4. おわりに

近年、OECD 諸国のクロスカントリー分析では、女性の労働力参加率と出生率には正の相関がみられ、女性の社会進出率が高い国ほど出生率が高いという逆転の現象が起きている。わが国においても、時系列データでみると女性の労働市場への参加率の上昇と共に出生率の低下が観察されているが、都道府県データを用いたクロスセクション分析では、1980 年代後半からは、女性の社会進出率が高い都道府県ほど出生率も高いという現象が観察されている。この集計されたマクロデータでみた場合と、地方disaggregateしたデータで観察される逆転現象が起きている。しかし、この逆転現象について明確な結論を与えられている研究が十分に存在しないのが現状である。これは、出生率を説明するのに、従来の女性の労働参加率や結婚率のみでモデルを構築し、説明するのが不十分だからではないのかと考えられる。そこで我々は、昨年度に引きつづき、出生率と労働投入の関係について、女性の社会進出のみではなく、消費行動、財の質やバラエティの関係を内生化することによって、出生率の変動を説明するモデルを構築した。本年度の研究では、分担研究報告の1で得られた結果に加えて新たな実証分析を行い、現実を反映した出生率と女性労働力化率の関係を検証することを目指した。

実証結果より、地域の観測されない異質性や消費のバラエティを考慮しない状態では、出生率と女性の労働力化率は無相関あるいは正の相関が観察されたが、われわれの方法で分析を行った結果、有意に負の関係があることが観察された。さらに**分担研究1**では消費財の質やバラエティ変数が有意にならなかったが、この点も改善できた。この結果は経済理論で考えられる出生率と女性労働力化率の関係と整合的であり、さらにわが国の現状とも整合的な結果が得られた。

厚生労働科学研究費補助金（政策科学総合研究研究事業）
「社会保障と経済の相互関係に関する研究」
各論

「人口減少、超高齢化と就業構造との関係について」

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研究要旨　相対的に低スキルながら状況に応じた対人対応が求められる非定型手仕事業務が増加した背景を、高齢化、世帯規模の縮小等の人口動態等傾向的な要因、一時的な経済環境（所得）の影響、需要者としての高スキル就業者の増加等、需要面から分析した。

世帯の個票及び都道府県別の職業別就業者のデータを用いた分析によると、人口動態上の変化や高スキル就業者の増加が非定型手仕事型の個人向けサービスの需要を高めたことが示唆された。

A．研究目的

情報化やグローバル化が進展するなかで、高スキル（専門知識や技能）を要する業務と同時に低スキルで機械化されにくい手仕事の業務も増え、中間的な業務が減少するという労働市場における「業務の二極化」が観察されている。本研究は、非定型手仕事業務増加の背景を、人口動態、経済環境、需要者として機会費用の高い高スキル就業者の存在等の需要面から分析する。

B．研究方法

国勢調査の職業小分類を「非定型分析業務」「非定型相互業務」「定型認識業務」「定型手仕事業務」「非定型手仕事業務」の５つに分類し、過去20年に高スキル型の「非定型分析業務」と低スキル型の「非定型手仕事業務」の双方が増加していることを確認した。

さらに、全国消費実態調査、就業構造基本調査等で利用可能なデータから上記５業務に概念が近いものを使用することとし、高スキル型就業者を「専門的・技術的・管理的職業」で、非定型手仕事業務は「サービス職業」で近似し、非的計手仕事業務の増加の背景を、以下のように需要面から分析した。

第一に、個人向けサービス消費の需要について、世帯の個票を用いて家計の属性（所得水準、世帯主年齢、家族構成や規模等）との関係を見るとともに、二時点の差について要因分解する。

第二に、都道府県のデータを用いて、サービス就業者比率と地域の属性及び高スキル就業者比率との関係を見た上で、世帯の場合と同様に二時点の差を要因分解する。

C．研究結果

第一に、世帯の消費支出に占める個人向けサービス（及び内訳）支出割合は、概ね所得階層が高まるにつれて、また、世帯人員数の減少につれて高まる場合が多い。その際、一部例外はあるが世帯主が 60 代以上で支出割合が高まる場合が多い。1994 年と 2004 年の二時点の差については、人口動態要因が支出割合に対して重要な説明要因となっており、一般世帯では支出割合の上昇に寄与している。

第二に、都道府県の有業者に占めるサービス就業者比率（サービス全体、生活関連サービス、飲食・給仕サービス）については、2007 年のクロスセクションで世帯人員が少ない地域また高スキル就業者比率の高い地域において、サービス全体と飲食・給仕サービス従事者の比率が高い。また 1997 年と 2007 年の差については、高齢者人口（生活関連）や世帯人員（サービス全体と飲食・給仕）などの人口動態要因が比率の上昇に寄与すると同時に、サービス全体と飲食・給仕において高スキル就業者比率の上昇の寄与もみられた。

