1 Introduction

People discount future events both by preferring to obtain an immediate gain (even if it is smaller than the one that may be obtained in the future), and by trying to avoid an immediate loss, even if it is smaller than the one that may incur in the future (Frederick, Loewenstein, & O' Donoghue, 2002). In this paper we propose that this phenomenon is likely to take different forms across cultures because of cultural biases in attention allocation.

Consistent with recent work on cultural psychology (Markus & Kitayama, 1991; Masuda & Nisbett, 2001), we assume that people engaging in Western cultures (Westerners) tend to focus their attention on the magnitude of a reward in *lieu* of its context, including both a delay until receipt and a distant object. Accordingly, these individuals may be much more strongly affected by a proximal object than by the distant one with respect to the near future. Moreover, they may be relatively impervious to the exact length of the time interval between the two objects, because they may tend to focus on each temporal fragment (a divided portion of the time interval between more delayed rewards) separately, rather than focusing on the undivided time-interval between rewards. In contrast, individuals engaging in Eastern cultures (Easterners) tend to allocate their attention more holistically to both an immediate and a distant object, as well as to the time interval between them. When compared to Westerners, Easterners may then be expected to discount the future less, and moreover, even when they do it, they may do so as a steady function of the length of the time interval.

In the present paper, we first formulate these conceptual predictions in terms of a mathematical model of future discounting based on Tsallis' statistics, and we report a cross-cultural experiment designed to test the specific derivations of the model. Finally, we propose future directions in cultural neuroeconomics employing neurocomputational models based on Tsallis' statistics.

2 Intertemporal choice model based on Tsallis' statistics and psychophysics of time

When given a choice between two possible times at which an outcome can occur, people usually prefer to receive it earlier if it is a good outcome, and later if it is a bad one (Frederick et al., 2002). This phenomenon of temporal discounting has been the subject of much research in neuroeconomics and neuropsychopharmacology, which has revealed how the strength of the preference for earlier outcomes over later ones is influenced by factors including the magnitude and sign of the outcomes (Frederick et al., 2002) and temporal cognition (Takahashi, 2005, 2006; Wittmann & Paulus, 2008). Specifically, (i) people are patient with respect to the distant future but impulsive with respect to the near future, when they choose between smaller sooner rewards and larger later ones (preference reversal due to "hyperbolic discounting", also referred to as "present bias"; for details see Soman et al., 2005), and (ii) people make more impulsive choice when the length of delay is perceived as a sum of shorter time-intervals ("subadditive discounting"; see Read & Roelofsma, 2003). We provide examples of time-inconsistency and impulsivity in temporal discounting in Appendix A. These inconsistencies in intertemporal choice cannot be accounted for by a conventional model of temporal discounting in microeconomic theory ("exponential discounting"; see Frederick et al., 2002). As a consequence, both impulsivity (strong discounting) and inconsistency in temporal discounting (i.e., hyperbolic and subadditive discounting) have extensively been investigated in neuroeconomic studies by employing neuroimaging techniques (Boettiger et al., 2007; Hariri et al., 2006; Kable & Glimcher, 2007; McClure et al., 2004; McClure et al., 2007; Monterosso et al., 2007; Wittmann, Leland, & Paulus, 2007), stimulating thereby much further research and debate.

Recently, behavioral neuroeconomic and econophysical studies established discount models in order to better describe neural and behavioral correlates of impulsivity and inconsistency in intertemporal choice. In order to analyze human and animal intertemporal choice behavior in a manner that would allow for a dissociation between impulsivity and inconsistency, recent econophysical studies (Cajueiro 2006; Takahashi, Oono, & Radford, 2007) proposed and examined the following q-exponential discount function for subjective value $V(D)$ of a delayed reward:

$$V(D) = \frac{A}{\exp_q(k_q D)} = \frac{A}{[1+(1-q)k_q D]^{1/(1-q)}} \tag{1}$$

where $\exp_q(x) := [1+(1-q)x]^{1/(1-q)}$ is a q-exponential function, D is a delay until receipt of a reward, A is the value of a reward at $D = 0$, and k_q is a parameter of impulsivity at delay $D = 0$ (q-exponential discount rate). We can easily see that this generalized q-exponential function approaches the usual exponential function in the limit of $q \to 1$. The q-exponential function has extensively been utilized in econophysics, where the application of Tsallis' non-extensive thermostatistics (Tsallis et al., 2003) may possibly explain income distributions following power functions (Michael & Johnson, 2003). It needs to be noted here that when $q = 0$, the equation (1) becomes the same as the "hyperbolic" discount function (i.e., $V(D) = A/(1 + k_q D)$), while in the limit of $q \to 1$, it reduces to the "exponential" discount function (i.e., $V(D) = A\exp(-k_q D)$). In exponential discounting (when $q \to 1$ in equation (1)), intertemporal choice is consistent, because the discount rate $:= -(dV/dD)/V = k_q$ is time-independent when $q \to 1$. The q-exponential discount function is capable of continuously quantifying human subjects' inconsistency in intertemporal choice (Takahashi et al., 2007). Namely, human agents with smaller q values are more inconsistent in intertemporal choice. If q is less than 0, the intertemporal choice behavior is more inconsistent than hyperbolic discounting. Thus, $1 - q$ can be utilized as an inconsistency parameter. Moreover, it is possible to examine neuropsychological modulation of k_q (impulsivity in temporal discounting) and q (dynamic consistency) in the q-exponential discount model. It is now important to note that in any continuous time-discounting functions, a discount rate (preference for sooner rewards over later ones) is defined as $-(dV(D)/dD)/V(D)$, independently of functional forms of discount models, with larger discount rates indicating more impulsive intertemporal choice. In the q-exponential discount model, the q-exponential discount rate q_{EDR} ("impulsivity") is then defined as:

$$q_{EDR} = \frac{k_q}{[1 + k_q(1-q)D]} \tag{2}$$

We can see that when $q = 1$, the discount rate is independent of delay D, corresponding to the exponential discount model (consistent intertemporal choice);

while for $q < 1$, the discount rate is a decreasing function of delay D, resulting in preference reversal over time. This can be seen by a direct calculation of the time-derivative of the q-exponential discount rate:

$$(d/dD)\ q_{EDR} = -\frac{k_q^2(1-q)}{[kq(1-q)D+1]^2} \tag{3}$$

which is negative for $q < 1$, indicating "decreasing impatience" for q smaller than 1. Also, impulsivity at delay $D = 0$ is equal to k_q irrespective of q. Therefore, k_q and q can parameterize impulsivity and consistency, respectively, in a distinct manner.

Regarding the neuropsychological processing underlying the q-exponential discounting (i.e., inconsistent intertemporal choice), Takahashi (2005) proposed that exponential discounting with logarithmic time-perception, $\tau(D) = \alpha \log(1+\beta D)$, may explain dynamic inconsistency in intertemporal choice. If a subject tries to discount a delayed reward exponentially with the logarithmic time-perception (i.e., Weber-Fechner law in psychophysics), then $F(\tau) = \exp(-k\tau) = 1/(1+\beta D)^{k\alpha}$, which has the q-exponential functional form. Intuitively, subjects try to discount exponentially (rationally and consistently), but actual intertemporal choice behavior may be hyperbolic and dynamically inconsistent, due to a distortion in time-perception. This may also explain subadditive discounting, because $\tau(D)$ is concave in delay D (i.e., the subjective delay length is larger when the delay is divided into shorter time-intervals than when the delay is perceived as a single time-interval; for details see Takahashi, 2006). Therefore, it can be expected that the non-linear psychophysical effects of temporal cognition on intertemporal choice may be reflected in the q parameter in the q-exponential discount function. However, to our knowledge, no study has yet examined how psychological factors, such as attention to a time-interval between sooner and later rewards, modulate intertemporal choice behavior by utilizing the q-exponential function, although recent studies reported attention effects on time modulated dynamic consistency in temporal discounting (Ebert & Prelec, 2007; Zauberman et al., 2008).

In the present study, we address the question of how cultural differences in attention allocation (i.e., "analytic" versus "holistic" allocation) modulate intertemporal choice behavior between American and Japanese decision makers.

3 Cultural neuroscience of attention and thought

In recent years, cultural psychologists have begun to show that there are systematic cultural variations in human (neuro)psychological processes (Markus & Kitayama, 1991). These researchers assume that neuropsychological processes are by nature socially driven. According to their theories, the neuropsychological processes are shaped through their interaction with cultural, social and environmental factors. Based on this assumption, it has often been examined how particular cognitive processes (e.g., attention allocation) could be manifested in particular cultural contexts and how different cultural environments in turn lead to the development of different patterns of ability. These studies reported that East Asians' patterns of attention were in general "context dependent", whereas Westerners' patterns of attention were "context independent".

