

clearing condition,

$$L_s^D(\xi) = L_s^S(\xi).$$

The unskilled labor market has cleared by Walrus Law.

### 3.2 Comparative statics

We first see how the equilibrium labor supply and relative wage change with quality.

**Claim 3.** (i)  $L_s^S$ ,  $L_u^S$  and  $L_s^D$  are increasing and  $L_u^D$  are decreasing in  $Q$ .

(ii) Equilibrium relative wages and level of skilled labor are increasing in quality. That is,  $\partial\xi^*(Q)/\partial Q > 0$  and  $\partial L_s^*(Q)/\partial Q > 0$ .

(See Figures 6 and 7. Proof is in the Appendix.) Higher quality makes consumption attractive for skilled workers and also increase proportion of all workers that consume the high quality product. Thus both demand and supply of skilled labor is increasing in quality. The same effect increases the supply of unskilled workers and reduces demand for low quality good. The latter effect implies demand for unskilled workers decreases when quality improves.

Skilled labor supply is increasing in population,  $\partial L_s^S/\partial N > 0$ , from (12) and demand is also increasing in population,  $\partial L_s^D/\partial N > 0$ , from (10). (See proof of Claim 3 in the Appendix.) This implies

**Claim 4.** Both equilibrium skilled and unskilled labor will increase when population increases,  $\partial L_s^*/\partial N > 0$  and  $\partial L_u^*/\partial N > 0$ .

Again, using the proof of Claim 3 in the Appendix, both demand and supply of skilled labor is also increasing in proportion of skilled consumers,  $\partial L_s^S/\partial\theta > 0$ , from (12) and  $\partial L_s^D/\partial\theta > 0$ , from (10).

**Claim 5.** Equilibrium skilled labor and equilibrium relative wage are increasing in the proportion of skilled consumers,  $\partial L_s^*/\partial\theta > 0$  and  $\partial\xi^*/\partial\theta > 0$ .

## Birthrate

Individual number of children are,

$$\begin{aligned}
 n_s^H(\xi) &= n_\sigma^*(w_s, w_s; Q) = \frac{\bar{\ell}}{Q^\sigma + 1}, \quad \sigma < \hat{\sigma}, \\
 n_s^L(\xi) &= n_\sigma^*(w_u, w_s; 1) = \frac{\bar{\ell}}{\xi^{\sigma-1} + 1}, \quad \sigma > \hat{\sigma} \\
 n_u^H(\xi) &= n_\sigma^*(w_s, w_u; Q) = \frac{\bar{\ell}}{Q^\sigma \xi^{1-\sigma} + 1}, \quad \sigma < \hat{\sigma}, \\
 n_u^L(\xi) &= n_\sigma^*(w_u, w_u; 1) = \frac{\bar{\ell}}{2}, \quad \sigma > \hat{\sigma}.
 \end{aligned}$$

It is clear that for given wage level, those that consume high quality good devoted even more resources for consumption and thus reduce number of children when quality improves. Since the equilibrium relative wage is increasing in quality, we can say the following,

**Claim 6.** (i) *Skilled consumers have less children. That is,  $n_s^H < n_u^H$  for  $\sigma < \hat{\sigma}$  and  $n_s^L < n_u^L$  for  $\sigma > \hat{\sigma}$ .*

(ii) *Skilled consumers have less children when quality of product improves. That is,  $dn_s^H/dQ < 0$  for  $\sigma < \hat{\sigma}$  and  $dn_s^L/dQ < 0$  for  $\sigma > \hat{\sigma}$ .*

(iii) *Unskilled consumers that consume low quality product have the same number of children when quality improves. That is,  $dn_u^L/dQ = 0$  for  $\sigma > \hat{\sigma}$ .*

Although there is an income effect, the substitution effect dominates and skilled workers that consume low quality reduce number of children. For unskilled consumers that bought high quality good, improvement makes consumption more attractive (reduce children) but their relative wage becomes lower and the substitution effect works in the opposite direction. The total effect is not clear.

