4.2 Recursive Formulation

Notice that the problem of the individual can be recast in a recursive formulation. In addition, the dynamic problem ends in a finite horizon. Thus, the model can be solved backward, starting from the terminal decision period T. At this last age, the continuation value is exogenously given as a function of the state variable at that period. I do not normalize it to be zero because if I do so the individual consumes all the income in this last period, which may significantly affect the pattern of the optimal path.[31] The details are as follows.

First, let j-th element of the choice set in each period be denoted by

$$\begin{aligned} d_t^j &\in \{Zero,\ SE\} \\ &\times \{Zero,\ Part\text{-}time\ PE, Full\text{-}time\ PE\} \\ &\times \{\underline{\Delta a}, ..., \overline{\Delta a}\} \end{aligned}$$

and the utility associated with that choice as u_t^j. In addition, letting the state space at t be denoted by S_t, state point in period t, $s_t \in S_t$, is given by

$$s_t = ((h_t^s, \tau_t^s, h_t^w, l_{t-1}^w, a_t), (educ,\ type,\ \underline{age}),\ (\epsilon_t^{ls},\ \epsilon_t^{lw},\ \epsilon_t^{ys},\ \epsilon_t^{yw})),$$

where the generic element of the *predetermined* part of S_t is written by $\overline{S}_t$ whose generic element is

$$\overline{s}_t = ((h_t^s, \tau_t^s, h_t^w, l_{t-1}^w, a_t), (educ,\ type,\ \underline{age})).$$

Note that the part $(h_t^s, \tau_t^s, h_t^w, l_{t-1}^w, a_t)$ is a result of past decisions (up to $t-1$), and that the element $(educ,\ type,\ \underline{age})$ is the part of the state points that is permanently fixed.[32] Note also that actual age age_t is implicitly included in $\overline{s}_t$ because it is determined by t and the age in the first decision period $(\underline{age})$, that is, $age_t = \underline{age} + (t-1)$. Exogenous to the decisions but moving across t's are $(\epsilon_t^l, \epsilon_t^y)$ and age_t.

Thanks to the Bellman representation, the value function at any period t, V_t, is written in a recursive way by

$$\begin{aligned} V_t(s_t) &= \max_{d_t^j} u_t^j + \beta E_t[V_{t+1}(s_{t+1})|s_t] \\ &= \max[V_t^1(s_t), ..., V_t^J(s_t)] \end{aligned}$$

where E_t is the expectations operator at the beginning of period t, and

$$V_t^j(s_t) = u_t^j + \beta E_t[V_{t+1}(s_{t+1})|d_t^j = 1, s_t]$$

for $j = 1, 2, ..., J$. The expectation is taken over the joint distribution of the stochastic shocks in *next* period, $\widetilde{\epsilon}_{t+1}^l = (\widetilde{\epsilon}_{t+1}^{ls}, \widetilde{\epsilon}_{t+1}^{lw})$ and $\widetilde{\epsilon}_{t+1}^y = (\widetilde{\epsilon}_{t+1}^{ys}, \widetilde{\epsilon}_{t+1}^{yw})$. This alternative-specific value function assumes that future choices are optimally made for any given current decision.

[31] In the actual implementation, I use the quasi-terminal period, T^*, which is set to be 30 for all individuals, not T, to ease computational burden. Under this simplification, model individuals live up to age 49 (for those with $\underline{age} = 20$) to 54 (for those with $\underline{age} = 25$). As explained in Appedix B, the highest age observed in the data age is 39, so this simplification does not lose information from the empirical data.

[32] In the data, $(h_t^s,\ h_t^w,\ l_{t-1}^s,\ l_{t-1}^w,\ a_t)$ is not always (across t's and the sample individuals) observed, and $(educ, \underline{age})$ is observed for all of the sample individuals. Note that *type* is the variable to caputure unobserved hetertogeneity.

For $t = 1, ..., T$, let the part $E_t[V_{t+1}(s_{t+1})|d_t^j = 1, s_t]$ be denoted by $Emax_t$. Notice that this is a function that assigns each element of the predetermined state space *and* decisions (i.e, $\overline{s}_t \in \overline{S}_t$ and d_t^j) to some value.[33]

When the individual in the model (as well as the econometrician) wants to optimally choose a decision element in period t, he needs to know this function to compare $\{V_t^j(s_t)\}$ across j. He can do so in the following way. Consider the last period T. Then, for *each* $s_T \in S_T$, he has the following system of J equations:

$$\begin{cases} V_T^1(s_T) = u_T^1 + \beta V_{T+1}(d_T^1 = 1, s_T) \\ \dots \\ V_T^J(s_T) = u_T^J + \beta V_{T+1}(d_T^J = 1, s_T), \end{cases}$$

where $V_{T+1}(d_T^j,\ s_T)$, or $Emax_T$, is the terminal value that he obtains by choosing $d_t^j = 1$ when the state is s_T.[34] So, if this terminal value is given for *all* d_T^j and *all* s_T, it is then possible to compute $Emax_{T-1}$ by taking expectations of $V_T(s_T) = \max[V_T^1(s_T), ..., V_T^J(s_T)]$, given the distribution of ϵ_T. He can then solve for $Emax_t$ for all t by recursively solving the simple static optimization problems of discrete choice that is a system of linear equations. Once $Emax_t$ functions are known, the optimal path of decisions, $\{(l_t)^*,\ (\Delta a_{t+1})^*\}_{t=1}^T$, can be determined as follows: conditional on the deterministic part of the state space S_t, the probability that an individual is observed to choose option j takes the form of an integral over the region of the space of the five errors such that j is the preferred option.

As the decision period approaches the final period, however, the dimension of the predetermined state space $\overline{S}_t$ becomes too huge for the econometrician to obtain the optimal decision path in a computationally reasonable manner (in terms of both memory allocation and running time), especially if there are many total number of decision periods as in this study.

To deal with this problem, I use an approximation method that was proposed by Keane and Wolpin (1994) and applied by the same researchers (1997, 2001) and many others, in which the $Emax_t$ functions are expressed polynomials of the state variables.[35] Specifically, starting with T, for each *type*, I randomly select many points, $\{h_T^s, \tau_T^s,\ h_T^w, l_{T-1}^w,\ a_T,\ educ,\ \underline{age}\}$,[36] and for each of these points, I calculate $V_T(s_T)$, given $Emax_T$.[37] I obtain estimates for the polynomial coefficients by regressing $\{V_T(s_T)\}$ on the polynomial, and then interpolate the $Emax_{T-1}$ polynomial by using these estimated coefficients. This interpolated $Emax_{T-1}$ is used to calculate the one in period $T-1$. After period $T-1$ and on, for each $t \in \{T-1, .., 2\}$, I use Monte Carlo integration over the distribution of the disturbance in period t ($\widetilde{\epsilon}_t^l = (\widetilde{\epsilon}_t^{ls}, \widetilde{\epsilon}_t^{lw})$ and $\widetilde{\epsilon}_t^y = (\widetilde{\epsilon}_t^{ys}, \widetilde{\epsilon}_t^{yw})$) for a randomly selected subset of S_t to obtain the approximated expected value of the maximum of the alternative-specific value functions at those state points,$Emax_{t-1}$.[38] This procedure continues to decision period 2, where the

[33] In determining the decision in period t, he observes initial shocks ϵ_t, and he uses this information. However, because of the independence between ϵ_t and ϵ_{t+1}, this information does not affect $Emax_t$.

[34] Note that there is no need to take expectations over the next period's shocks in the last period.

[35] I use the second degree polynomial, including all interactions between the state variables. The variables ϵ_t^{ls}, ϵ_t^{lw}, ϵ_t^{ys} and ϵ_t^{yw} do not have to be incorporated in Emax calculation because of their serial uncorrelation.

[36] Notice that age_t takes only one particular value given $\underline{age}$ and t, so it cannot be a component of the randomly chosen subset.

[37] This function has a parametric form and its parameters are the target of estimation. The actual parametric form is given in Appendix A.7.

[38] I use 1500 state points and 49 (22 if $t = 2$) variables for the approximations of the $Emax_t$ functions. The number of random draws for Monte Carlo integration is 30. The goodness of fit is assessed by the adjusted coefficients of determination: with the estimated parameter values they range from 99.84 to 99.98.

interpolated $Emax_1$ is calculated.

To computationally implement the above procedure, I need to specify parametric functional and model distributional assumptions. Appendix A shows the exact functional forms. I now turn attention to the data that is used for estimation.