D．考察

世帯の消費支出に占める個人向けサービスの支出割合の二時点の差に対しては、高齢化、世帯人員数の減少、単身世帯での女性割合の増加など人口動態要因の支出割合の変化がかなり頑健な寄与を示した。また、一般世帯ではこうした人口動態要因は全般的に支出を高める方向い働いた。一方、両年の世帯構成の差だけでは説明できない要因（介護保険制度の導入や新たなサービスの登場等）も無視できない寄与を示している。

E．結論

非定型手仕事業務増加の背景には高齢化の進展、世帯規模の縮小という人口動態上の変化や高スキル就業者の増加があることが示唆された。

F．健康危険情報
該当しない。

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H．知的財産権の出願・登録状況
該当しない。

厚生労働科学研究費補助金（政策科学総合研究研究事業）
「社会保障と経済の相互関係に関する研究」
各論

「財の品質差別のモデル化の新しい試み」

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研究要旨　薬品や医療行為のように品質差別が重要である財やサービスにおいて、よく見られるのが「標準品」と「特注」または「カスタム製」などの品質差別である。これらの差別は単に「高」と「低」品質だけでなく、多数の消費者がある程の効用をみいだす「標準品」と極少数の消費者が非常に高い効用をみいたす「特注品」といった区別がよく見られる。インターネットにより情報の交換が容易になることは「特注品」の生産費の低下であり、このような品質差別は多くなってきた。この区別を反映することができる「スポーク型」品質差別のモデルを新たに構築する。独占と寡占における製品の均衡における製品の種類を分析する。独占企業もしくは2企業寡占の場合の一方の企業は「標準品」を提供するが、3企業の場合はどの企業も「特注品」を提供するので、「標準品」は存在しなくなる。これは保育や医療サービスの市場でそれぞれの子供や患者にあった医療を提供するためにはある程度のサービス提供者が市場に存在しなければ不可能なことを示唆している。

A．研究目的

技術進歩によって、財やサービスのみならず、それらの提供方法が変化している。一方少子化と核家族化により、消費者のニーズは多様化している。結果的に品質差別は「高」と「低」よりも個人のニーズや好みに対応する「特注品」と大衆むけの「標準品」の区別に移行している。標準品は全ての人が特注品よりは劣ると評価しているが、特注品より多くの人がそのように評価しているという特徴がある。本研究は「特注品」と「標準品」の区別をより多くの消費者にある程度アピールするか、それとも少数の消費者に非常に高く評価されるといった区別を明確にモデルに導入して、市場構造と均衡において供給される財の質と価格を分析する。

市場構造と財の質と価格の関係を把握することにより、医療サービスに対する政策が市場をどのように考慮する必要があるのかが明らかになる。これは少子化や高齢化により、必要とされる医療その他のケア・サービスや財のニーズが多様化し、いかに個人のニーズに答えるかが問題になる。それぞれのニーズに対応する「特注品」の提供するために政策が目指すべきサービス市場を理解するあしがかりになる。

B．研究方法

消費者が３本の枝があるスポークに一様に分布しているホテリング・モデルの拡張を考える。企業は製品の質と価格をそれぞれ選ぶ。スポークの中心を選ぶことは「標準品」の選択を現す。スポークから枝の一本を外側にいくにつれ、製品の特注度が増すころになる。同じスポークの消費者にとってはより自分の選好に近いものになる、他のスポークの消費者にとっては自分のニーズからほど遠いものである。標準品は多くの消費者のニーズをある程度満たしている。
企業がひとつである独占企業のほかに、企業が２つと３つの寡占の場合も考える。寡占の

場合は企業は順番に参入して財の特化度と価格を選ぶ。分析をするのは3企業の場合までであるが、その場合の議論が一般にN企業の場合も通用することも示す。

C．研究結果

独占企業はスポークの中心を選択する。つまり、独占企業は標準製品を供給するのである。特注製品を供給して高い価格で極少数の消費者に売るよりも、標準製品を多くの消費者に売った方が特ということである。2 企業の場合は、最初に参入した企業が標準製品を売り、後に参入した企業がどれかのスポークの枝の先に近いところに位置する。つまり、ある程度特注した製品を供給する。これは企業数がスポークの枝の数よりも少ない場合はつねに均衡で起こることで、必ず標準製品がある。

