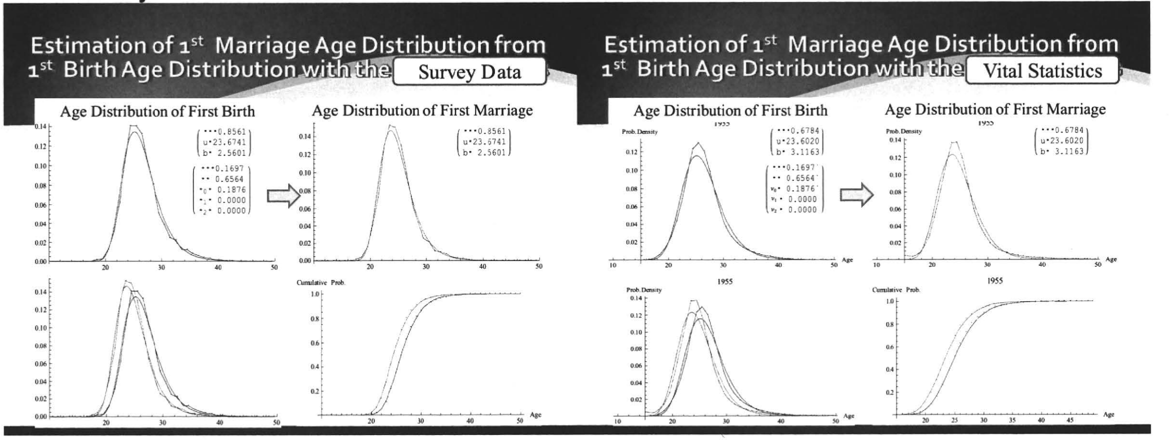


Figure 9 indicates the results of estimation of age distribution of the first birth from the second with the vital statistics data by applying the same procedure as case for the survey data assuming that the waiting time distribution observed in the survey sample does not vary a lot in population of the vital statistics. The fit is fairly good though it is not as good as the case applied to the survey data from which the information on waiting time is derived.

I applied the same procedure to estimate age distribution of the first marriage from that of the first birth. Figure 10A shows results of the application for the survey date, which indicate again that the estimation represents the actual data fairly well. Figure 10B shows results of the application for the Vital Statistics. It indicates that the estimation is moderately accurate though it does not represent the actual data as well as the case for the survey data. The central tendency and dispersion may be well approximated by the model.

Figure 10 Estimation of Age Distribution of the First Marriage from the Distribution of the First Birth

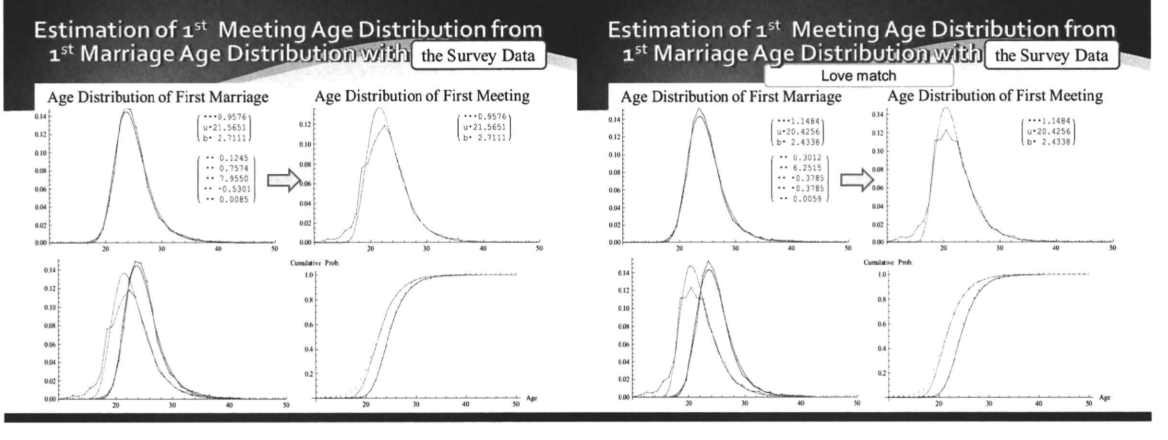
A. Survey data　　　　　　　　　　　B. Vital statistics



Finally, I conducted an estimation for age distribution of the first meeting with present spouse from data on age distribution of the first marriage. In this case, remember that the waiting time is heavily dependent on age at meeting with spouse. Then I used the dependent convolution framework explained before for this case. The result is shown in Figure 11A. Though the distribution of age at meeting with present spouse itself shows somewhat irregular shape, it is observed that the estimated model catches the main properties of the distribution.