5 Data

The data for estimation of the life-cycle model is constructed from the 1979-2000 waves of the 1979 youth cohort of the National Longitudinal Survey of Youth (NLSY79). Conducted every year for 1979 to 1993 and once two years for 1994-2004, the NLSY79 contains a nationally representative sample of 12,686 individuals (with 6,403 of them being males) who were 14-21 years old as of January 1, 1979. It contains a core random sample and oversamples of blacks, Hispanics, economically disadvantaged non-black/non-Hispanics, and members of the military. As of the 2000 interview round, all the individuals became 35-43 years old. In this study, I use the white male part in the core random sample.[39] This reduces the initial sample size 12,686 to 2,439. The further restriction on the dimension of individuals is explained in the following subsection.[40]

5.1 Data Construction

I first exclude individuals who have military experience (268 individuals) and then those who are judged to be professionals or farmers (102 individuals). Both professionals and farmers characterized by high rates of self-employment. Why I exclude these people is that the workings of labor markets for them may be quite different from those for nonprofessional, nonfarmers, and hence the decision to become a farm or professional self-employer may depend on different factors than the decision to become a nonfarm, nonprofessional self-employer. I then follow each white man of these 2,068 individuals after the (calender) year when he is considered to have finished schooling, no matter how long it takes for him to finish it. I drop, however, those who are judged to have started working too late (i.e. 26 years old or older) or too early (i.e. 14 years old). The total number of these people is 82. Also excluded are those who are judged to have temporally left for adult schooling in the midst of their work career (24 individuals). Some individuals have to be excluded if it is difficult to determine the first decision period, or if no survey years are covered when working (47 individuals). All money values in this paper are expressed in 2000 dollars. My final sample consists of 1,916 white males with a total of 32,166 person-year observations (which is an unbalanced panel).

Let the constructed data be denoted by $X = \{X_i\}_{i=1}^{N}$, where N is the number of individuals in the data and X_i is data for individual i. Using the notation in the dynamic model presented in Section 3, I can write the actual form of X_i as

$$X_i = \{(l_{i,t}^s, l_{i,t}^w),\ (y_{i,t}^s, w_{i,t}),\ a_{i,t+1}, age_{i,t})_{t=1}^{\widehat{T}_i},\ a_{i1},\ educ_i\},$$

where $\widehat{T}_i$ is the last period when individual i's information is available (note the difference between $\widehat{T}_i$ and T_i), $age_{i,1}$ is actually equal to $\underline{age_i}$ (so that both variables will be used inter-

[39]Future research would include studying issues of self-employment among non-whites (racial discrimination) and women (fertility and child rearing).

[40]Appedix B describes the details on the sample inclusion criteria and on how variables in my data are created.

changeably), and N is the number of individuals in the sample. Note also that consumption can be calculated up to period $\widehat{T}_i - 1$. So, essentially, I do not use data observations in period $\widehat{T}_i$. For the schooling variable $educ_i$, I consider only two categories, "High-school dropouts or graduates (H)" (called *non-college educated* hereafter), and "Some college degree or higher (C)" (called *college educated* hereafter), mainly because of the small numbers of the self-employment experienced. While $educ_i$ and age_i are observed for any i, the amount of initial asset $a_{i1}$is not necessarily observable for all i's.[41] The hourly wage, $w_{i,t}$, is observed constructed only when individual i worked as a paid worker. Similarly, $y^s_{i,t}$ is observed only when individual i worked as a self-employer in period t.[42]

The NLSY79 has detailed information on the self-employed themselves, but very limited information on the businesses they run.[43] This limited information on the financial side of self-employment, however, would not be too restrictive because modeling that part is kept to minimum in this study. We also no information on how many workers each self-employer employs in his firm. Remember, however, that I have assumed that production contribution by the self-employed is separable from that by his employees, so this data limitation is not restrictive to this study, either.

5.2 Descriptive Statistics

In this subsection, I explain key descriptive statistics of the constructed sample X.

5.2.1 Initial Conditions (age_i, $educ_i$ and a_{i1}) and Information on Individual-Period Observations in the Pooled Data

Panel 1 in Table 3 shows the initial conditions of the sample individuals. First, remember that the earliest age for decisions is set to 20.[44] About 60 percent of the individuals start decisions at age 20, and 94 percent of them start decisions until age 23. Next, as for schooling attainment, 63 percent of the individuals are non-college educated and the remaining individuals are college educated. In the joint distribution of initial age and schooling (not shown), nearly 90 percent of the individuals in the non-college educated group start decisions at age 20, while about 50 percent of the college educated individuals start decisions at age 22

[41] Regarding the risk-free interest rate (r), I first computed for each year from 1979 to 2000 the difference between the nominal annual rate of federal funds and the next year's realized inflation rate (as a substitute for the expected inflation rate). I then impute the yearly average, 3.5%, to r ($r = 0.035$). I also use a constant rate of business capital depreciation (δ, and it is taken as a data input: as in Cagetti and De Nardi (2006), it is set to be 6.0% ($\delta = 0.060$).

[42] I carefully constructed "income from self-employment" in my data to capture the "returns to capital" as precisely as possible. In particular, I compared Income Information from the "Income Section" with from the "Employer Supplement Section" in the NLSY79. The downside of using the "Income Section" is that after 1995 income information is obtained once in every two years, which reduced the number of observed income. However, by comparing labor earnings calculated from the "Employer Supplement Section" with total income (the sum of wage/salary income and business income) calculated from the "Income Section", I found, for income from self-employment, the former appears to have downward bias especially for higher percentiles, while for income from paid-employment, both are surprisingly similar. So, I use the "Income Section" to calculate income from self-employment while the "Employer Supplement Section" is used to calculate income from paid-employment. See Appendix B for the details.

[43] Currently, at the US Census Bureau, effort are undertaken to integrate business and household data (the Longitudinal Employer-Household Dynamics (LEHD) program) and employer-employee data (the Integrated Lonitudinal Business Database (ILBD)). See Davis, Haltiwanger, Jarmin, Krizan, Miranda, Nucci, and Sandusky (2007) for details.

[44] Note also that the ealiest age when information on asset is available is age 20.

Table 3: Summary Statistics (NLSY79; White Males; Aged 20-39)

Variable	No.Obs.	Mean	Std.Dev.	Median	Max	Min
(Panel 1. Initial conditions)						
Age at the first decision period (%)						
20	1916	0.627	-	-	1	0
21	1916	0.114	-	-	1	0
22	1916	0.112	-	-	1	0
23	1916	0.083	-	-	1	0
24	1916	0.042	-	-	1	0
25	1916	0.022	-	-	1	0
Educational attainment (%)						
Non college-educated	1916	62.58	-	-	1	0
College-educated	1916	37.42	-	-	1	0
Net worth at the initial age of decisions	387	11940.8	39499.7	3952	576000	-38308
(Panel 2. Pooled)						
Age	32166	29.03	5.29	29	20	39
Labor supply (%)						
Self-employed	31494	0.069	-	-	1	0
Paid-employed, full-time	31494	0.705	-	-	1	0
Paid-employed, part-time	31494	0.099	-	-	1	0
Non-employed	31494	0.101	-	-	1	0
SE & full-time PE	31494	0.013	-	-	1	0
SE & part-time PE	31494	0.012	-	-	1	0
Experience of paid-employed work (years)	32166	6.27	4.65	6	19	0
Annual Income from self-employment	2079	51350.8	56971.4	36900	884801	64
Annual Income from paid-employment						
Full-time	22295	32932.4	23900.2	28560	990057	184
Part-time	3417	14256.8	10885.6	11900	228000	143
Net worth	17169	57312.4	118703.6	20008	2673988	-72600

Note 1: "Non college-educated" individuals are highschool dropouts and highschool graduates, and "College-educated" are individuals with some college education and more.
Note 2: "Years of paid-employed work experience" counts 1-year experience if an individual works as a full-time as a full-time wage worker, and 0.5-year experience if he works as a part-time wage worker.
Note 3: Monetary values are in terms of year 2000 dollars.

or 23. With respect to net worth that each individual owns at his first age of decisions, the considerable difference between the mean and the median suggest the skewness of the wealth distribution even in early 20s. As is expected, the joint distribution of initial net worth and schooling (not shown), both the mean (13,062 versus 9,505 dollars) and the median (5,495 versus 1840 dollars) are higher for the college educated.