これに対して、枝の数が 3 本である場合は 3 企業の寡占であると、全ての企業が特注製品を供給する。一般に企業の数よりも枝の数が多い場合に均衡でおこるころである。企業の数が消費者の多様性（スポークの枝の数）よりも多い場合は、全ての企業が特注製品を供給し、市場には標準品は出回らない。

D．考察

大衆向けと思われる標準品は企業の数が多くなると市場から消えてしまう。全ての消費者が特注品を購入することになる。それに対して、独占企業は必ず標準品を供給する。標準製品の存在と市場が競争的であるということが単調な関係ではないことがわかる。

E．結論

消費者のニーズが多様な場合は、複数の企業の参入が望ましい。それぞれの消費者のニーズにあった製品が供給される。逆に標準品の有無を市場の競争、つまり経済効率性の指標とは考えるべきでない。

F．健康危険情報
該当しない。

G．研究発表

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該当しない。

H．知的財産権の出願・登録状況
該当しない。

Product Customisation in the Spokes Model

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April 1, 2009

Abstract

We analyse firms' incentives to customise their products using the spokes model. Consumers are located along three rays with a common origin. The product space features a mass of consumers in the centre and gives a natural interpretation of product customisation. By locating close to the centre, the firm offers a more standardised product which appeals to the mass consumers. As the firm moves away from the centre, the degree of customisation increases. Our results indicate that a monopolist always offers the standard product. For duopoly firms, one firm always offers the standard product. With three firm oligopoly, for a wide range of parameter ranges, the standard product is not offered by any firm. Furthermore, the equilibrium prices may increase when the number of firms increases. The welfare implication from such price increase is however not straightforward since with more firms, consumers incur less travelling cost.

JEL Classification: L11, L13.
Keywords: product differentiation, product costomisation, spatial oligopoly.

1 Introduction

In this paper, we analyse firms' incentives to customise in a spokes model. The standard literature on product customisation balances firms' incentives to customise against the cost of customisation. However, as the cost of customisation declines due to developments such as E-commerce, such analysis becomes less relevant. Alternatively, the literature also looks at the Hotelling model. In the typical Hotelling model, each consumer has an ideal product and the notion of standardised product versus customised product is not captured. We propose a model of product customisation entirely driven by consumer preferences by using a modified Hotelling model. The product space specification looks like Chen and Riordan (2006), but the interpretation and focus is different.

Chen and Riordan (2006) use a spokes model to analyse differentiated oligopoly. In their model, consumers are uniformly distributed on a N spokes network, with each spoke representing a product variety. There are $n \leq N$ firms, each locating at the end point of a spoke. Consumers desires at most two varieties. If the spoke where the consumer is located has a local firm at the end of the spoke, the consumer would desire this local brand. All other brands are of equal distance to the consumer and are bought each with probability $\frac{1}{N-1}$. Chen and Riordan fix firms' locations at the end of the spokes and analyse only price competition.

We analyse a three-spokes model and argue that the results extend to the general case of $n \leq N$, where n is the number of firms and N is the number of spokes in the product space. For the product space, the three rays have a common origin. This product space looks similar to the one in Chen and Riordan (2006). However, firms compete by choosing prices as well as locations in our model. Since there is a central point with mass consumers, this point represents the standard product. By moving further away from the centre, the degree of customisation increases. Examples of this type of product include for example, general sports shoes which appeal to most sports needs and specialised sports shoes such as running shoes and tennis shoes which cater for more specific markets. While firms' locations are fixed at the end point of the spokes in Chen and Riordan, it is an important endogenous variable in our model.

A paper which addresses a similar problem is Doraszelski and Draganska (2006). They also look at standard versus niche market. There are two types of consumers, A and B, each prefer one type of good. Apart from the two customised goods A and B, there is also a general good available, S. For given prices, a type A of consumer would prefer good A to good S and good B is the least prefered option. Doraszelski and Draganska include cost of customisation in the model.

In our model, customisation is driven by spatial competition, not the cost of specialization. We show that if there is only a monopoly, the monopolist always locates at the centre and offers the standard product. For duopoly competition or the more general case $n < N$ with sequential entry, one firm offeres the standard product and the other firm customises in equilibirum. When the market structure moves from duopoly to 3 firm oligopoly, or more generally, $n = N$, as long as firms do not act as unconstrained monopolists, the standard product is not offered anymore. Furthermore, the equilibirum prices may increase with 3 firm oligopoly compared with the duopoly setting. Although there is the possibility of price increase upon entry, the welfare results is not straightforward. With one more firm in the market, consumer welfare may increase due to better match of products despite the price increase.

The rest of the paper is organised as follows. Section 2 presents our model set up. Section 3 analyses the monopoly equilibrium. Section 4 presents the analysis for duopoly competition with sequential move. Section 5 studies 3 firm oligopoly game. We discuss some welfare analysis in Section 6, while the final seciton contains the concluding remarks.