Figure 11 Estimation of Age Distribution of the First Meeting from the Distribution of the First Marriage

A. Survey data (Total)　　　　　　　　　B. Survey data (Love-match)



One of causes of irregularity in the observed distribution of age at meeting is derived from the mixture of different type of marriage, i.e. arranged marriage and love match. Then I applied same procedure to only marriage from love match. The result in Figure 11B shows modest improvement, though distribution itself is still irregular.

A summary for model fits in terms of the mean and the standard deviation are presented in Table 1. The results indicate that the models represent the central tendency and dispersion of age distribution of those latent events fairly well.

Table 1 Actual and Estimated Mean and S.D. of Age at Onset of Process

Target event	Mean	S.D.
Survey data		
Age at first meeting (est. from 1st marriage)		
Actual	22.95	3.73
Model	23.05	3.41
Δ	**0.10**	**- 0.32**
Age at first marriage (est. from 1st birth)		
Actual	24.87	3.04
Model	24.90	3.08
Δ	**0.03**	**0.04**
Age at first birth (est. from 2nd birth)		
Actual	26.12	3.01
Model	26.12	2.96
Δ	**0.00**	**- 0.05**
Vital statistics		
Age at first marriage (est. from 1st birth)		
Actual	24.37	3.73
Model	24.16	3.25
Δ	- 0.21	- 0.48
Δ*	**- 0.08**	**- 0.24**
Age at first birth (est. from 2nd birth)		
Actual	26.34	3.77
Model	25.95	3.27
Δ	- 0.39	- 0.50
Δ*	**- 0.01**	**- 0.15**

A possible application of these procedures is an estimation of age distribution of the union formation from the distribution of the first birth in a society where cohabitation is common and relevant statistics is limited.

Tentative Conclusion

In this paper, the CM model is enhanced as a behavioral model so as to describe latent processes behind occurrences of first marriage such as entering into marriage market and searching for a mate.

The CM model is viewed as a multistage model, by which we mean a process that consists of multiple processes required for a target event (*i.e.* marriage in this case) to occur. Its convolution structure can be regarded as an expression of a multistage process consisting of attainment of marriageable age, waiting time for meeting the eventual spouse, dating stage, and engagement period. This interpretable nature of the CM model is carefully examined and the corresponding equivalent convolution structure of the GLG model is presented. Various other convolution models for first marriage process are also examined. The critical problem associated with the multistage view of the CM model is that the assumption of the model over independence between the sub-processes is not satisfied in the reality. The analysis on a national representative survey of the first marriage process in Japan detected the substantial dependency between age at first meeting of a couple and subsequent waiting time for the first marriage (Kaneko 1991a, Kaneko 1999). This is likely the case for other societies. Hence we proposed a multistage model in which the independence among the sub-processes is not assumed (here, we call it non-independent "convolution" model) by introducing liner relationships among parameters so that realistic measurement of the sub-processes can be obtained. Though it may spoil the elegance to some extent, the model acquires additional applicability to the actual process. Our verification of the new model indicates reasonable agreements with the empirical data. Actually applications to the vital statistics date in Japan with waiting time distribution from survey indicates good estimates to latent preceding events like first marriage, or first meeting with present spouse. It demonstrates the outstanding potential of the CM model to represent the behavioral process of first marriage and conceivably other demographic events as well such as birth.

The new model is to be applicable in many ways to behavioral studies of such vital events as marriage and birth separated by birth order. For example, it can be applied to estimation of size of marriage market, i.e. population size of marriageable people in a society. The size is possible to measure by estimating age distribution of people's becoming marriageable, which is major part of the model. The model should be effective to detect latent processes of births, since they are viewed as multistage processes. For application, it should be useful for estimating the distribution of "real" age at onset of sexual union by applying the model to the first birth for the society in which marriage is no longer viewed as the onset of sexual relationships among couples. Some of these applications should be demonstrated in the complete version of the present paper.