Panel 2 in Table 3 displays information on individual-period observations in the pooled data. The average (and the median) age is 29. As is mentioned in Introduction, of all the observations on labor supply decisions, 7 percent are provided as self-employed work while 80 percent are as either full- or part-time paid-employed work. The average accumulated years of experience as a wage worker is 6.3 (excluding years as a self-employer). The mean income from self-employment (51,351 dollars) is considerably higher (56 percent higher) than that from full-time paid-employment (32,932 dollars). The median difference is much smaller: the median income from self-employment is 29 percent higher than that from full-

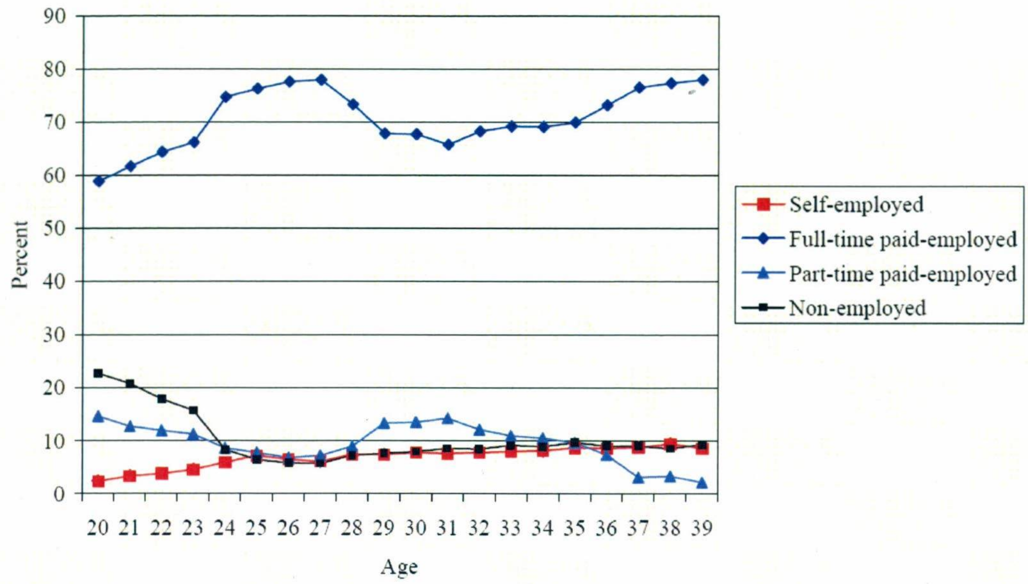

Figure 1: Distribution of Labor Supply Decisions by Age (White Males; NLSY79)

time paid-employment (36,900 versus 28,560 dollars). The mean income from part-time paid employment is 57 percent lower than that from full-time paid-employment. Lastly, the average net worth is 57,312 dollars while the median is 20,008 dollars.

5.2.2 Labor Supply Decisions: Age Profiles ($\{l^s_{i,t}, l^w_{i,t}\}$), Transitions, and Entry into and Exit from Self-Employment

Table 4 and Figure 1 show the marginal distribution of labor supply decisions by age. At age 20, only 2.4 percent of the white men are self-employed. Then, the rate increases rapidly until age 25 (7.3 percent). After that, it remains stable with a slight increase (9.5 percent at age 38). The rates of full-time paid employment are highest and stable over all the ages. Starting with 58.9 percent, the percentage grows to 77.9 percent at age 27. After that age, the number declines slightly (65.7 percent at age 31), and then it grows again. Corresponding to the slight decline in full-time paid employment, the rate of part-time paid-employment starts go up at age 27 after the decline since age 20, reaching 14.3 percent at age 31. Lastly, the percentage of the non-employed decreases rapidly in their early 20s: 22.8 percent at age 20 to 5.8 percent at age 27. Then, after age 28 the rates are stable with a slight increase (between 7.3 and 9.7).

Some key differences of self-employment by schooling have been already presented in Tables 1 and 2. In what follows, we look at details of life-cycle aspects of labor supply decisions. Table 5 shows the percentages of the individuals for the numbers of entries into self-employment. First, we find *self-employment experience is not rare*: 28.3 percent of individuals (543 out of 1916 individuals) have at least one year of self-employment experience. Second, we do not observe too many trials by the same young individual, however: 94.1 percent of them enters *only once or twice* in the data periods. As was already mentioned, the non-college educated is more likely to have self-employment experience than the college educated do.[45] Figure 2 shows an important difference in the timings of first entries into

[45] Remember that my data contains only nonprofessional white males in nonagricultural sectors. Excluded

Table 4: Marginal Distribution of Labor Supply Decisions by Age

Age (No.Obs.)	Self-employed	Paid-employed Full-time	Paid-employed Part-time	Non-employed	Dual-employed SE & Full-time PE	Dual-employed SE & Part-time PE
20 (1172)	2.4% 28	58.8% 589	14.5% 171	22.9% 268	0.9% 10	0.5% 6
21 (1393)	3.4% 47	61.7% 859	12.8% 178	20.8% 290	0.6% 9	0.7% 10
22 (1609)	3.9% 62	64.3% 1035	12.0% 193	17.9% 288	1.1% 18	0.8% 13
23 (1751)	4.6% 81	66.1% 1157	11.3% 197	15.9% 278	1.0% 18	1.1% 20
24 (1820)	6.0% 109	74.7% 1359	8.6% 157	8.4% 153	1.2% 22	1.1% 20
25 (1848)	7.3% 134	76.5% 1409	7.8% 144	6.5% 121	1.2% 23	0.9% 17
26 (1829)	6.6% 120	77.6% 1419	6.8% 125	5.9% 108	2.0% 37	1.1% 20
27 (1826)	6.1% 112	77.9% 1422	7.3% 133	5.8% 106	1.6% 30	1.3% 23
28 (1817)	7.5% 136	73.3% 1331	9.0% 164	7.3% 133	1.8% 32	1.2% 21
29 (1795)	7.5% 134	67.9% 1218	13.3% 239	7.7% 138	1.9% 35	1.7% 31
30 (1780)	7.8% 139	67.7% 1205	13.5% 240	8.0% 143	1.4% 25	1.6% 28
31 (1750)	7.7% 134	65.8% 1151	14.2% 249	8.6% 151	1.8% 31	1.9% 34
32 (1735)	7.8% 136	68.2% 1183	12.2% 211	8.5% 148	1.6% 28	1.7% 29
33 (1708)	8.0% 137	69.1% 1181	11.0% 188	9.3% 158	1.5% 25	1.1% 19
34 (1686)	8.2% 138	69.0% 1164	10.6% 178	9.0% 152	1.5% 25	1.7% 29
35 (1636)	8.7% 143	69.9% 1143	9.7% 159	9.8% 160	1.0% 16	0.9% 15
36 (1446)	8.6% 125	73.1% 1057	7.5% 108	9.1% 132	0.8% 11	0.9% 13
37 (1199)	8.8% 106	76.6% 918	3.2% 38	9.1% 109	1.1% 13	1.3% 15
38 (956)	9.6% 92	77.2% 738	3.5% 33	8.6% 82	0.6% 6	0.5% 5
39 (738)	8.8% 65	77.9% 575	2.2% 16	9.3% 69	0.8% 6	0.9% 7

Note: Percentages and number of observations.

Table 5: Distribution (percent) of the Number of Entries into Self-Employment

Number of entries into self-employment	All individuals	(Non-college)	(College)
0	69.8	68.2	72.5
1	19.5	20.8	17.4
2+	10.7	11.0	10.1
	100.0	100.0	100.0
(No.Obs.)	(1916)	(1199)	(717)

Note: Measured at the last periods observed in the data.

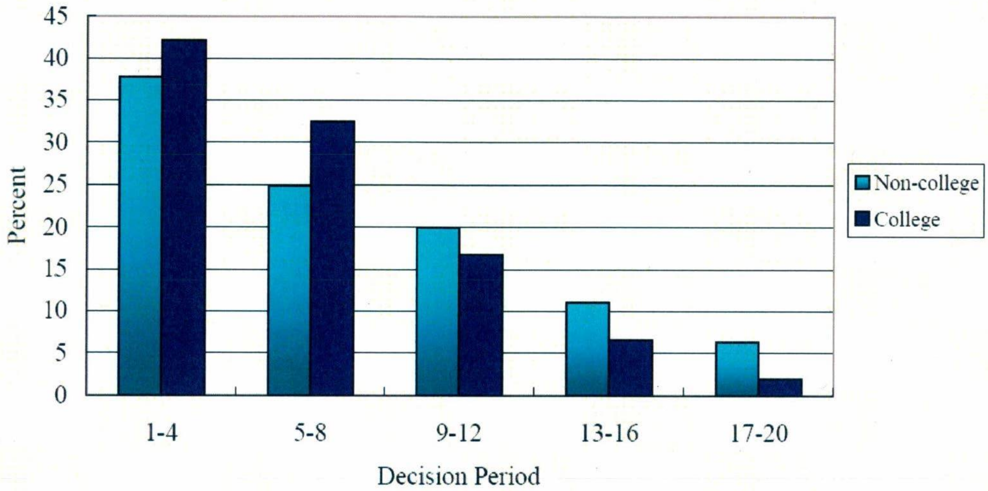

Figure 2: Distribution of Labor Supply Decisions by Age (White Males; NLSY79)

by schooling. Although the means and the medians of the first entries for both types of schooling are quite similar (8.2 (mean) and 7 (median) for the non-college educated, and 8.6 (mean) and 8 (median) for the college educated), the two distributions do not look similar: the highest percentage is attained at decision periods 5-8 for the college educated, while it is attained at decision periods 1-4 for the non-college educated.