Accordingly, Westerners are more likely to focus on some salient objects or contents ("analytic" attention), whereas East Asians are more likely to attend to the global context ("holistic" attention) of an object, and its broad spectrum of perceptual and conceptual fields, in addition to its local characteristics (see e.g. Masuda & Nisbett, 2001; Kitayama, Duffy, Kawamura, & Larsen, 2003; Chua, Boland, & Nisbett, 2005).

Chiao & Ambady (2007) have recently proposed a "cultural neuroscience" approach in order to integrate biological perspectives into endeavors of cultural psychology. This approach employs both biological (e.g., neurophysiological, neurogenetic, and neuroendocrinological methods) and cultural psychological experiments, in a manner similar to neuroeconomics unifying biopsychology and economics (Glimcher & Rustichini, 2004; Lee, 2005; Loewenstein et al., 2008; Sanfey et al., 2006; Zak, 2004). Furthermore, a recent neuroimaging study (Hedden et al., 2008) identified neural correlates of cultural differences in attention control in simple visual attention tasks. Therefore, it is highly important for further neuroeconomic investigations to incorporate neurocomputational processes mediating attention in order to establish neuroeconomically plausible models of decision-making.

4 Attention and perception in neural valuation of delayed rewards

In neuroeconomic studies of the valuation of delayed rewards, it has been reported that (i) immediate rewards activate midbrain regions (McClure et al., 2004, 2007), and (ii) subjective value of the delayed reward is encoded as the midbrain dopaminergic activities (Kable & Glimcher, 2007). Regarding the role of temporal cognition in intertemporal choice, Wittmann and colleagues reported that the psychological time is represented in the striatum (Wittmann et al., 2007); while no neuroimaging study to date examined the neural correlates of attention allocation during intertemporal choice.

Recent behavioral economics studies (Ebert & Prelec, 2007; Zauberman et al., 2008) have demonstrated that modulation of attention to time perspectives (time-sensitivity) changes the human intertemporal choice behavior by shifting the functional form of the psychophysical time-perception from a logarithmic to a linear function. This is consistent with the psychophysical account of hyperbolic discounting (Takahashi, 2005, 2006). Together, these studies suggest that control of attention allocation to time explains both hyperbolic and subadditive discounting.

Specifically, (i) if a subject pays more attention to the delayed reward but less attention to the time-length of delay ("time-insensitivity"), her/his temporal discounting may be inconsistent due to non-linearly distorted time-perception (i.e., hyperbolic discounting), and (ii) if a subject focuses her/his attention on each temporal "segment" along the future time (i.e., "analytic" temporal cognition) rather than overviews the future time perspective as a whole (i.e., "holistic" temporal cognition), her/his temporal discounting may be exaggerated (i.e., subadditive discounting). In both cases, it can be predicted that narrower allocation of attention should be associated with more impulsive and inconsistent temporal discounting behavior.

In social psychology literature, the "temporal construal" theory has been proposed for explaining time-inconsistency in discounting behavior (Trope & Liberman, 2003). This theory claims that temporal horizons change people's responses to future events by changing the manner they psychologically represent those future events. More specifically, people may form more abstract representations ("high-level construals") of distant-future events than near-future events. High-level construals consist of decontextualized and central features ("content" in terms of cultural neuroscience) that convey the essence of information about future events (e.g., the type and size of a delayed reward), while low-level construals include more contextual and peripheral details ("context" in terms of cultural neuroscience). Hence, a subject with narrow attention allocation (i.e., primarily paying her/his attention to either "content" or "context") may experience preference reversal in decision over time (e.g., procrastination of formerly planned actions), whereas a subject with wide attention allocation (i.e., paying attention to both "content" and "context") may not change her/his preference in decision over time.

Taken together, these behavioral economic and social psychological theories and findings hypothesize that narrower allocation of attention may be associated with more impulsive and inconsistent temporal discounting. With respect to cultural differences in temporal discounting, we propose that Westerners are more impulsive and inconsistent in inter-temporal choice behavior in comparison to Easterners, for cultural neuroscience studies have demonstrated that Westerners have more analytic attention allocation than Easterners. This prediction is also supported by psychophysical accounts of hyperbolic and subadditive discounting, as stated above (i.e., association between "analytic", rather than "holistic", attention allocation and hyperbolic/subadditive discounting).