2 The Model

Consumers are located uniformly along three rays with a common origin. Each ray represents a different type of customisation (different attribute of the product). Since the three rays intersect in the centre, by locating close to the origin, a firm sells a product which appeals to more customers. Along any given ray, the degree of customisation increases as the distance from the central point increases. By specialising in one attribute of the product, the product becomes less attractive to consumers who value other attributes of the product. Consumers and products are identified by both the ray and the distance from the origin. A firm i is identified by,

$$i = (r_i, i),$$

where r_i indicates the ray and i is the distance of this firm from the center. Consumer i is the consumer whose ideal product (most preferred product) is offered by firm i. For two points $x_1 = (r_{x1}, x_1)$ and $x_2 = (r_{x2}, x_2)$, the distance $\Delta(x_1, x_2)$ is defined by,

Definition 1

$$\Delta(x_1, x_2) = \begin{cases} |x_1 - x_2| & \mathit{if}\ r_{x1} = r_{x2} \\ x_1 + x_2 & \mathit{if}\ r_{x1} \neq r_{x2} \end{cases}$$

The product space is illustrated in Figure 1.

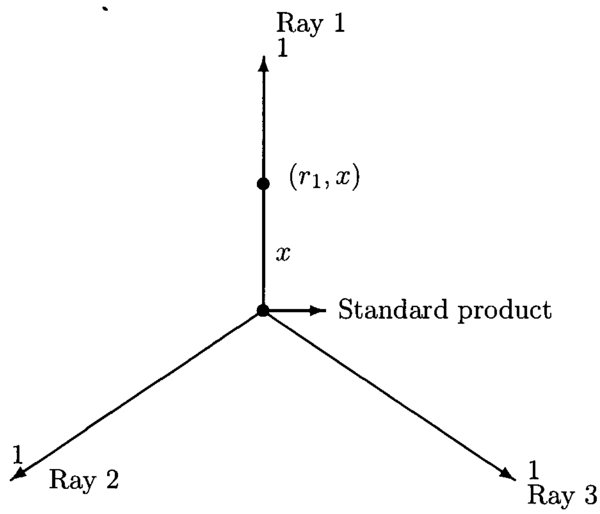

Figure 1: Product space.

It can be verified that $\Delta(x_1, x_2)$ is a metric in this product space. We can apply the standard analysis of product differentiation. That is, consumer $t = (r_t, t)$'s evaluation of a product $x = (r_x, x)$ with price p_x is $v_t(x, p_x)$:

$$v_t(x, p_x) = V - \tau\Delta(t, x) - p_x,$$

where V is some inherent value from consumption of one unit of the ideal product, and τ measures the marginal disutility of moving a unit distance away from the ideal product. A consumer will buy one unit of product if and only if the evaluation is non-negative. In the case of oligopoly, consumers buy the product that yields the highest non-negative evaluation. We assume no cost of production and customisation.

3 Monopoly

We first characterise the demand function for the monopolist's product x, $x \geq 0$. For any given p_x, demand will be a line segment which includes x. The marginal consumers $\bar{t} = (r_{\bar{t}}, \bar{t})$ and $\underline{t} = (r_{\underline{t}}, \underline{t})$ are those who satisfy $v_t(x, p_x) = 0$ or,

$$\Delta(t, x) = \frac{V - p_x}{\tau}.$$

We define $\bar{t}$ to be the consumer further away from the origin than x on the same ray ($r_{\bar{t}} = r_x$, $\bar{t} > x$). If p_x is high, then $\underline{t}$ will also be on the same ray ($r_{\underline{t}} = r_x$, $\underline{t} < x$). If p_x is sufficiently low, then consumers who actually prefer other forms of customisation will buy x ($r_{\underline{t}} \neq r_x$). We define the upper bound for such a price as p_0:

$$V - \tau(x - 0) - p_0 = 0 \quad \Leftrightarrow \quad p_0 = V - \tau x.$$

For $p_x \geq p_0$, demand is,

$$D(p_x) = D = \bar{t}_x - \underline{t}_x = 2\frac{V - p_x}{\tau}.$$

When price is lower, $p_x < p_0$, the marginal consumers are $\bar{t}_x > x$ and $\underline{t}$. There are two marginal consumers, $\underline{t}$, one each on rays i and j, where $i \neq j \neq x$. In this case, the marginal consumers $\underline{t}$ are defined by

$$V - \tau(x + \underline{t}) - p_x = 0.$$

For $p_x < p_0$, demand is,

$$D(p_x) = \bar{t}_x + 2\underline{t} = 3\frac{V - p_x}{\tau} - x.$$

The monopolist solves the problem

$$\max_{p_x, x} D(p_x, x)\, p_x.$$

We characterise the optimal monopoly pricing and corresponding profit for given x below.