The left panel of 6 shows one-period transition rates of labor supply decisions for both schooling levels.[46] The first number in each cell is the percentage of transitions from origin to destination (row %) while the second is the percentage in a particular destination who started from each origin (column %). The table shows *persistence in self-employment and in full-time paid-employment*: 75.4 (73.6) percent of the non-college (college) educated self-employers in one year do self-employment the next year, and 85.4 (89.5) percent of the

are 40 lawers/accountants and 23 doctors. This seems the reason of a low self-employment rate among category "College or higher" because college degree is necessary to be a professional of these kinds. If these 63 individuals are added to the self-employment cell, then the rate of self-employment rate for the college educated will be 31.4% (=(182+63)/(717+63)).

[46] I define year of *entry* into self-employment t by $l^s_{i,t} = SE$ and $l^s_{i,t-1} = Zero$, and define year of *exit* from self-employment t by $l^s_{i,t} = SE$ and $l^s_{i,t+1} = Zero$. The duration of a SE spell is defined by the difference between the exit year and the entry year.

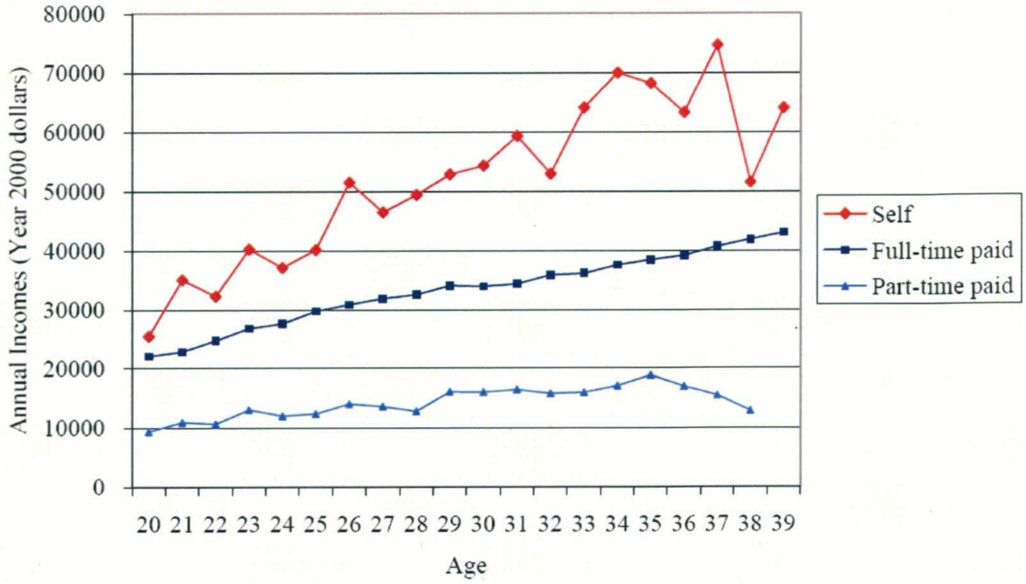

Figure 3: Age Profiles of Mean Annual Incomes

non-college (college) education who worked full-time as a wage worker in one year work as a full-time wage worker the next year. The age pattern of the self-employment and full-time paid employment is also worth attention. The left panel of 7 implies that self-employment in twenties is likely to end earlier than that in thirties: the transition rate of self-employment in twenties is 69 percent while that in thirties is 80 percent.

5.2.3 Age Profiles of Income ($\{y^s_{i,t}, y^w_{i,t}\}$) and of Net Worth ($\{a_{i,t}\}$)

As is already seen in Table 2, the self-employed earn more, on average, than the paid-employed do, and across the two groups of education levels, income from self-employment is higher than income from paid-employment (both for the mean and for the median). Table 8 and Figure 3 display age-specific mean real incomes from self-employment, from full-time and from part-time paid-employment. Real incomes rise with age in all the three modes of employment. The percentage difference between income and income from full-time paid-employment at early twenties is about 40 to 50 percent. It grows with age: at late thirties it becomes about 60 to 70 percent.

Table 9 and Figure 4 show the age profile of the mean and median net assets of all individuals. As is seen, the mean grows faster then the median does, and as a result, he mean net worth at late thirties is about 14 times larger than that at early twenties. The wealth distribution is thus more skewed in later ages.

6 Estimation Method

Using data X that is explained in the previous section, I estimate the parameters of the life-cycle model of employment mode decisions and wealth accumulation. Now, given the approximated values for $Emax_t$, it is possible to simulate individual behavior from the first decision period (one year after he finished schooling) to age 65, with an arbitrary pair of model parameters. I simulate individual choices (choice on labor and asset) and income

Table 6: Transition Matrices for Labor Supply Decisions (aged 20-39) by Schooling

Actual
Non-college

Labor supply (t-1)		Labor supply (t) SE	PE Full-time	PE Part-time	NE
SE					
	Row %	75.4	7.2	2.6	6.2
	Column %	91.4	4.1	4.8	7.9
PE, Full-time					
	Row %	1.0	85.4	7.5	4.5
	Column %	1.2	49.2	14.1	5.7
PE, Part-time					
	Row %	1.7	51.1	28.0	18.4
	Column %	2.0	29.4	52.2	23.3
NE					
	Row %	4.4	29.9	15.5	49.7
	Column %	5.4	17.2	28.9	63.1

Predicted
Non-college

Labor supply (t-1)		Labor supply (t) SE	PE Full-time	PE Part-time	NE
SE					
	Row %	83.0	10.3	0.9	5.8
	Column %	91.1	6.5	1.1	8.3
PE, Full-time					
	Row %	4.1	81.9	9.0	4.9
	Column %	4.5	52.3	11.0	7.1
PE, Part-time					
	Row %	2.2	49.0	43.3	5.5
	Column %	2.4	31.3	52.6	7.9
NE					
	Row %	1.8	15.5	29.0	53.7
	Column %	2.0	9.9	35.3	76.7

Actual
College

Labor supply (t-1)		Labor supply (t) SE	PE Full-time	PE Part-time	NE
SE					
	Row %	73.6	8.4	3.3	4.2
	Column %	88.8	4.2	6.3	7.6
PE, Full-time					
	Row %	0.9	89.5	5.9	2.2
	Column %	1.1	45.5	11.3	4.0
PE, Part-time					
	Row %	2.9	56.0	27.8	12.5
	Column %	3.5	28.5	53.5	22.9
NE					
	Row %	5.5	42.7	15.0	35.7
	Column %	6.6	21.7	28.9	65.5

Predicted
College

Labor supply (t-1)		Labor supply (t) SE	PE Full-time	PE Part-time	NE
SE					
	Row %	85.8	8.7	1.1	4.3
	Column %	89.7	4.3	3.2	6.5
PE, Full-time					
	Row %	4.3	91.6	2.8	1.2
	Column %	4.5	45.4	7.9	1.9
PE, Part-time					
	Row %	4.1	69.7	19.2	7.0
	Column %	4.3	34.5	54.0	10.5
NE					
	Row %	1.4	31.9	12.4	54.3
	Column %	1.5	15.8	34.9	81.1

Note 1: NE stands for non-employed, SE for self-employed, and PE for paid-employed.
Note 2: Rows labeled by "Row" contain the distribution of destinations (period t+1) conditional on origin (period t). Rows labeled by "Column" contain the distribution of origins conditional on destination. conditional on destination.
Note 3: "SE and full-time PE" and 'SE and part-time PS" are omitted, so the sums of the numbers across row or column are not 100.0.