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死亡・寿命の数理モデル

5　年齢シフトモデルの応用・発展に関する研究

石井　太

はじめに

国立社会保障・人口問題研究所の「日本の将来推計人口（平成 18 年 12 月推計）」(国立社会保障・人口問題研究所 2007) の将来生命表作成にあたっては、現在国際的に標準的な方法とされ、平成 14 年 1 月推計でも用いたリー・カーター・モデルを採用しつつ、これに対して世界の最高水準の平均寿命を示すわが国の死亡動向の特徴に適合させるため、新たな機構を加えたモデル（以下、「年齢シフトモデル」と呼ぶ）により死亡率の投影を行った。具体的には、過去の死亡率曲線にロジスティック曲線を当てはめて、その年齢シフト量と勾配に関するパラメータを推定し、これによる高齢死亡率の年齢シフトを考慮した上でリー・カーター・モデルを適用することによって、死亡率改善の著しいわが国の死亡状況に適合させるものである。

本研究は、この年齢シフトモデルについて、より幅広い観点から有効性を再検証するとともに、さらなる応用・発展の可能性を検討するものである。

1. ロジスティックモデルと年齢変換

「日本の将来推計人口（平成 18 年 12 月推計）」の将来生命表の推計において採用された年齢シフトモデルは、過去の死亡率曲線にロジスティック曲線を当てはめて、その年齢シフト量と勾配に関するパラメータを推定し、これらのパラメータを利用して死亡率に年齢軸上の変換を施した上でリー・カーター・モデルを適用するものである (石井 2008a)。ここでは、まず、その理論的根拠となっているロジスティックモデルの性質と、死亡率に対する年齢軸上の変換との関係について簡単に整理を行う。

年齢シフトモデルにおいてパラメータ推定に用いられている 3 パラメータロジスティックモデルは、

$$\mu_{x,t} = \frac{\alpha_t \exp(\beta_t x)}{1 + \alpha_t \exp(\beta_t x)} + \gamma_t$$

で表される。Bongaarts (2005) では、第 1 項を senescent mortality ($\mu^s_{x,t}$) , 第 2 項を background mortality (μ^b_x) と呼んでいる。高齢部においては senescent mortality の影響が大きいものとなるので、まずこれに着目する。

$\mu^s_{x,t}$ は S_t という新たなパラメータを用いて以下のように書き変えることができる。

$$\begin{aligned}\mu^s_{x,t} &= \frac{\alpha_t \exp(\beta_t x)}{1+\alpha_t \exp(\beta_t x)} \\ &= \frac{\exp(\log\alpha_t + \beta_t x)}{1+\exp(\log\alpha_t + \beta_t x)} \\ &= \frac{\exp(\beta_t(x-(-\frac{\log\alpha_t}{\beta_t}))}{1+\exp(\beta_t(x-(-\frac{\log\alpha_t}{\beta_t}))} \\ &= \frac{\exp(\beta_t(x-S_t))}{1+\exp(\beta_t(x-S_t))}\end{aligned}$$

ここで $S_t = -\frac{\log\alpha_t}{\beta_t}$ である。この表現は、任意の時刻 t における senescent mortality の死力関数は、ある基準時刻 t_0 の死力関数に年齢軸上の線形変換を施すことによって得られることを示している。そこで、本稿ではこのような年齢軸上の変換を年齢変換 (age-transformation) と呼ぶこととし、上記のロジスティックモデルの性質を年齢変換の概念を用いて表してみよう。

まず、年齢変換について、以下のように定義しておく。

定義 1 $x, y \in [0,\infty)$ を、ともに年齢を表す座標とする。$f_t : y \to x$ という連続な狭義単調増加関数によってこれらの座標間の一対一対応関係が示されたとき、f_t を時刻 t における x から y への年齢変換と呼ぶ。