### 3.3 Endogenous Quality and Easterlin Hypothesis

Assume that level of quality is increasing in the size of the skilled labor. That is,  $Q = Q_T(L_s)$  is an increasing function of  $Q$ . Subscript  $T$  refers to “technology” which is what this relationship reflects. We will denote the inverse relationship between the market equilibrium supply of skilled labor and quality of  $L_s^*(Q)$  by  $Q = Q_M(L_s)$ , which is an increasing function from Claim 3. The equilibrium level of labor  $L_s^*$  and equilibrium level of quality,  $Q^* = Q_M(L_s^*) = Q_T(L_s^*)$ , is the intersection of the two curves.

When marginal increase in quality from labor is very large, then the equilibrium is unstable. Graphically, this would mean slope of  $Q_T$  is steeper than  $Q_M$  ( $Q'_T > Q'_M$ ). This is the case around equilibrium  $E_1$  in Figure 8. A perturbation away from  $E_1$  results in either spiral increase in quality and skilled labor supply or decrease of quality and skilled labor supply. When technology is mature so that marginal quality improvement is very small, then equilibrium is stable ( $Q'_T < Q'_M$ ). This is equilibrium  $E_2$  in Figure 8. There may be multiple equilibria, some stable and others unstable. A slight perturbation from low quality with small skilled labor force will start a spiral of labor and quality improvement until  $E_2$  is reached.

Now using Claim 4, we analyze the effect of declining population. The claim implies that the  $Q_M(L_s)$  function will shift upward in the  $L_s - Q$  space (Figure 9).

**Claim 7.** (i) *If the technology is in its infancy, then equilibrium quality and skilled labor supply increase when population declines. That is ,*

$$Q'_T > Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial N} < 0, \quad \frac{\partial L_s^*}{\partial N} < 0.$$

(ii) *If the technology is mature, then equilibrium quality and skilled labor supply decrease when the population decreases. That is ,*

$$Q'_T < Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial N} > 0, \quad \frac{\partial L_s^*}{\partial N} > 0.$$

When the technology is mature, then declining population results in “con-

traction” of the economy. That is, quality and supply of skilled labor are reduced. Claim 6 suggests that lower quality will increase the birthrate. Recall that all but unskilled consumers that consumed high quality product will increase birthrate when quality improves. This situation is consistent with a cohort effect.

The situation is different when the technology still has not exhausted increasing marginal returns. The new equilibrium results in more skilled labor and higher quality. Products are more polarized, skilled labor has higher relative wages and work more. Utility is derived from more consumption and there is less children. The cohort effect does not hold because the economy adjusts to the lower level of population according to the available technology.

Now we consider the effect of more skilled workers, using Claim 5. The claim implies that the  $Q_M(L_s)$  function will shift downward in the  $L_s - Q$  space (Figure 10). Immediately we have the following,

**Claim 8.** (i) *If the technology is in its infancy, then equilibrium quality and skilled labor supply decrease when the proportion of skilled workers increase. That is ,*

$$Q'_T > Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial \theta} < 0, \quad \frac{\partial L_s^*}{\partial \theta} < 0.$$

(ii) *If the technology is mature, then equilibrium quality and skilled labor supply increase when the proportion of skilled workers increase. That is ,*

$$Q'_T < Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial \theta} > 0, \quad \frac{\partial L_s^*}{\partial \theta} > 0.$$

Equilibrium quality will decrease (increase) when technology is in its infancy (maturity). When proportion of skilled consumers increase, each skilled worker needs to supply less labor to maintain the same quality. When marginal quality from labor is very large, quality must be lower to accommodate it. Lower quality (and lower wage) likely to imply higher birthrate. Thus when technology is sufficiently productive, the increasing skilled workers will increase the birthrate. On the other hand when the marginal product of labor is low, then higher labor implies higher quality. This may reduce

the birthrate.

Claims 7 and 8 suggest that increasing the proportion of skilled labor can be effective in reversing decline in birthrate whenever the cohort effect may not hold. This was the case when marginal return from increasing skilled labor is large. On the other hand, when the technology is mature, Esterlin Hypothesis is likely to hold and the same policy will prevent the feedback mechanism that otherwise will function.

## 4 Concluding Remarks

We have presented an alternative explanation of the positive relationship between total fertility rate (TFR) and female labor participation rate (FLPR) observed in a cross section of OECD countries in recent years.