Table 7: Transition Matrices for Labor Supply Decisions by Age Group

Actual Aged 20-29

Labor supply (t-1)	SE	PE Full-time	PE Part-time	NE
SE				
Row %	69.8	10.7	3.5	5.3
Column %	89.8	5.8	6.3	7.8
PE, Full-time				
Row %	1.3	85.5	7.1	4.5
Column %	1.7	46.0	12.9	6.6
PE, Part-time				
Row %	2.5	52.0	26.8	17.9
Column %	3.2	27.9	48.9	26.5
NE				
Row %	4.2	37.9	17.4	40.0
Column %	5.4	20.4	31.8	59.1

Predicted Aged 20-29

Labor supply (t-1)	SE	PE Full-time	PE Part-time	NE
SE				
Row %	87.3	9.4	2.3	1.1
Column %	94.3	6.0	2.6	1.6
PE, Full-time				
Row %	2.5	77.4	13.4	6.7
Column %	2.7	50.0	15.4	10.2
PE, Part-time				
Row %	2.1	50.4	47.1	0.4
Column %	2.3	32.6	54.1	0.6
NE				
Row %	0.7	17.6	24.4	57.4
Column %	0.7	11.4	28.0	87.6

Actual Aged 30-39

Labor supply (t-1)	SE	PE Full-time	PE Part-time	NE
SE				
Row %	79.4	4.7	2.2	5.8
Column %	91.3	2.8	4.4	7.0
PE, Full-time				
Row %	0.6	88.9	6.7	2.5
Column %	0.6	52.2	13.4	3.1
PE, Part-time				
Row %	1.6	53.4	29.3	14.9
Column %	1.8	31.4	58.3	18.3
NE				
Row %	5.4	23.4	12.0	58.5
Column %	6.3	13.7	23.9	71.6

Predicted Aged 30-39

Labor supply (t-1)	SE	PE Full-time	PE Part-time	NE
SE				
Row %	83.8	9.7	0.1	6.4
Column %	82.2	3.9	0.4	32.3
PE, Full-time				
Row %	5.6	92.3	1.1	1.0
Column %	5.5	36.7	4.2	4.9
PE, Part-time				
Row %	6.4	75.6	16.6	1.5
Column %	6.3	75.6	61.4	7.5
NE				
Row %	6.2	73.8	9.2	10.9
Column %	6.0	29.4	34.0	55.3

Table 8: Age Profiles of Mean Incomes by Labor Supply Decisions

	Self-employed			Paid-employed					
				Full-time			Part-time		
Age		(std.dev.)	(no.obs.)		(std.dev.)	(no.obs.)		(std.dev.)	(no.obs.)
20	25479.4	3346.0	38	22086.9	336.1	694	9302.1	354.7	171
21	35048.4	5636.7	55	22819.5	316.8	863	10891.8	417.9	183
22	32257.9	3008.8	78	24716.4	325.3	1044	10601.5	366.4	200
23	40238.2	3843.4	105	26791.4	582.8	1164	13016.0	1313.4	215
24	37131.3	2607.6	129	27602.3	355.0	1363	11992.7	1043.6	176
25	40122.7	2776.5	153	29703.4	624.2	1415	12313.6	683.4	158
26	51498.5	7828.1	155	30816.8	683.0	1435	14001.5	875.2	142
27	46444.7	3553.9	140	31865.1	779.3	1433	13582.5	964.9	151
28	49377.0	4034.2	163	32530.4	578.2	1344	12751.8	465.4	182
29	52852.2	4196.9	166	34036.2	1015.2	1230	16071.5	704.7	266
30	54313.3	8016.5	142	33911.9	780.0	1206	16020.7	591.6	265
31	59283.8	6854.9	141	34311.0	584.3	1160	16416.3	795.8	275
32	52942.5	6308.6	134	35868.1	671.1	1183	15775.6	622.7	234
33	64065.8	6837.5	100	36102.0	584.0	1180	15953.5	846.3	202
34	69952.1	7038.6	113	37554.4	716.2	1161	17069.6	903.0	202
35	68146.2	11395.2	71	38395.4	694.1	1143	18845.6	879.1	165
36	63237.1	7187.5	69	39146.6	714.6	1060	16963.7	1038.1	119
37	74635.8	11327.1	52	40691.1	864.4	917	15458.6	2032.3	51
38	51499.3	5752.5	38	41868.1	987.1	730	12833.2	1144.4	38
39	64036.2	7761.4	37	42990.4	1315.7	570	-	-	-

Note 1: Numbers are in year 2000 dollars.
Note 2: Mean part-time income for age 39 is not shown
because the number of observations is small (22).

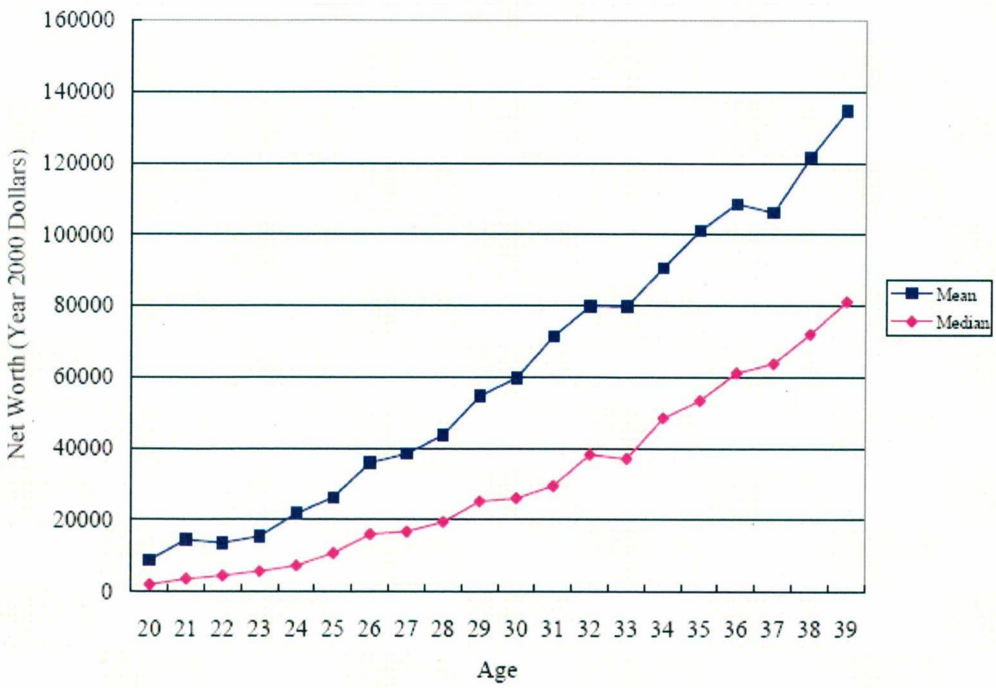

Figure 4: Actual Mean and Median of Net Worth

Table 9: Age Profile of Net Worth

Age (No.Obs.)	Mean	Median	25%	75%	Min	Max	Percent Negative
20 (138)	8829.1	2080	160	6840	-8480	184160	20.3
21 (311)	14437.0	3611	628	11200	-21038	576000	19.9
22 (588)	13556.3	4560	785	13860	-48165	769302	19.6
23 (834)	15453.4	5768	970	18240	-52560	364802	19.2
24 (1078)	21731.9	7300	1274	22338	-56836	814722	17.9
25 (1259)	26170.6	10640	2376	28576	-72336	679442	14.8
26 (1255)	35819.5	15985	3562	39672	-52820	742922	13.5
27 (1360)	38413.3	16800	3208	43916	-71438	912915	14.4
28 (1312)	43644.7	19382	4049	51778	-63510	1128144	13.0
29 (1327)	54481.7	25258	4620	61612	-60800	1905040	11.7
30 (1126)	59657.9	26101	5587	70702	-59796	1136710	12.1
31 (1123)	71103.3	29304	6838	73780	-59860	2594100	10.4
32 (953)	79519.8	38080	6991	87115	-56100	2231799	10.5
33 (925)	79702.4	37120	6100	93432	-59003	2579427	12.1
34 (836)	90289.1	48452	10681	111863	-69020	1217687	9.0
35 (842)	100622.3	52910	13807	118985	-63800	2023543	10.2
36 (651)	108198.7	60900	14160	141541	-49000	2673988	8.1
37 (516)	105425.2	62948	14329	143285	-56500	1421460	11.4
38 (436)	121347.6	71525	23723	160490	-55120	796098	6.7
39 (297)	134295.6	81000	20465	188400	-48760	816200	9.8

opportunities (including ones not chosen by the individual). The level of education and the initial amounts of net worth are taken as exogenous. At the initial decision period, any individual has no experience both for self-employment and for paid-employment.

Conditional on the deterministic part of the state space $\overline{S}_t$, the solution of the dynamic programming problem gives the conditional probability that an individual chooses option d, as the product of the type probabilities and a five-dimensional integral over the vector of shocks so that choice d is indeed optimal. If all variables in the state space were observed, then the conditional likelihood could be constructed as the the product, over time and individuals, of these probabilities.