一般に、年齢座標 x において、年齢と時間の関数 $m_{x,t}$ が定義されているとき、年齢変換 f_t により、年齢座標 y における関数 $\tilde{m}_{y,t}$ が、

$$\tilde{m}_{y,t} \underset{\text{def}}{=} m_{f_t(y),t}$$

により定義される。

次に、先のロジスティックモデルの性質を年齢変換を用いて表すこととする。x: 変換前の年齢座標、y: 変換後の年齢座標とし、$f_t : y \to x$: 時刻 t における線形な年齢変換を以下で定義する。

$$x = f_t(y) \underset{\text{def}}{=} \frac{\beta_{t_0}}{\beta_t}(y - S_{t_0}) + S_t$$

さらに、変換後の年齢座標における死力関数を、$\tilde{\mu}^s_{y,t} \underset{\text{def}}{=} \mu^s_{f_t(y),t}$ により定義すれば、以下の命題により変換後座標における死力関数は不変であることがわかる。

命題 1

$$\tilde{\mu}^s_{y,t} = \tilde{\mu}^s_{y,t_0}$$

Bongaarts (2005) はパラメータ β_t の時系列変化が概ね一定であることに着目し、シフティング・ロジスティックモデルを提案した。これは、移動量が $S_t - S_{t_0}$ である年齢軸上の平行移動という、年齢変換の特別なケースであると見ることができる。

次の二つの命題は、S_t と β_t の性質を述べたものである。

命題 2

$$\left.\frac{\partial^2}{\partial x^2}\mu_{x,t}\right|_{x=S_t} = 0$$

命題 3

$$\left.\frac{\partial}{\partial x}\log\mu_{x,t}\right|_{x=S_t} = \frac{1}{2+\gamma_t}\beta_t$$

命題 2 は死力関数の傾きが年齢 S_t を境に逓減を始めるポイントであることを意味する。一方、命題 3 は、$\gamma_t \ll 2$ が近年の先進諸国のデータで成立することから、年齢 S_t における死力関数の対数が概ね $\frac{\beta_t}{2}$ に近いことを示す。ここから、パラメータ S_t と β_t を次のように解釈することが可能である。すなわち、S_t の近傍においては、ロジスティック曲線は S_t を基点とした横方向への動きと、β_t で表される曲線の勾配の変化によって特徴づけられており、β_t が一定で S_t が増加する場合には高齢方向へのシフティングが起きており、S_t が一定で β_t が増加する場合には矩形化が起きていると解釈することができる。

そこで、これら S_t、β_t を実データに基づき観察することとする。ここでは、基礎データとして Human Mortality Database の m_x を用い、グレビルの式により平滑化した m_x を $\mu_{x+0.5}$ の近似として 3 パラメータロジスティック曲線に当てはめることによりパラメータ S_t、β_t を推定した。パラメータ推定にあたっては、より高齢死亡率に焦点を当てる方法を試みることとした。すなわち、石井 (2008a) では Bongaarts (2005) を参考として、25 歳以上の対数死亡率に対して最小二乗法を用いてパラメータ推定を行ったが、本稿においては 50 歳以上の死亡率に限定し、対数を取る前の死亡率に対して最小二乗法を適用した。

図 1、2 は、日本のデータによる S_t と β_t の推計値である。図 3、4 は、1950、 1975、2000 年における $\log(m_x)$（実線）に、S_t（点）と S_t における接線（破線）を示したものである。また、図 5〜8 は、米国のデータに対して同様に示したものである。両者の主な違いは、わが国に関してはこの間、S_t が大きく増加してきているのに対し、米国では S_t の変化がそれほど大きいものではないという点である。したがって、先のパラメータの解釈に基づけば、この観察期間において、日本では高齢方向へのシフティングから高齢死亡率改善が起きてきた傾向が強いと考えられる一方、米国では矩形化により高齢死亡率改善が起きてきた傾向が強いのではないかと考えられることになる。

このように、S_t と β_t を用いることにより、高齢部の死亡率を年齢変換により表現することができ、死亡率のモデリングに関する手がかりを得ることができる。しかしながら一

方で、これらは高齢部分の死亡率の性質のみしか表現していないため、全年齢の死亡率のモデリングを行うためには若年層も含んだ年齢変換を考えることが必要となる。そこで、次節においてこれを考察する。