$$p^m = \begin{cases} \frac{3V - x\tau}{6} \\ \frac{V}{2} \end{cases}, \quad \pi^m = \begin{cases} \frac{(3V - x\tau)^2}{12\tau} & x \leq \frac{(3-\sqrt{6})V}{\tau} \\ \frac{V^2}{2\tau} & x > \frac{(3-\sqrt{6})V}{\tau} \end{cases}.$$

When product is very specialized (large x), then it is sold only to one type (ray) of consumers. With more standard product (small x), it is sold to all types of consumers. Both price and profit are decreasing in x in this case. Any customization for a single type comes at the cost of becoming less attractive for the other two types (rays).

Proposition 1 *A monopolist should sell the standard product. That is, choose* $x = 0$.

Demand by each type is $\frac{V}{2\tau}$ and total demand is $\frac{3V}{2\tau}$.

4 Duopoly Competition

Firms move sequentially. Firm x first chooses price and location and upon observing x's choice, firm y chooses its location and price.[1] Since the follower can always choose to locate at the same position and undercuts the first mover marginally, in equilibrium, $\pi_y \geq \pi_x$. We solve the game backwards and start with the analysis for firm y, taking as given x and p_x.

Assumption 1 *Each ray is of length 1.*

Without this assumption, in equilibrium the firms would always act as local monopolists.

4.1 The follower's decision

Take (x, p_x) as given, we analyse in the following order

1. $x = 0$
   (a) local optimum with $y > x = 0$
   (b) local optimum with $y = 0$
2. $x > 0$
   (a) $y > x$ and $r_y = r_x$ is never an equilibrium
   (b) $y > 0$ and $r_y \neq r_x$ is never equilibrium
   (c) local optimum with $x \geq y \geq 0$ and $r_y = r_x$

[1] Due to the mass consumers in the centre, there does not exist pure strategy equilibrium when firms move simultaneously.

4.1.1 Firm x chooses $x = 0$

When firm x locates at the center, firm y can either chooses to locate off the center and sets its price optimally given p_x or it can choose to locate at the center. If firm y locates at the centre, its optimal strategy is to undercut p_x marginally whenever $p_x \leq \frac{V}{2}$. For $p_x > \frac{V}{2}$, the optimal $p_y = \frac{V}{2}$.

local optimum with $y > x = 0$ Note that with $x = 0$, for consumers from the other two rays to purchase from firm y, it must be the case that even the consumer located at the center purchases from firm y. Since x is the consumer located at 0's ideal bundle, if this consumer buys from y, it implies that $D_x = 0$. This clearly would not occur in equilibirum. First, even if $p_y = 0$, firm x can still charge a low but positive price and induce the consumer located at the center to buy its good. Second, with D_y coming from the other two rays and $D_x = 0$, firm y's best location would be to locate at $y = 0$. Therefore, for the analysis here, we only focus on the case that D_y comes only from r_y.

To characterise D_y, we define two critical p_y levels. Let p_1 be the price such that $\bar{t}_y = 1$.

$$V - \tau\left(\bar{t}_y - y\right) - p_1 = 0$$

$$\bar{t}_y = \frac{V - p_1}{\tau} + y = 1 \Leftrightarrow p_1 \equiv V - \tau\left(1 - y\right) \tag{1}$$

The second critical value is the p_y such that $\underline{t}_y = \bar{t}_x$. Let this price level be p_2.

$$y - \frac{V - p_2}{\tau} = \frac{V - p_x}{\tau} \Leftrightarrow p_2 \equiv 2V - y\tau - p_x. \tag{2}$$

Note that $p_1 \geq p_2$ if $p_x \geq V - 2y\tau + \tau$. For $p_x < V - 2y\tau + \tau$, $p_1 \leq p_2$:

$$\begin{aligned} D_y &= 1 - t_{xy} & p_y &\leq p_1 \\ D_y &= \bar{t}_y - t_{xy} & p_1 \leq p_y &\leq p_2 \\ D_y &= \bar{t}_y - \underline{t}_y & p_y &\geq p_2. \end{aligned}$$

For $p_x \geq V - 2y\tau + \tau$, $p_1 \geq p_2$:

$$\begin{aligned} D_y &= 1 - t_{xy} & p_y &\leq p_2 \\ D_y &= 1 - \underline{t}_y & p_2 \leq p_y &\leq p_1 \\ D_y &= \bar{t}_y - \underline{t}_y & p_y &\geq p_1. \end{aligned}$$

Given $(p_x, x = 0)$, firm y charges the local monopoly price, $\frac{V}{2}$, if there is enough space left on r_y. Or if

$$1 - \bar{t}_x = 1 - \frac{V - p_x}{\tau} \geq \frac{V}{\tau} \Leftrightarrow p_x \geq 2V - \tau.$$

For $p_x < 2V - \tau$, the two firms are in competition and in equilibirum, all consumers on r_y purchase one unit of good.