However, a serious problem is that endogenous state variables in S_t are not always observed. In particular, as explained in Section 5, the NLSY79 started collecting information on asset in 1985, and since 1994 it has been collecting the asset information biannually.[47] Calculating the conditional choice probabilities would require one to integrate out all possible choices over the distribution of the unobserved elements. This would, however, be comutationally burdensome. I therefore adopt the method of *simulated maximum likelihood* developed by Keane and Wolpin (2001).[48] Notably, this method allows one to avoid computing the conditional probabilities, and only unconditional probabilities are used in estimation. The idea is that all observed outcomes are measured with error and model parameters are so chosen that the "distance" between simulated ("true") and observed outcomes is minimized.[49,50]

Specifically, I first fix a trial vector of parameters $\theta \in \Theta$ and $type = 1,..,4$. In each sim-th simulation ($sim = 1,...,M$), a period-by-period random shock ϵ_t^{sim} is generated for each decision period t. As a solution of the dynamic choice problem, starting with the initial level of asset $\widetilde{a}_1^{sim}$ (see below), for each permanent state (except *type*) (*educ*, $\underline{age}$), I generate outcome histories of (i) choice realizations $\{(\widetilde{l^s}_{e,t}^{sim}, \widetilde{l^w}_{e,t}^{sim}), \widetilde{\Delta a}_{e,t+1}^{sim}\}_{t=1}^T$, and (ii) the resulting realizations of income $(\widetilde{y^s}_{e,t}^{sim}, \widetilde{y^w}_{e,t}^{sim})$, and asset realizations for the next period $\widetilde{a}_{e,t+1}^{sim}$.[51] I denote the sim-th simulated data (outcome history) for individual i in case his type is *type* by

$$\begin{aligned}\widetilde{X}_{i,type}^{sim} = (\{(\widetilde{l^s}_{i,type,t}^{sim}, \widetilde{l^w}_{type,t}^{sim}), (\widetilde{y^s}_{i,type,t}^{sim}, \widetilde{y^w}_{i,type,t}^{sim}), \widetilde{a}_{i,type,t+1}^{sim}, \widetilde{k}_{i,type,t}^{sim}, age_{i,type.t}^{sim}\}_{t=1}^T,\\ \widetilde{a}_{i,1}^{sim}, educ_i),\end{aligned}$$

[47]In addition, other endogenous variables (labor choice and income) are sometimes missing.

[48]See Keane and Sauer (2007) for technical issues of this method. In particular, they argue that the method is not only computationally practical but has good small sample properties. For an application of the method, see e.g. Keane and Sauer (2009).

[49]A byproduct of this method is that one does not have to discretize all continuous outcome variables. In this study, I do not have to discretize values for income. This is because in the presense of (normally distributed) measurement error any observed outcome history is able to be generated by any simulated outcome history with a nonzero probability.

[50]In constructing the log liklihood function, Rendon (2006) focuses only on the path of state variables afer the year 1985 (when collection on asset information started). In particular, his log likelihood function is constructed conditional on the observation in the year 1985. After obtaining the behavioral parameters, Rendon (2006) goes on to recover the initial asset distribution by using the data from the initial decision period to the year 1985. The way he does so is to update the uniform prior on initial assets by conditional on subsequent behavior.

[51]Notice here that the model components that have no counterpart in the actual data, realized income *opportunities* for the current period $\{\widetilde{f}_{e,t}^m, \widetilde{w}_{e,t}^m\}_{t=1}^T$ and the level of human capital in the next period $\widetilde{\Psi}_{e,t+1}^m$, are also generated by simulation.

where $age^{sim}_{i,type,t}$ is actually independent of sim or $tupe$ (determined by $\underline{age}$ and t). I assume that $educ_i$ and $\underline{age}_i$ (and hence $age_{i,t}$) are observed without error for any individual i.

Now, let the probability of the observed history of individual i conditional on the simulated history be $\Pr(X_i|\widetilde{X}^{sim}_{i,type})$.[52] The novel feature of the estimation method used in the present study is that the calculation of $\Pr(X_i|\widetilde{X}^{sim}_{i,type})$ does not depend on the state variables at any decision period t. This property enables me to construct the (unconditional) likelihood from the distributions of the measurement and classification errors (and the assumption that each error is independently distributed over individuals and time).

Specifically, I first obtain, by simulating M outcome histories, the unbiased simulator of the probability of X_i

$$\widehat{\Pr}(X_i|\theta) = \frac{1}{2M}\sum_{type=1}^{2}\sum_{sim=1}^{M}\Pr(X_i|\widetilde{X}^{sim}_{i,type})\frac{\Pr(type)}{M},$$

where $\Pr(type)/M$ is interpreted as the proportion of individuals with $type$ in all the simulated histories. The log likelihood is then given by

$$\log\mathcal{L}(\theta|\{X_i\}_{i=1}^{N}) = \sum_{i=1}^{N}\log(\widehat{\Pr}(X_i|\theta))$$

and the estimate for θ is so chosen that it maximizes the log likelihood.[53] Appendix C offers the actual functional form of $\log\mathcal{L}(\theta|\{X_i\}_{i=1}^{N})$. In the current implementation, I choose $M = 5N = 9580$. Standard errors are calculated using the outer product of numerical first derivatives.

7 Estimation Results

In this section, I discuss the fit of the estimated model to the key empirical moments as well as the interpretation of the estimated parameters.

7.1 Model Fit

To evaluate the fit of the estimated model, I artificially generated 9580 (5 times 1916) individual life-cycle paths to age 50 for each age of the first decision period (ages 20-26) using the estimated parameters.[54]

Table 10 compares the three key statistics about entry into and exit from self-employment in the actual data with those in the simulated data (the left part is a reproduction of Table 1). All the three characteristics are underpredicted both for the non-college and the college educated. In particular, entries into self-employment take place in later ages in the simulated

[52] With the notation here, what is explained in Footnote 46 is now stated that for an arbitrary $\widetilde{X}^m_e$, $\Pr(X_i|\widetilde{X}^m_e) > 0$ for any X_i thanks to (adequately modeled) classification and measurement errors.

[53] While some model parameters have their own structural relationships, thus are possible to be estimated independently from the other part of the model structure (e.g. the relationship between observed income and modeled income opportunities), the entire set pf model parameters enters the likelihood through the choice probabilities that are computed from the solution of the dynamic programming problem. Thus, I estimate all the parameters by maximixing the log likelihood function of probablities of outcome histories.

[54] In obtaining any information, simulated data for each invidual is used up to his last period that was covered.

Table 10: Three Characteristics on Entry into and Exit from Self-Employment: Actual and Predicted

	Actual		Predicted	
	Non-college educated	College educated	Non-college educated	College educated
Ever experience of self-employment (%)	31.78	27.48	26.76	24.43
First entry into self-employment occurs in less than or equal to first eight decision years (%)	62.72	74.62	59.34	69.20
Exit from self-employment in a year (%)	32.28	28.57	28.09	23.61

data than in the actual data. It, however, seems to well capture the differences by schooling on entry into and exit from self-employment.

Figures 5-8 compare simulated age profiles of labor supply decisions (self-employed, full-time paid-employed, part-time paid-employed and non-employed) with actual ones. The model does a good job in replicating the age pattern of self-employment: the rate of the self-employed increases until age 25 and then it becomes moderately stable in the remaining ages. As for the other modes of employment, the simulated profiles resemble the actual profiles reasonably well, except few ages around early thirties. In the right panel of Table 1, the predicted one-period transition matrix is presented. The diagonal four transition rates of staying in the same mode of employment are reasonably replicated, though the one for part-time paid employment for the non-college educated and the one for non-employment for the college educated seem relatively overpredicted. The observation that the percentage from full-time paid-employment to self-employment is lower than that from non-employment is not well captured by the model. The right panel of Table 7 display the two transition matrices that correspond to age group 20-29 and to age group. The predicted numbers well capture the stronger persistence of self-employment for ages 30-39, though they show that the estimated model is less successful in explaining the transitions around part-time paid employment and non-employment.

Figures 9-14 display age profiles of annual income for each mode of employment. The model does a good job in replicating the age patterns of income as well. Figure 15 shows the age profile of the mean net worth. The model well captures the growth of the mean by age, though it is under predicted for most of ages, and is also less successful in replicating the skewness of the wealth distribution.

Overall, the estimated model reasonably fits the main features of the actual data, though there are some discrepancies between the empirical observations and the model predictions. More improvement is expected in future work.

7.2 Parameter Estimates

A full list of the model's estimated parameters is given in Appendix D. Here I discuss main characteristics of the estimates.