5 Cultural differences in temporal discounting behavior

In order to examine the cultural differences in temporal discounting, we compared intertemporal choices for monetary gains and losses by American and Japanese subjects, by utilizing the q-exponential discount model based on Tsallis' statistics. For discounting behavioral data by Americans, we analyzed Estle et al's raw data obtained from students (N=27) at Washington University (Estle et al., 2006). Japanese subjects were students at the University of Tokyo and Hokkaido University (N=21).

In order to avoid the magnitude effect on temporal discounting (i.e., small rewards are more rapidly discounted than large ones), we compared time-discounting behavior for gains and losses of 100 dollars and 10,000 yen (about 100 US dollars) between American and Japanese subjects. Our experimental procedure was exactly the same as in our previous study (Takahashi, Ikeda, & Hasegawa, 2007; also see Appendix B for experimental details). In order to parameterize impulsivity and inconsistency in intertemporal choices, we employed k_q and q parameters in the q-exponential discount model (equation (1)). We fitted the q-exponential function to the behavioral data by utilizing a non-linear least square algorithm implemented in R statistical computing software (The R Project for

Statistical Computing). We note here that larger k_q and smaller q correspond to more impulsive and inconsistent temporal discounting. The major results are summarized in Table 1.

	Gain		**Loss**	
	American	Japanese	American	Japanese
k_q (impulsivity)	0.021	0.0053	0.073	0.0
q (consistency)	0.520	0.78	0.82	0.99

Table 1: Impulsivity and inconsistency in temporal discounting for gain and loss: Americans (N=27, Estle et al., 2006) discounted delayed outcomes more steeply and inconsistently than Japanese (N=21).

For both gains and losses, Americans discounted the delayed outcomes more steeply (larger k_q) and inconsistently (smaller $q < 1$ values). The present observations are consistent with predictions from cultural neuroeconomic theory, combining findings from behavioral neuroeconomics, cultural neuroscience, and social psychology.

6 Discussions and future directions

This study is the first one to (i) propose a cultural neuroeconomic theory of intertemporal choice based on cultural neuroscience theory of attention and neuroeconomics, and (ii) it demonstrates that Westerners tend to discount delayed outcomes more rapidly and inconsistently than Easterners. Our present findings are in line with (i) the reported role of attention allocation in neurocomputational processes involved in intertemporal choice and with (ii) the effects of attention allocation strategies (i.e., "analytic" versus "holistic") on temporal discounting. Although a previous study examined cross-cultural differences in discounting behavior by American, Chinese, and Japanese students in the United States, the study did not analyze time-consistency and impulsivity separately (Wanjiang, Green, & Myerson, 2002).

Incorporating cultural differences in neuroeconomic decision processes may be important for establishing more efficient economic policies, because the world has become a highly multicultural place these days. Within the context of the ongoing expansion of the European Union, future studies should focus on measurements and models of temporal and probability discounting in Western, Central, and Southeast European countries. One could thereby monitor the differences in impulsivity and inconsistency in inter-temporal choice behavior between the individuals coming from the old EU member states, from the recently included countries, and those who still have the status of a candidate member. The estimated values of k_q and q parameters would then provide the relevant information about the cross-cultural differences in impulsivity and inconsistency in choice behavior in Europe. This information could further be used when extending other computational models, such as neural networks, so as to enable process-based, continuous modeling of cultural aspects of economic decision making in Europe, and moreover, to provide more details on how these aspects affect the European economy at a more global level.

Some generalizations of neural network models à la Tsallis were already previously reported (Cannas, Stariolo, & Tamarit, 1996; Hadzibeganovic & Cannas, 2007; submitted). These generalizations are based on analogies between the properties of neural network models and those found in statistical physics and thermodynamics. As discussed by Hopfield (1982) and then applied to attractor networks by Amit and colleagues (1985), neural network models have direct analogies in statistical physics, where the investigated system consists of a large number of units each contributing individually to the overall, global dynamic behavior of the system. The characteristics of individual units represent the microscopic quantities that are usually not directly accessible to the observer. However, there are macroscopic quantities, defined by parameters that are fixed from the outside, such as the temperature $T = 1/\beta$ and the mean value of the total energy. The main aim of statistical physics is to provide a link between the microscopic and the macroscopic levels of an investigated system. An important development in this direction was Boltzmann's finding that the probability of occurrence for a given state $\{x\}$ depends on the energy $E(\{x\})$ of this state through the well-known Boltzmann-Gibbs distribution $P(\{x\}) = \frac{1}{Z} exp[-\beta E(\{x\})]$, where Z is the normalization constant $Z = \sum_{\{x\}} exp[-\beta E(\{x\})]$.