Remark 1 *For $y > x = 0$ with D_y coming only from r_y, firm y's local optimal location is to locate at $\bar{t}_x + \frac{V}{2\tau} \leq y \leq 1 - \frac{V}{2\tau}$ if $p_x \geq 2V - \tau$ and $\bar{t}_y = 1$ otherwise.*

Consider the case when $\bar{t}_y \geq 1$ and $\underline{t}_y < \bar{t}_x$, t_{xy} is defined by

$$V - \tau (y - t_{xy}) - p_y = V - \tau t_{xy} - p_x \Leftrightarrow t_{xy} = \frac{\tau y + p_y - p_x}{2\tau}.$$

The demand for firm y is

$$D^y = 1 - t_{xy} = 1 - \frac{\tau y + p_y - p_x}{2\tau}.$$

The demand which comes from the further end of the market is $1 - y$ and the demand which comes from the end closer to the centre is

$$y - t_{xy} = \frac{\tau y - p_y + p_x}{2\tau}.$$

For any given p_y, by moving closer to the centre, the marginal gain of market from the far end is 1 while the marginal loss of market due to more competition with firm x is $-\frac{1}{2}$. Therefore firm y always has the incentive to move closer to firm x. On the other hand, when $\bar{t}_y = 1$ and $\underline{t}_y \leq \bar{t}_x$, by moving closer to the centre, the marginal loss of demand is -1 while the marginal gain of demand is $\frac{1}{2}$. Therefore, the optimal location is the one such that $\bar{t}_y = 1$.

For $p_x < 2V - \tau$, y always locates in the position such that $\bar{t}_y = 1$.

$$\bar{t}_y = 1 \Leftrightarrow V - \tau (1 - y) - p_y = 0.$$

This gives

$$y = \frac{\tau - V + p_y}{\tau},$$

and

$$\begin{aligned} D^y &= 1 - t_{xy} = 1 - \frac{\tau y + p_y - p_x}{2\tau} = 1 - \frac{\tau \frac{\tau - V + p_y}{\tau} + p_y - p_x}{2\tau} \\ &= \frac{V + \tau + p_x - 2p_y}{2\tau}. \end{aligned}$$

Firm y solves

$$\max_{p_y} \pi^y = \frac{V+\tau+p_x-2p_y}{2\tau}p_y.$$

The FOC gives the optimal price and location,

$$p_y = \frac{V+\tau+p_x}{4} \text{ and } y = \frac{5\tau-3V+p_x}{4\tau}.$$

Note that $p_1 \geq p_2$ if $p_x \geq V - 2y\tau + \tau$. Substituting in the equilibrium price and location, $p_1 \geq p_2$ if

$$p_x \geq V - 2y\tau + \tau = V - 2\frac{5\tau-3V+p_x}{4\tau}\tau + \tau.$$

Or if

$$p_x \geq \frac{5}{3}V - \tau.$$

Since $\frac{5}{3}V - \tau < 2V - \tau$, for $\frac{5}{3}V - \tau \leq p_x < 2V - \tau$, $p_1 > p_2$.

For $p_x < \frac{5}{3}V - \tau$ and $p_1 < p_2$, $D_y = 1 - t_{xy}$. For $p_x \geq \frac{5}{3}V - \tau$, $p_1 \geq p_2$. Therefore, for $\frac{5}{3}V - \tau \leq p_x \leq 2V - \tau$, the solution is a corner solution with the optimal price and location determined by $\bar{t}_y = 1$ and $\underline{t}_y = \bar{t}_x$, or $p_y = p_1 = p_2$. This gives

$$y = \frac{V-p_x+\tau}{2\tau},\ p_y = \frac{3V-\tau-p_x}{2},$$

and

$$\pi_y = \frac{(\tau-V+p_x)(3V-\tau-p_x)}{2\tau}.$$

We summarise the firm y's best response in the following lemma.