We first argued that when consumption and childrearing require both time and goods, there will always be a negative relationship between consumption and number of children. However relationship between children and labor participation is not clear. We employed Japanese cross section from eight different points in time (every five years from 1970 – 2005), that have also shown a positive correlation between TFR and FLPR in recent years to test the theory. However, we found that FLPR has a significantly negative effect on TFR after dealing with unobservable heterogeneity, simultaneity or endogeneity problem and the measurement error problem by Fixed effect IV estimation. We note that the use of fixed effects instrumental variables model (FE-IV model) guarantees a consistent estimator. Furthermore, consumption variables are statistically significant and have negative impact on TFR. The results are consistent with our new model as well as traditional economic models of the relation between TFR and FLPR.

We showed how low fertility is associated with consumption of higher quality products using a general equilibrium model with vertical quality differentiation and heterogeneous labor. Higher quality product has two effects: it makes consumption more attractive but also increases wage for skilled workers. The second effect makes working more attractive and the resulting income effect implies having more children or consuming more higher quality

product or both. If the income effect dominates, higher labor participation and higher birthrate will be observed when income effect dominates. If the substitution effect dominates, the relationship will be negative. In both cases, there will be a negative relationship between birthrate and consumption.

The general equilibrium analysis suggests that if the technology is productive enough, the economy will adjust to smaller population and the cohort effect does not reverse the trend of declining population. We also showed that increasing the proportion of skilled consumers (potential workers) can increase birthrate and reverse the trend precisely when the cohort effect does not hold. We note that the same relationship between population size and proportion of skilled consumers means that changing the proportion can prevent the natural feedback mechanism from functioning when it would have functioned.

The two situations are characterized by if the technology has high marginal return from skilled labor (infant) or if this has been exhausted (mature). The economy will correct itself when it is mature, where we also observed the equilibrium to be stable. Therefore, another possible policy is to let the technology mature quickly.<sup>1</sup>

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<sup>1</sup>Immigration is another aspect of globalization. It is interesting to examine the impact of immigration on birthrate. These are tasks for our future research. Case of open economy is explored in Yomogida and Aoki (2005).

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## Appendix

### Optimization of $u(Qx, n)$

Denoting the Lagrange multiplier by  $\lambda$ , first-order conditions are,

$$u_n f_x = \lambda p, \quad u_n f_\ell = \lambda w, \quad u_n g_x = \lambda p, \quad u_n g_\ell = \lambda w,$$

and the budget constraint. This implies

$$\frac{f_x}{f_\ell} = \frac{g_x}{g_\ell} = \frac{p}{w}.$$

When  $w$  increases,  $\ell_c$  and  $\ell$  decrease while  $x$  and  $x_c$  increase.

### Proof of Claim 3

The demand and supply functions, (10), (11), (12), and (13), can be rewritten as,

$$\begin{aligned} L_s^S &= \theta N \bar{\ell} \int_1^\infty \frac{Q^\sigma}{Q^\sigma + \xi^{1-\sigma}} d\sigma + \theta N \bar{\ell} \int_{\hat{\sigma}}^\infty \left\{ \frac{Q^\sigma}{Q^\sigma + \xi^{1-\sigma}} - \frac{Q^\sigma}{Q^\sigma + 1} \right\} d\sigma \\ L_s^D &= \theta N \bar{\ell} \int_1^{\hat{\sigma}} \frac{Q^\sigma}{Q^\sigma + 1} d\sigma + (1 - \theta) N \bar{\ell} \int_1^{\hat{\sigma}} \frac{Q^\sigma}{Q^\sigma \xi + \xi^\sigma} d\sigma \\ L_u^S &= (1 - \theta) N \bar{\ell} \int_1^\infty \left\{ \frac{Q^\sigma \xi^{1-\sigma}}{Q^\sigma \xi^{1-\sigma} + 1} - \frac{1}{2} \right\} d\sigma + (1 - \theta) N \bar{\ell} \int_1^\infty \frac{1}{2} d\sigma, \\ L_u^D &= (1 - \theta) N \bar{\ell} \int_{\hat{\sigma}}^\infty \frac{1}{2} d\sigma + \theta N \bar{\ell} \int_{\hat{\sigma}}^\infty 1 \xi^{-1} + \xi^{-\sigma} d\sigma. \end{aligned}$$

The claim follows from noting that  $\hat{\sigma}$  is decreasing in  $\xi$  and increasing in  $Q$ , and that  $Q^\sigma \xi^{1-\sigma} > 1$  for  $\sigma < \hat{\sigma}$ .