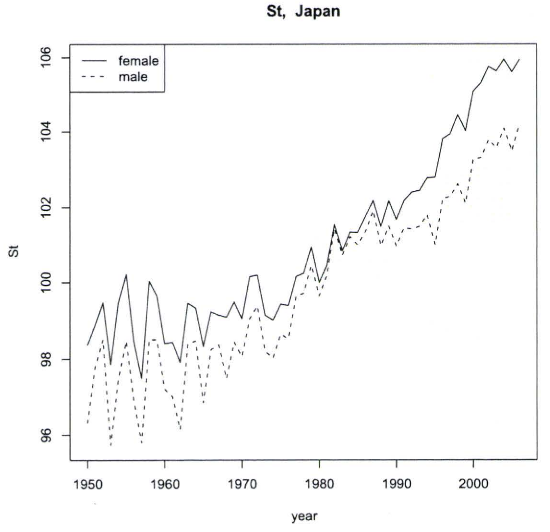

図 1　S_t (1950-2006, Japan)

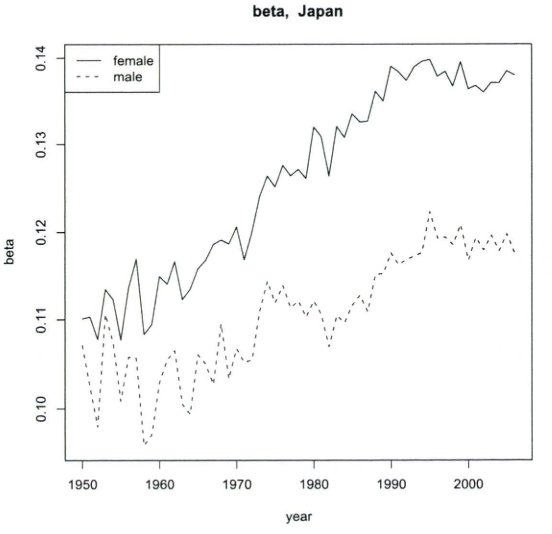

図 2　β_t (1950-2006, Japan)

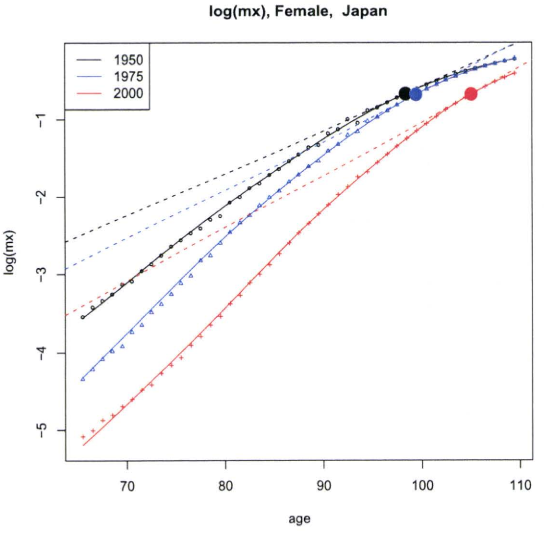

図 3　$\log(m_x)$ (1950,1975 and 2000, Female, Japan)

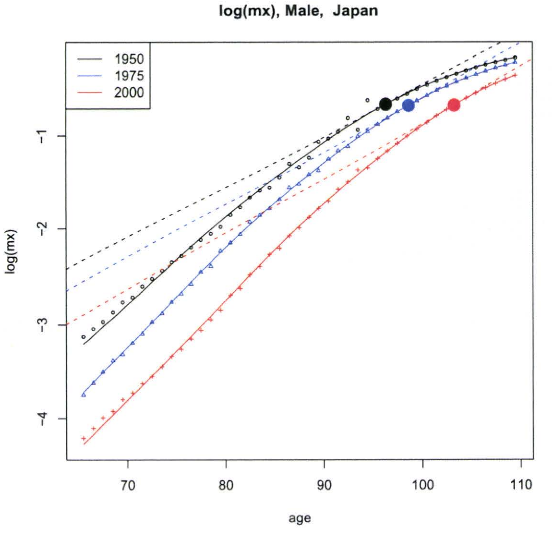

図 4　$\log(m_x)$ (1950,1975 and 2000, Male, Japan)