Lemma 1 *If $x=0$, $y>0$ and D_y comes only from r_y, firm y's local best response is:*

$$p_y = \begin{cases} \frac{V+\tau+p_x}{4} \\ \frac{3V-\tau-p_x}{2} \\ \frac{V}{2} \end{cases},\quad y = \begin{cases} \frac{5\tau-3V+p_x}{4\tau} \\ \frac{\tau+V-p_x}{2\tau} \\ y \geq \bar{t}_x + \frac{V}{2\tau} \end{cases},$$

$$\pi^y = \begin{cases} \frac{(V+\tau+p_x)^2}{16\tau} & p_x \leq \frac{5}{3}V-\tau \\ \frac{(\tau-V+p_x)(3V-\tau-p_x)}{2\tau} & \frac{5}{3}V-\tau < p_x < 2V-\tau \\ \frac{V^2}{2\tau} & p_x \geq 2V-\tau \end{cases}.$$

Note that we need the additional restriction that $y \geq 0$.

$$\frac{5\tau-3V+p_x}{4\tau} \geq 0 \Longrightarrow p_x \geq 3V-5\tau.$$

$$\frac{5}{3}V-\tau \geq 3V-5\tau \text{ if } V \leq 3\tau.$$

For $p_x < 3V-5\tau$, firm y's optimal response would be $y=0$.

local optimum with $y = 0$ Given $x = 0$, if firm y chooses $y > 0$ and p_y so that it sells to other rays, as discussed previously, this implies $D_x = 0$. The optimal location is therefore $y = 0$. For any given p_y, by moving from $y > 0$ closer to $y = 0$, the increase in D_y comes from the other two rays while the decrease in D_y is only from r_y. Note that if we compare two equilibrium: $(x = 0, y = 0)$ and $(x = 0, y > 0)$, it may be the case that $\pi_y (y = x = 0) > \pi_x (x = 0, y > 0)$. By undercutting firm x a little bit, firm y practically gets the same profit as firm x, except that there is no additional firm out there. Therefore, as long as $v(t_{xy}) > 0$, $\pi_y (y = x = 0) > \pi_x (x = 0, y > 0)$.

Lemma 2 *When $x = 0$, y's local best response so that D_y comes from all rays is $y = 0$ and*

$$p_y = \begin{cases} \textit{Not defined} & p_x \le \frac{V}{2} \\ \frac{1}{2}V & p_x > \frac{V}{2} \end{cases};$$

$$\pi^y = \begin{cases} \textit{Not defined} & p_x \le \frac{V}{2} \\ \frac{3V^2}{4\tau} & p_x > \frac{V}{2} \end{cases}.$$

For $p_x \le \frac{V}{2}$, firm y would like to charge p_y as close to p_x but not equal to p_x. For continuous price setting, the best response is not defined.

4.1.2 Firm x chooses $x > 0$

We analyse in this section the case where firm x chooses $x > 0$. We first argue that the following two cases would never occur in equilibrium: (1) $y > x$ and $r_y = r_x$; (2) $y > 0$ and $r_y \neq r_x$. Therefore, the only case we need to consider is $x \ge y \ge 0$ and $r_y = r_x$. These three cases completes the analysis for $x > 0$.

1. $y > x$ and $r_y = r_x$ is never optimal

   For $x > 0$ and $r_y = r_x$, firm y would never locate at $y > x > 0$. Apart from the trivial case where both firms act as monopolists, for any given p_y, firm y is better off by locating at $r_y \neq r_x$.

2. $x > 0$, $y > 0$ and $r_y \neq r_x$ is never an equilibrium

   If $y > 0$, $r_y \neq r_x$, and t_{xy} locates on r_x, then by moving towards 0, the loss of demand from r_y is compensated by the gain of demand from r_i, $r_i \neq r_x \neq r_y$. Therefore, total demand from $r_y \neq r_x$ remains the same while demand from r_x increases. Thus, it is never optimal to choose $y > 0$ and $r_y \neq r_x$. If $y > 0$, $r_y \neq r_x$, and t_{xy} locates on r_y, then by the same argument, it is never optimal for x to choose $x > 0$.

Therefore, the only case we need to analyse here is $x \geq y \geq 0$ and $r_y = r_x$. We proceed by analysing D_y and optimal (p_y, y). To facilitate our discussion of D_y, we first define a few critical price levels:

$$\underline{p}_y \text{ solves } \bar{t}_y = \underline{t}_x \Rightarrow \underline{p}_y = 2V - \tau(x-y) - p_x.$$
$$\tilde{p}_y \text{ solves } \underline{t}_y = 1 \Rightarrow \tilde{p}_y = V - \tau(1+y).$$
$$p_y^0 \text{ solves } \underline{t}_y = 0 \Rightarrow p_y^0 = V - \tau y.$$

By definition, $\tilde{p}_y < p_y^0$ and $\tilde{p}_y < \underline{p}_y$. $\underline{p}_y \geq p_y^0$ if $y \geq \frac{x\tau - V + p_x}{2\tau}$.