Table 1: Description of Variables

Var. Name	Description	Data
TFR	Total Fertility Rate	the Vital Statistics
FLPR	Female Labor Force Participation Rate	the Labour Force Survey
Marriages	# of married couple at the year per 1000 Pop.	the Vital Statistics
One-person Household	# of one-person households / # of Private households	the Population Census
Automobile Ownership	# of automobiles / the working population	Automobile Inspection & Registration
Leisure & Entertainment	Reading and recreation / Living expenditure	the Family Income and Expenditure Survey
Department Store	# of department store / 1000	the Unincorporated Enterprise Survey

Table 3: Estimation Coefficients of FLPR

year	1975	1980	1985	1990	1995	2000	2005	1975-2005
Coefficient	-0.002	0.007	0.003	0.007	0.006	0.006	0.007	0.066**
Std. Err.	(0.006)	(0.005)	(0.004)	(0.005)	(0.005)	(0.006)	(0.006)	(0.002)

Significant Level: \*\* : 5%

Table 4: Estimation Results

	Pooled OLS Model 1	Fixed Effect Model 2	Fixed Effect Model 3	IV Fixed Effect Model 4
Variables	Coef. (S.E.)	Coef. (S.E.)	Coef. (S.E.)	Coef. (S.E.)
FLPR	0.007*** (0.00)	-0.005 (0.00)	-0.025*** (0.00)	-0.036*** (0.01)
Marriages	0.053*** (0.01)	0.171*** (0.01)	0.044*** (0.01)	0.076*** (0.02)
One-person households	-1.174*** (0.19)		-0.626** (0.25)	0.867** (0.44)
Automobile ownership	-0.415*** (0.06)		-0.820*** (0.06)	-1.168*** (0.13)
Leisure & Entertainment	-4.162*** (0.74)		-1.597*** (0.51)	-3.133*** (1.18)
Department- store	-1.290*** (0.20)		0.297 (0.27)	0.601 (0.88)
Const.	1.889*** (0.14)	0.861*** (0.17)	3.237*** (0.14)	3.630*** (0.29)
adj. R2	0.806	0.673	0.937	0.901
Hausman Test	N/A	35.6***	51.5***	24.9***
Obs.	329	375	329	282

Significant Level: \*\* : 5% \*\*\* : 1%

# On the Persistence of Low Birthrate in Japan

Reiko Aoki

Center for Intergenerational Studies, Hitotsubashi University \*

“Fertility and Public Policy: How to Reverse the Trend of Declining Birth Rates”

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## Abstract

We first show that quality of consumption is an important determinant of fertility and labor supply. Taking this observation into account and using a general equilibrium model with vertical quality differentiation and heterogeneous labor, we show how low fertility may persist. This occurs because product quality and skilled labor supply adjust, never realizing the change in labor productivity necessary to reverse declining fertility.

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\*Center for Intergenerational Studies, Institute of Economic Research, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan, [aokirei@ier.hit-u.ac.jp](mailto:aokirei@ier.hit-u.ac.jp)

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# 1 Introduction

This paper consists of two parts. First, we present a model of consumer choice where children and consumption experience require both goods and time. We demonstrate how change in marginal utility of consumption and change in wages generate different relationship between fertility and labor participation, i.e., possible source of the difference between cross section and time series. In the second half, we embed a simplified version of this consumer into a general equilibrium model with heterogenous labor and vertically differentiated products. Through comparative statics, we analyze the cause and implications of low birthrate in the long run. We show that the feedback mechanism of the economy may not reverse the declining birthrate, contradicting an implication of the Easterlin Hypothesis cohort effect. This is because the labor market structure and product market adjusts to change in birthrate and thus the cohort effect never materializes.