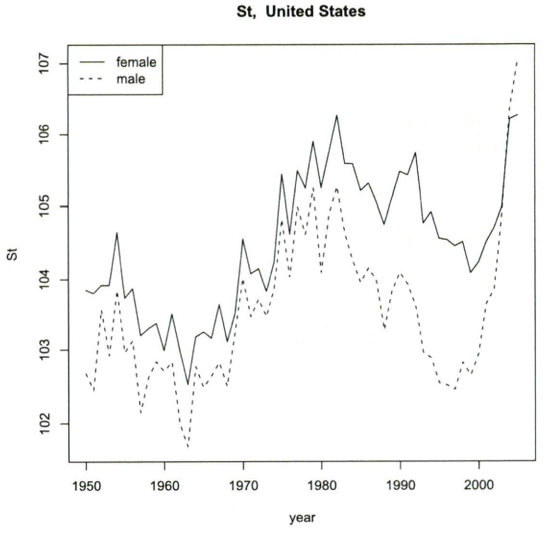

図 5　S_t (1950-2006, USA)

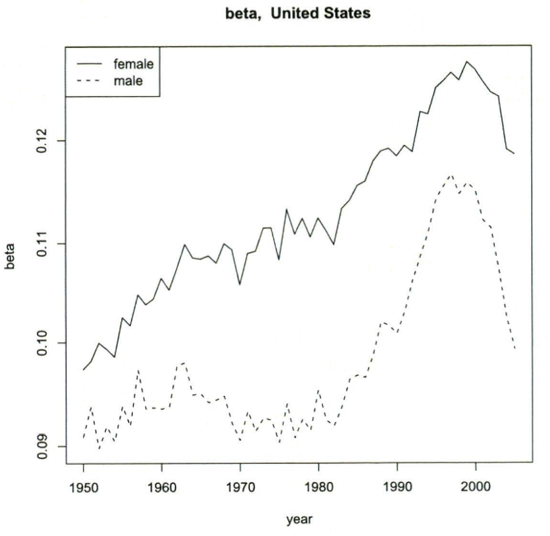

図 6　β_t (1950-2006, USA)

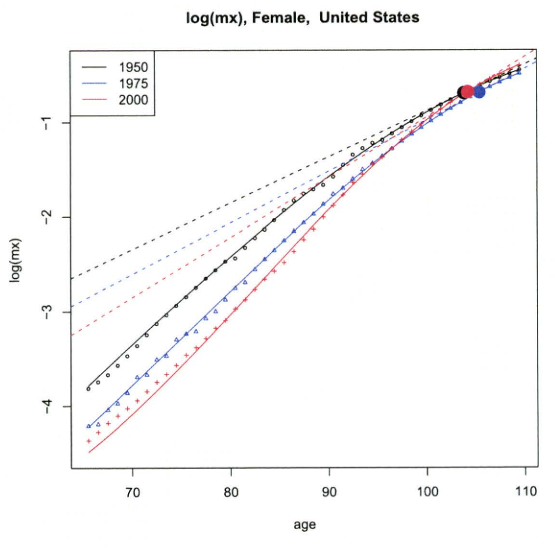

図 7　$\log(m_x)$ (1950,1975 and 2000, Female, USA)

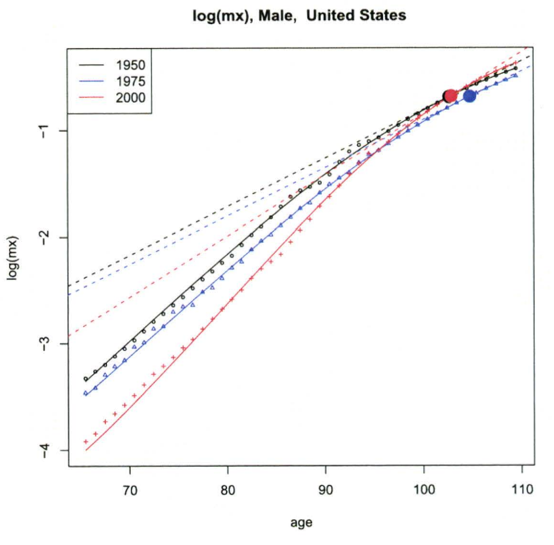

図 8　$\log(m_x)$ (1950,1975 and 2000, Male, USA)