In the context of neural networks, statistical physics can be applied to study learning behavior in the sense of a stochastic dynamical process of synaptic modification (Watkin, Rau, & Biehl, 1993). In this case, the dynamical variables $\{x\}$ represent synaptic couplings, while the error made by the network (with respect to the learning task for a given set of values of $\{x\}$) plays the role of the energy $E(\{x\})$. The usage of a gradient descent dynamics as a synaptic modification procedure leads then to a stationary Boltzmann-Gibbs distribution for the synapses (Watkin et al., 1993). However, the gradient descent dynamics corresponds to a strictly *local* learning procedure, while non local learning dynamics may lead to a synaptic couplings distribution different from the Boltzmann-Gibbs one (Stariolo, 1994; Cannas, Stariolo, & Tamarit, 1996).

Here, we briefly report an implementation of the Tsallis entropy[1] formalism in a simple neural network model which has been used for simulation of learning behavior in adults. In this model, a generalization of the gradient descent dynamics is realized via a nonextensive cost function (Stariolo, 1994) defined by the map

$$\overline{V} = \frac{1}{\beta(q-1)} \ln [1 + \beta(q-1)V] \tag{4}$$

where the index q is an arbitrary real number such that $q \geq 1$; $\overline{V}$ is a monotonically increasing function of V, and therefore it preserves its minima structure. The *Langevin* equation, which governs the (local) gradient descent dynamics that is usually applied in neural networks, is here replaced by:

$$\frac{dJ_{ij}}{dt} = -\frac{1}{1+\beta(q-1)V} \frac{\partial V}{\partial J_{ij}} + \eta_{ij}(t). \tag{5}$$

[1]Tsallis' entropy $S_q = \frac{1-\sum_i p_i^q}{q-1}$ $(q \in \mathcal{R})$ is a nonlogarithmic (generalized) entropy, which reduces to standard Boltzmann-Gibbs-Shannon entropy $S_{BGS} = -\sum_i p_i \ln p_i$ as the nonextensive entropic index q approaches unity. See Appendix C for a proof and details.

The advantages of the presented q-generalized learning rule in a neural network model (Cannas, Stariolo, & Tamarit, 1996; Hadzibeganovic & Cannas, 2007; submitted) span beyond classical learning applications. The model may also help in studying other problems in cognitive (neuro)science such as neurological impairments. Moreover, the model could serve as an example of how to generalize and improve other neural networks that have regularly been used in several different areas of economics.

By means of estimating the index q in the presented q-exponential discount model, the inconsistency in choice behavior may be expressed in a continuous manner (where a whole spectrum of q indices may be obtained corresponding to different inconsistencies in choice; with smaller q values indicating more inconsistent choices). Future studies should also examine and model the behavior of alcohol or drug addicted patients, people with orbitofrontal lesion, pathological gamblers, and other individuals who were previously shown to have impaired decision-making behavior in inter-temporal choice. By utilizing the q-exponential discount function, one could diagnose the degree of inconsistency in choice in these patients with greater sensitivity and accuracy than with many currently available methods.

Finally, we note that no neuroeconomic theory of temporal discounting is going to be complete until it can fully incorporate the cultural aspects of impulsivity and inconsistency in decision making, the underlying cognitive and neurocomputational processes, emotionally driven choice aspects, and other (neuro)biological properties in humans that may drive the dynamics of economic behavior.

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A Appendix

There are two distinct behavioral tendencies in intertemporal choice: Impulsivity and inconsistency. First, suppose the following Example 1 for demonstrating impulsivity. Agent A who prefers "one glass of beer available one year later" over "two glasses of beer available (one year plus one week) later" is more impulsive than agent B who prefers "two glasses of beer available (one year plus one week) later" over "one glass of beer available one year later". In this Example 1, most people tend to behave as the patient (less impulsive) agent B. It is to be noted that both impulsive agent A and patient agent B may be "rational" in terms of economic theory, because, in this example alone, there is no inconsistency even in impulsive agent A's behavior. Next, suppose the following Example 2 for intertemporal choice. There are two options: "one glass of beer available now" and "two glasses of beer available one week later". In Example 2, most people (who planned to choose the larger but more delayed option in Example 1) simultaneously tend to prefer smaller but immediate option: "one glass of beer available now" over "two glasses of beer available one week later". This phenomenon is referred to as "preference reversal" over time, and an instance of time-inconsistency in decision over time. It is important to note that impulsivity and inconsistency corresponds to a large time-discount rate and time-dependency of the time-discount rate, respectively.