This paper is in the spirit to papers in growth and trade that take into account the reaction of the economy in the long run (Acemoglu (1998), Flam and Helpman (1987), Thoenig and Verdier (2003)). Acemoglu (1998) showed that while in the short run, labor input is reduced in response to scarcity of skilled labor and high wages, skilled labor supply increase in response triggers technological change that makes skilled labor even more productive, raising skilled labor wage in the long run. Our analysis suggests that a similar long term adjustment of the economy will prevent a natural feedback mechanism from working. That is, smaller population will increase marginal product of labor more productive in the short run but consumption pattern will change in the long run reducing such an advantage.

## 2 Re-examination of female labor participation - birthrate relationship

Large Time series among many OECD countries show negative relationship between female labor participation and TFR (Figure 1) , while cross country in 2005 (average of years 1985-1996 as well as year 2000, Sleebos (2003), d'Addio and d'Ercole (2005), Da Rocha and Fuster (2006)) show a positive relationship. In Japan, although time series relationship has been negative for 1980 - 2000 (Figure 1) , cross section among prefectures show positive relationship in 1987 and 2002 (Figure 2). Obviously conditions that differ across regions in Japan are different from difference between two points in time. We also note that countries with high per capita GDP have low birthrates (Figure 3), suggesting low fertility may be correlated with high consumption. In this section we introduce a consumer optimization model to capture differences in income difference and quality of consumption.

We assume that a utility of a household depends on number of children,  $n$ , consumption of a good  $x$ . Both child rearing and consumption of a good requires time. Number of children is determined by amount of good  $x_c$ , and time devoted,  $\ell_c$ ,

$$n = f(x_c, \ell_c), \quad f_x > 0, f_{\ell} > 0.$$

Subscripts on functions denote partial derivatives. The utility of consumer is actually determined by amount of  $z$ , which is consumption experience that depends on amount of the good,  $x$ , and time devoted,  $\ell$ ,

$$z = g(x, \ell), \quad g_x > 0, g_{\ell} > 0.$$

Utility function is,

$$u(n, z), u_n > 0, u_z > 0.$$

Budget constraint depends on price of good and wage, and labor endowment,  $\bar{\ell}$ ,

$$px + px_c + w\ell + w\ell_c = w\bar{\ell}.$$

Figure 4 demonstrates the optimization problem. The opportunity set is

defined as,

$$\{(z, n) | n = f(x_c, \ell_c), \quad z = g(x, \ell), \quad p(x + x_c) + w(\ell + \ell_c) = w\bar{\ell}\}.$$

The frontier is downward sloping (see Appendix). It reflects the budget constraint as well as the technologies,  $g$  and  $f$ .

We further index consumption (consumption experience) by quality,  $Q$ . Utility function is

$$u(Qz, n)$$

where  $z$  measures quantity of consumption. First-order condition for utility maximization are,

$$\frac{f_x}{f_\ell} = \frac{g_x}{g_\ell} = \frac{p}{w}, \quad (1)$$

$$\frac{u_n}{u_z} = Q \frac{g_x}{f_x}. \quad (2)$$

Equation (1) implies less labor intensive consumption and child rearing method will be used when wage increase. The time series of female wage has been rising in Japan would lead to less labor intensive methods which means greater labor participation. Equation (2) implies better quality of consumption leads to more consumption and less children.

Higher wage but not significantly higher quality means positive relationship. However with the same higher relative wage and higher quality consumption means negative relationship between labor participation and fertility. Availability of consumption goods, such as entertainment and restaurants, is much greater in larger cities. This means higher  $Q$ , meaning less children and more consumption in cities.<sup>1</sup>

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<sup>1</sup>For instance, there are 191 Tokyo restaurants listed in the Michelin restaurant guide, compared to 64 in Paris and 42 in New York (Robinson (2007)). Same hours spend at a Tokyo restaurant yields higher  $Qz$  on the average compared to other locations in Japan.



### 3 General Equilibrium with high quality product and heterogenous labor

In this section we analyze a general equilibrium in which consumers have a utility function that reflect the previous analysis, although somewhat simplified. Consumers differ by two attributes, their preference and quality of labor. Consumers choose either to consumer high quality product or standard (low quality) product. Child bearing choice differ according to which product they choose, as well as if they are skilled or not. Skilled workers produce the high quality product and the labor supply level determine the level of quality.