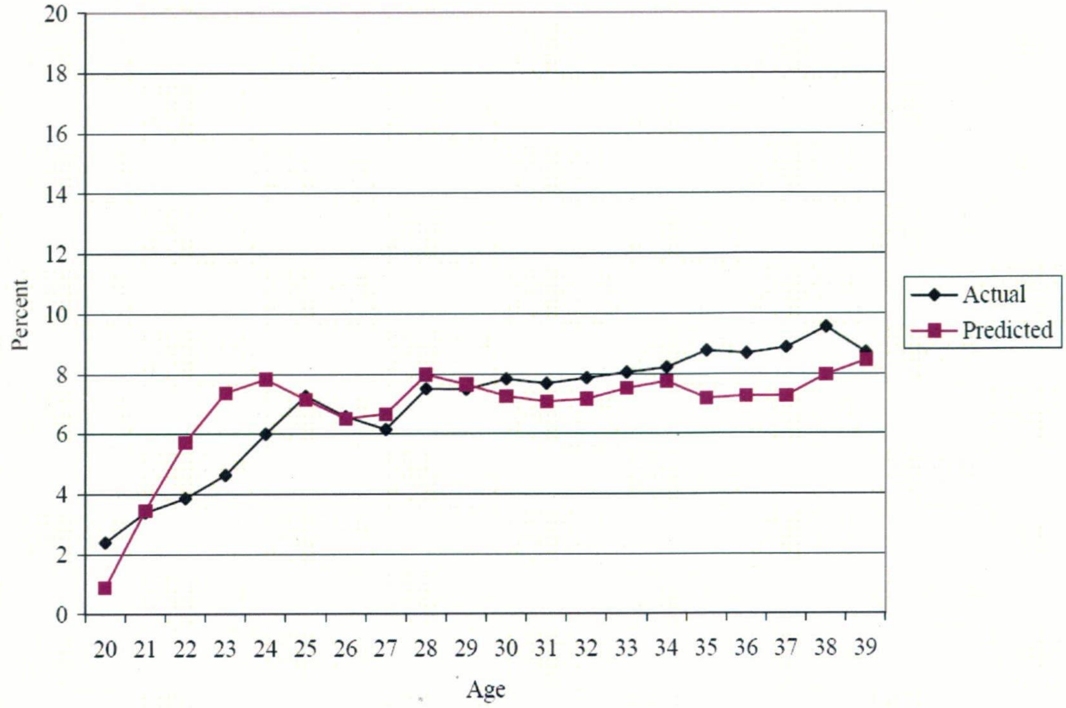

Figure 5: Age Profiles of the Self-Employed

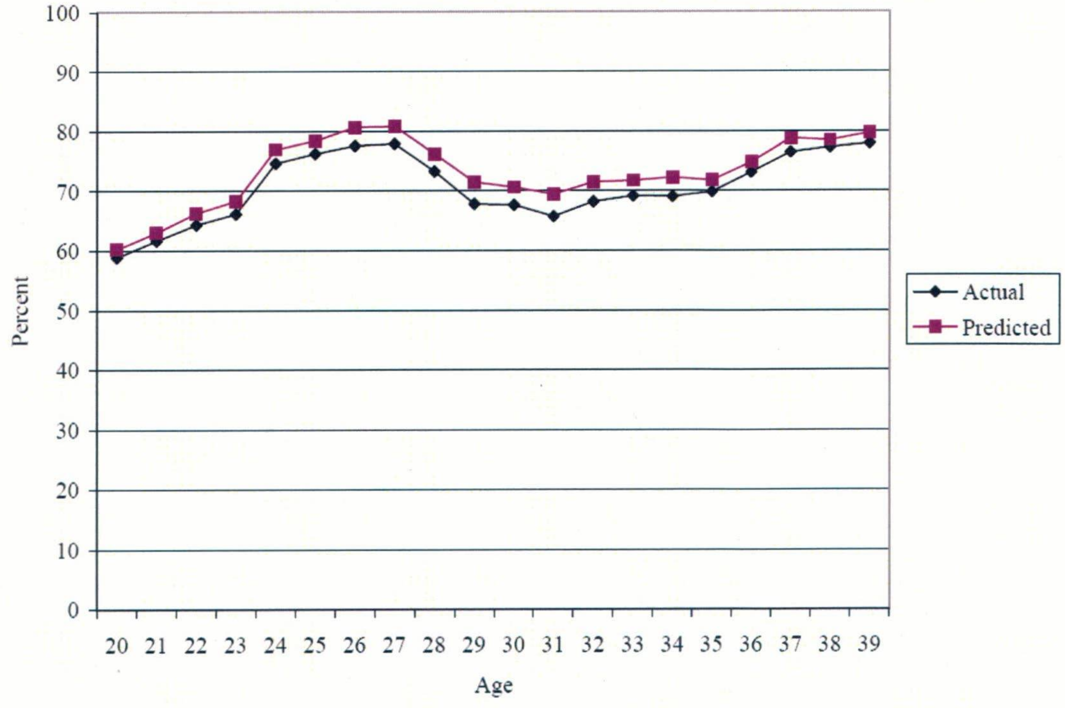

Figure 6: Age Profiles of the Full-Time Paid-Employed

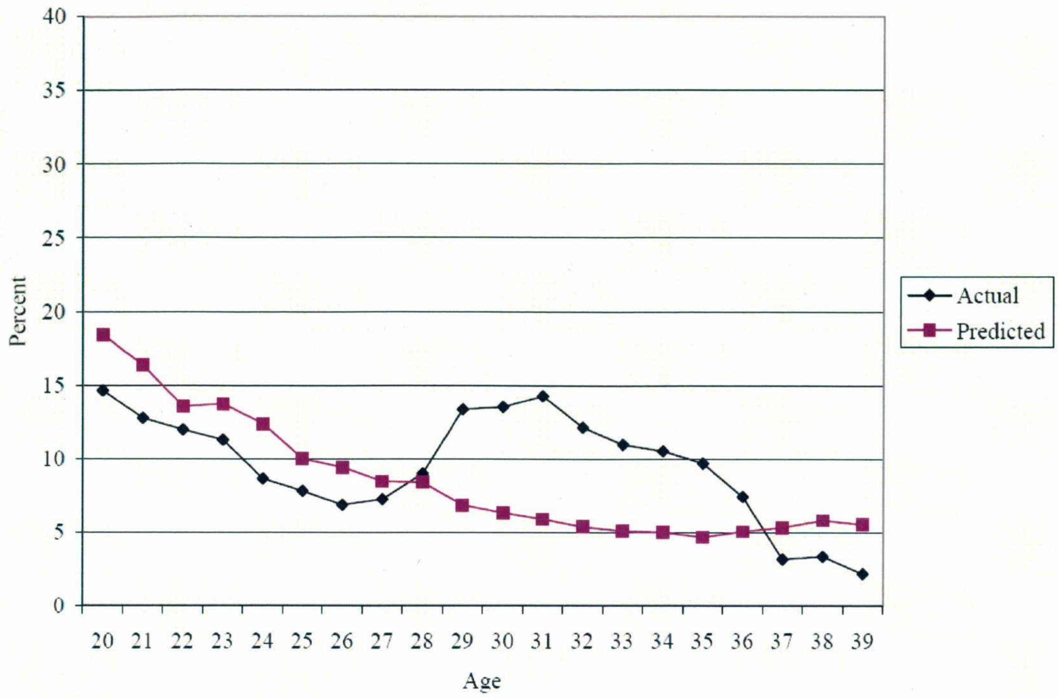

Figure 7: Age Profiles of the Full-Time Paid-Employed

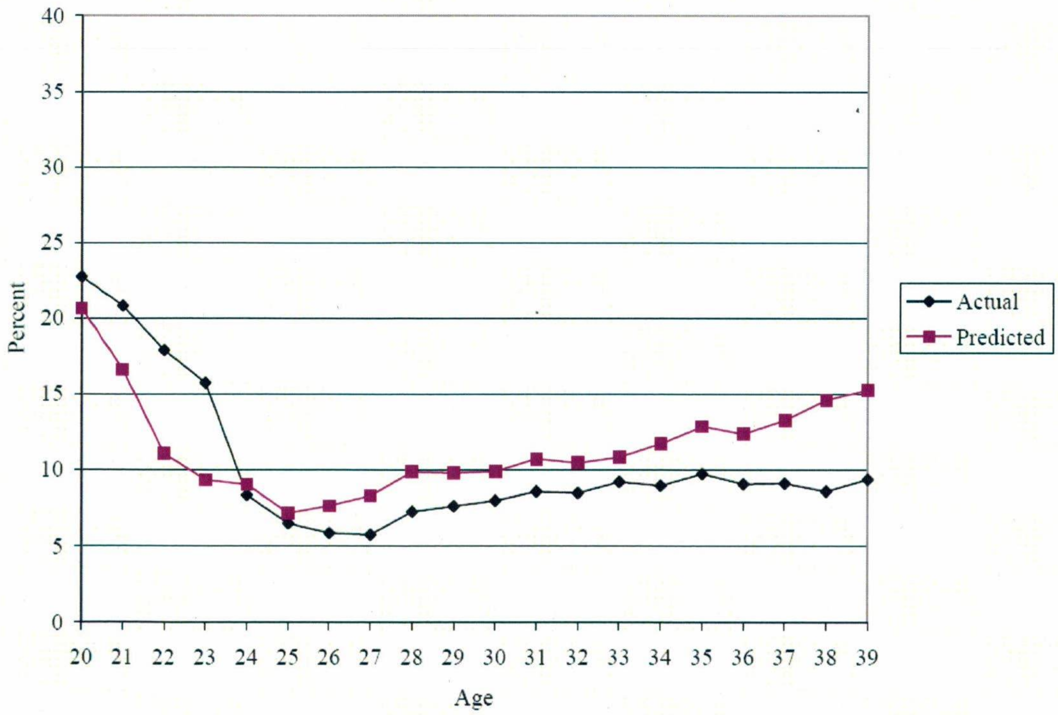

Figure 8: Age Profiles of the Non-Employed

Figure 9: Age Profiles of the Mean Income from Self-Employment (Non-college)