B Appendix

Participants (N=9 male, N=12 female, mean age=21.4) were requested to select among alternatives based solely on their free will, as if choices were about real money. Instructions were written on the top of each page of the questionnaire and expressed the temporal distance of delay (i.e., delays of 1 week, 2 weeks, 1 month, 6 months, 1 year, 5 years, 25 years, where each page included all delays presented in exactly this order). Two columns of hypothetical money amounts were listed below the instructions. The right column (standard amount) contained 40 rows of a fixed magnitude of monetary gain or loss (10,000 yen, i.e., about 100 US dollars). The left column (adjusting amount) listed ascending or descending magnitudes of money in 2.5% increments (= 10,000 yen × 0.025 = 250 yen) of the alternative in the right column. Participants were instructed to choose between the two alternatives in each row of the questionnaire. Furthermore, participants were directed to attend to the directions on the top of each page (containing each delay) of the questionnaire, as the temporal distance would change over the course of the experiment. Thus, subjects chose between the delayed-standard amount and the immediate-adjusted amount of money. The order of the descending and ascending conditions was counterbalanced. The indifference points of delay discounting tasks were defined as the means of the largest adjusting value in which the standard alternative was preferred and the smallest adjusting value in which the adjusting alternative was preferred. Next, the mean of the indifference point in ascending and descending adjusting amounts were calculated for the delay conditions (gain and loss) for each participant.

C Appendix

Researchers with a professional background that is different from (statistical) physics cannot easily spot the correspondence between the two basic equations that were crucial in the formulation of Tsallis' entropy: The standard Boltzmann-Gibbs-Shannon (BGS) entropy formula and the generalized nonextensive entropy formula. It is therefore our intention here to mathematically clarify this relationship. More specifically, it will be shown that the Tsallis' entropy $S_q = \frac{1-\sum_i p_i^q}{q-1}$ ($q \in \mathcal{R}$) reduces to BGS entropy $S_{BGS} = -\sum_i p_i \ln p_i$ as the nonextensive entropic index q approaches unity. The proof is based on (Bernoulli-)L'Hôpital's rule.

The rule named after the French mathematician Guillaume de l'Hôpital employs derivatives to calculate limits with *indeterminate* forms. In this sense, using this rule, one can convert an *indeterminate form* (e.g. $\frac{0}{0}$ or $\frac{\infty}{\infty}$) into a determinate form with an easy computation of the limit.

Let

$$S_q = \frac{1-\sum_i p_i^q}{q-1}.$$

When $q \to 1$, the numerator of S_q tends to $1 - \sum_i p_i = 0$ ($\sum_i p_i = 1$). Since the denominator also tends to 0, S_q has the indeterminate behavior $\frac{0}{0}$ as $q \to 1$. Therefore, L'Hôpital's Rule can be applied to the limit $\lim_{q\to 1} S_q$:

$$\lim_{q\to 1} S_q = \lim_{q\to 1} \frac{(1-\sum_i p_i^q)'}{(q-1)'}, \tag{1}$$

where $'$ indicates the derivative with respect to q, i.e. $' = \frac{d}{dq}$. Since we can differentiate term by term, we obtain

$$\left(1-\sum_i p_i^q\right)' = (1)' - \sum_i (p_i^q)' = -\sum_i p_i^q \ln p_i$$

and

$$(q-1)' = (q)' - (1)' = 1.$$

We use above the following differentiation rules: $(1)' = 0$, $(q)' = 1$, and the rule for differentiating the exponential function a^q (the base a is a constant and the variable q is in the exponent), which reads $(a^q)' = a^q \ln a$ (in our case $a = p_i$). Therefore, from (1) we get

$$\lim_{q\to 1} S_q = \lim_{q\to 1} \frac{-\sum_i p_i^q \ln p_i}{1} = -\sum_i p_i \ln p_i.$$