#### Consumers

We simplify the consumer's problem so that she chooses between consumption ( $x$ ) and childbearing ( $n$ ). Her preference is represented by the following utility function which also depends on the quality of the good consumed,  $Q$ ,

$$U_\rho(n, x) = (Qx^\rho + n^\rho)^{\frac{1}{\rho}}, \quad 0 < \rho < 1. \quad (3)$$

Consumers preference,  $\rho$ , is distributed uniformly over  $[0,1]$ . Consumption good is either the standard (low quality)  $Q = 1$  or high quality  $Q > 1$ . Consumer's labor endowment is  $\bar{\ell}$  and wage is  $w$  which is also the opportunity cost of children. Denoting price of the good by  $p$ , consumer chooses consumption and number of children to maximize (3) with respect to the budget constraint,

$$px + wn = w\bar{\ell}.$$

Each consumer's consumption and number of children given quality  $Q$  is determined by the utility maximization given the budget constraint,

$$x_\sigma^*(p, w; Q) = \frac{Q^\sigma \bar{\ell}}{\left(\frac{p}{w}\right)^\sigma \left(Q^\sigma \left(\frac{p}{w}\right)^{1-\sigma} + 1\right)}, \quad n_\sigma^*(p, w; Q) = \frac{\bar{\ell}}{Q^\sigma \left(\frac{p}{w}\right)^{1-\sigma} + 1}, \quad (4)$$

$$\text{where } \sigma \equiv \frac{1}{1-\rho} > 1.$$

Consumption is increasing and number of children is decreasing in quality, as in the previous section. The indirect utility is,

$$v_\sigma(p, w; Q) = \bar{\ell} \left( Q^\sigma \left( \frac{w}{p} \right)^{\sigma-1} + 1 \right)^{\frac{1}{\sigma-1}}.$$

The consumer must choose which quality to consume. If her marginal utility from more consumption is relatively large, she devotes less resources to children and has fewer children. If the quality is low and not as beneficial, she derives utility by having many children. She compares the utility levels from consuming each quality and buys whichever yields higher utility. We denote the prices of the goods with different qualities by  $p_H$  and  $p_L$ . Consumer will buy the high quality good when

$$v_\sigma(p_H, w; Q) > v_\sigma(p_L, w; 1).$$

This condition is equivalent to,

$$\sigma < \hat{\sigma} \equiv \frac{\ln \frac{p_H}{p_L}}{\ln \frac{p_H}{p_L} - \ln Q}. \quad (5)$$

Since  $\sigma > 1$ , there will be no demand for the low quality good if  $\ln \frac{p_H}{p_L} < \ln Q$ . This occurs if low quality product is more expensive ( $p_L \geq p_H$ ) since  $Q > 1$  and  $p_H > p_L$  but the price premium for the high quality is small relative to difference in quality. It does not depend on the level of income.

Consumer's labor supply is the hours not devoted to raising children,

$$\ell_\sigma(p, w; Q) = \bar{\ell} - n_\sigma^*(p, w; Q) = \frac{Q^\sigma}{Q^\sigma + \left( \frac{p}{w} \right)^{\sigma-1}}. \quad (6)$$

## Markets

The labor each consumer supplies is either skilled ( $s$ ) or unskilled ( $u$ ). There are total of  $N$  consumers, and  $\theta \in (0, 1)$  of the consumers are skilled. Labor endowment,  $\bar{\ell}$ , is the same for both types. We denote wages for skilled and unskilled by  $w_s$  and  $w_u$ . Production technology is constant returns to scale

in labor: one unit of skilled labor produces one unit of high quality product and one unit of unskilled labor produces one unit of the standard product. Furthermore we assume both products are supplied competitively. Thus we have  $p_H = w_s$  and  $p_L = w_u$ .