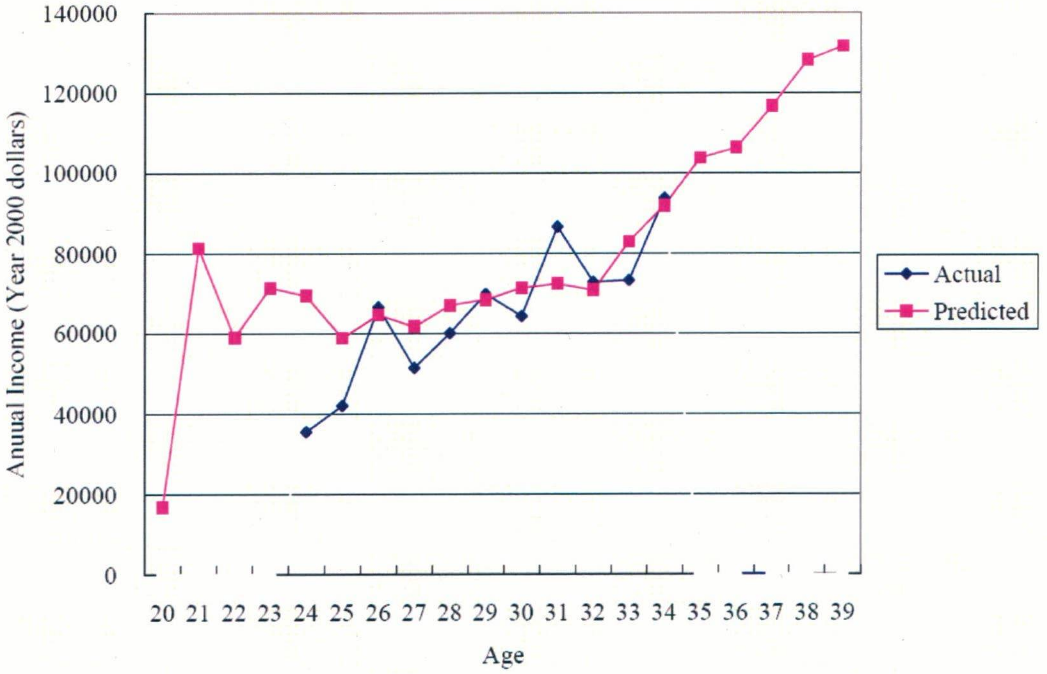

Figure 10: Age Profiles of the Mean Income from Self-Employment (College)

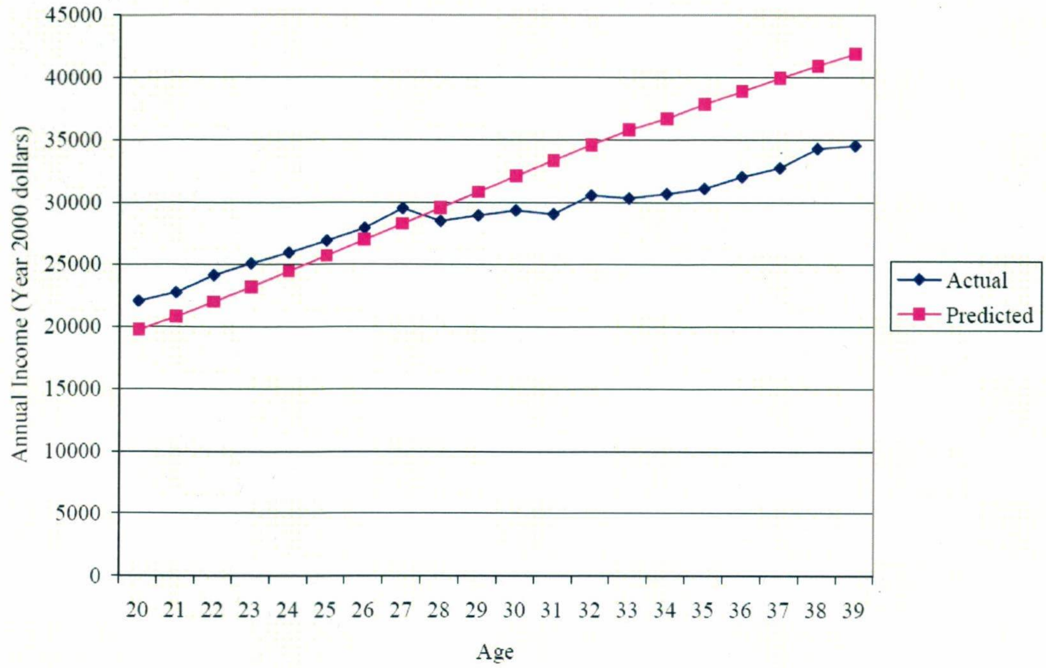

Figure 11: Age Profiles of the Mean Income from Full-time Paid-Employment (Non-college)

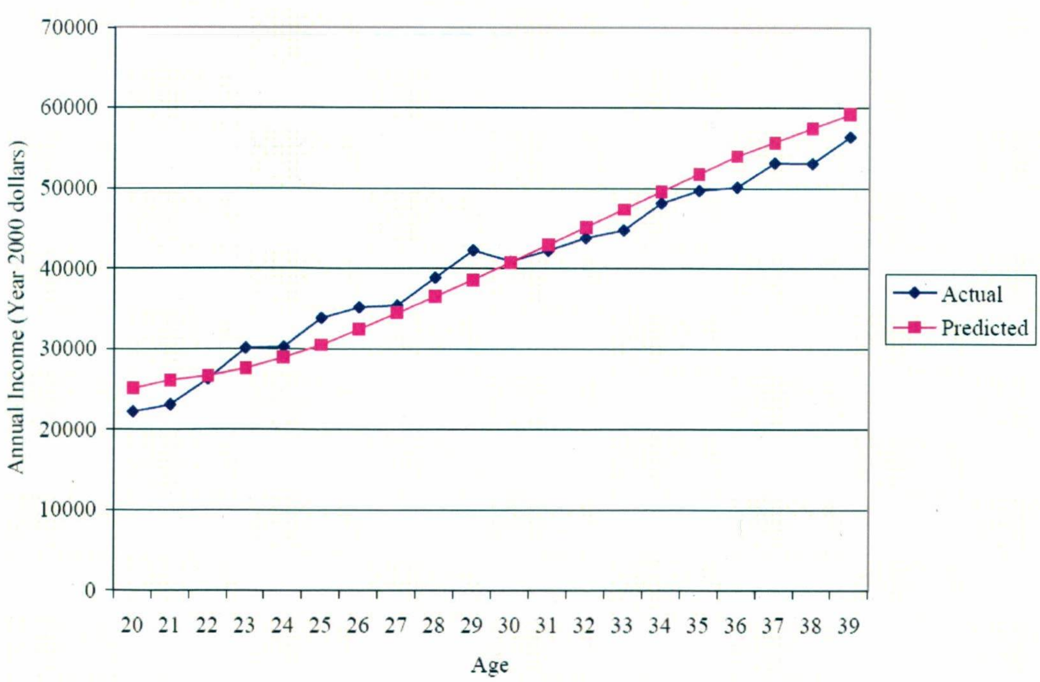

Figure 12: Age Profiles of the Mean Income from Full-time Paid-Employment (College)