One skilled worker's demand for high quality product is , denoting relative wage by  $\xi = \frac{w_s}{w_u} > 1$  and using (4),

$$x_s^H(\xi) = x_\sigma^*(w_s, w_s; Q) = \frac{Q^\sigma \bar{\ell}}{Q^\sigma + 1}, \quad \sigma < \hat{\sigma} = \frac{\ln \xi}{\ln \xi - \ln Q},$$

and demand for low quality is,

$$x_s^L(\xi) = x_\sigma^*(w_u, w_s; Q) = \frac{\bar{\ell}}{\xi^{-\sigma}(\xi^{\sigma-1} + 1)}, \quad \sigma > \hat{\sigma}.$$

There will be positive demand for the low quality only if  $\xi > 1$  since  $\xi = \frac{p_H}{p_L}$ . We make the following observation

**Claim 1.** *High skilled consumers consume more of both quality,  $x_s^H(\xi) > x_u^H(\xi)$  and  $x_s^L(\xi) > x_u^L(\xi)$ .*

Total demands from all the skilled workers for high quality product and low quality product are ,

$$X_s^H(\xi) = \theta N \int_1^{\hat{\sigma}} x_s^H(\xi) d\sigma, \quad X_s^L(\xi) = \theta N \int_{\hat{\sigma}} x_s^L d\sigma.$$

Similarly for unskilled workers, we have the individual demands for high quality good,

$$x_u^H(\xi) = x_\sigma^*(w_s, w_u; Q) = \frac{Q^\sigma \bar{\ell}}{\xi^\sigma (Q^\sigma \xi^{1-\sigma} + 1)}, \quad \sigma < \hat{\sigma} = \frac{\ln \xi}{\ln \xi - \ln Q},$$

and demand for low quality good,

$$x_u^L(\xi) = x_\sigma^*(w_u, w_u; Q) = \frac{\bar{\ell}}{2}, \quad \sigma > \hat{\sigma}.$$

Total demands for each quality from all unskilled workers are,

$$X_u^H(\xi) = \int_1^{\hat{\sigma}} x_u^H(\xi) d\sigma, \quad X_u^L(\xi) = \int_{\hat{\sigma}} x_u^L(\xi) d\sigma.$$

Since production of one unit of good requires one unit of labor, demand for skilled and unskilled labor,  $L_s^D$  and  $L_u^D$  are,

$$L_s^D(\xi) = \theta N X_s^H(\xi) + (1 - \theta) N X_u^H(\xi), \quad (7)$$

$$L_u^D(\xi) = \theta N X_s^L(\xi) + (1 - \theta) N X_u^L(\xi). \quad (8)$$

Labor supply is constructed in a similar manner from individual supplies. Individual labor supply as function of relative wage is , using (6) ,

$$\ell_s^H(\xi) = \ell_\sigma^*(w_s, w_s; Q) = \frac{Q^\sigma \bar{\ell}}{Q^\sigma + 1}, \quad \sigma < \hat{\sigma},$$

$$\ell_s^L(\xi) = \ell_\sigma^*(w_u, w_s; 1) = \frac{\bar{\ell}}{\xi^{1-\sigma} + 1}, \quad \sigma > \hat{\sigma}$$

$$\ell_u^H(\xi) = \ell_\sigma^*(w_s, w_u; Q) = \frac{Q^\sigma \bar{\ell}}{Q^\sigma + \xi^{\sigma-1}}, \quad \sigma < \hat{\sigma},$$

$$\ell_u^L(\xi) = \ell_\sigma^*(w_u, w_u; 1) = \frac{\bar{\ell}}{2}, \quad \sigma > \hat{\sigma}.$$

Aggregation yields the total labor supply of each type,

$$L_s^S = N \bar{\ell} \int_1^{\hat{\sigma}} \left\{ \theta \frac{Q^\sigma}{Q^\sigma + 1} + (1 - \theta) \frac{Q^\sigma}{Q^\sigma + \xi^{\sigma-1}} \right\} d\sigma, \quad (9)$$

$$L_u^S = N \bar{\ell} \int_{\hat{\sigma}}^\infty \left\{ \theta \frac{Q^\sigma}{Q^\sigma + \xi^{1-\sigma}} + (1 - \theta) \frac{1}{2} \right\} d\sigma. \quad (10)$$

It is easy to show, from (5), that  $\hat{\sigma}$  is decreasing in  $\xi$  that  $L_s^D$  and  $L_u^S$  is decreasing in  $\xi = \frac{w_s}{w_u}$  and  $L_s^S$  and  $L_u^D$  are increasing in  $\xi$ . Equilibrium relative wage for a given quality level,  $\xi^*(Q)$ , is determined by the skilled labor market clearing condition,

$$L_s^D(\xi) = L_s^S(\xi).$$

The unskilled labor market has cleared by Walrus Law.