Note first that $p_y \geq p_y^0$ would never occur in equilibrium. If this is the case, given any price, y is better off locating at $r_y \neq r_x$. We analyse the cases for $p_y \leq p_y^0$ here. Therefore, for $y \leq \frac{x\tau - V + p_x}{2\tau}$, $p_y^0 \geq \underline{p}_y > \tilde{p}_y$, and the demand for firm y is

$$D_y = \begin{cases} 2 + t_{xy} = \frac{4\tau + \tau(x+y) + p_x - p_y}{2\tau} & p_y \leq \tilde{p}_y \\ 2\underline{t}_y + t_{xy} = \frac{4V + p_x - 5p_y + \tau(x-3y)}{2\tau} & \tilde{p}_y \leq p_y \leq \underline{p}_y \\ \bar{t}_y + 2\underline{t}_y = 3\frac{V-p_y}{\tau} - y & \underline{p}_y \leq p_y \leq p_y^0 \end{cases}.$$

For $y \geq \frac{x\tau - V + p_x}{2\tau}$ we have $\underline{p}_y \geq p_y^0 > \tilde{p}_y$, and

$$D_y = \begin{cases} 2 + t_{xy} = \frac{4\tau + \tau(x+y) + p_x - p_y}{2\tau} & p_y \leq \tilde{p}_y \\ 2\underline{t}_y + t_{xy} = \frac{4V + p_x - 5p_y + \tau(x-3y)}{2\tau} & \tilde{p}_y \leq p_y \leq p_y^0 \end{cases}$$

First, notice that for the demand specification, firm y would only prefer to choose a bigger y in the region where the (y, p_y) combination gives $p_y(y) \leq \tilde{p}_y$. In other cases, firm y would like to locate as close to the centre as possible. This gives $y^* = 0$.

We discuss each of the demand specifications in turn.

Case 1 For $p_y \leq \tilde{p}_y$, π_y increases in y. Firm y would like to increase y provided that the conditions $y \leq x$ and $p_y \leq \tilde{p}_y$, or equivalently, $y \leq \frac{V-\tau-p_y}{\tau}$, are satisfied. Therefore, $y^* = \min\left\{x, \frac{V-\tau-p_y}{\tau}\right\}$.

$$x \leq \frac{V - \tau - p_y}{\tau} \text{ if } p_y \leq V - \tau(1+x).$$

Therefore, the optimal (y, p_y) combination is that for $p_y \leq V - \tau(1+x)$, firm y locates at $y = x$. For $p_y > V - \tau(1+x)$, the optimal (p_y, y) satisfies $p_y = \tilde{p}_y$ or $y = \frac{V - p_y - \tau}{\tau}$.

For $p_y \leq V - \tau(1+x)$ and $y = x$,

$$D_y = \begin{cases} 3 & \text{if } p_y < p_x \\ 0 & \text{if } p_y > p_x. \end{cases}$$

We assume that firms share the market if they charge the same price. This case, however, will not occur in equilibrium. For $p_y \leq V - \tau(1+x)$, the local best response is $p_y = \min\{V - \tau(1+x), \text{Not defined}\}$. For the not defined part of the best response, firm y would like to charge p_y as close to p_x as possible with $p_y < p_x$. As noted previously, for the model with continuous price setting, the best response is not defined.

For $p_y > V - \tau(1+x)$, $p_y = \widetilde{p}_y = V - \tau(1+y)$. This gives

$$\pi_y = \left(\frac{3\tau + \tau x + V + p_x - 2p_y}{2\tau}\right) p_y.$$

The FOC gives

$$p_y = \frac{3\tau + \tau x + V + p_x}{4}.$$

Note that with the restriction $y \geq 0$, the solution is only relevant for

$$p_y \leq V - \tau.$$

Or

$$p_x \leq 3V - 7\tau - x\tau.$$

For $p_y \geq V - \tau$, the corner solution occurs at $y = 0$ and $p_y = V - \tau$.

The solution is interior if

$$\frac{3\tau + \tau x + V + p_x}{4} > V - \tau(1+x)$$

or if

$$p_x > 3V - 7\tau - 5\tau x.$$

Compare the critical values:

$$3V - 7\tau - 5\tau x \geq V - \tau(1+x)$$

if

$$x \leq \frac{V - 3\tau}{2\tau}.$$

For $x \leq \frac{V-3\tau}{2\tau}$, the local best response in the area $p_y \leq \widetilde{p}_y$, is

$$p_y = \begin{cases} \text{Not defined} & p_x \leq V - \tau(1+x); \\ V - \tau(1+x) & V - \tau(1+x) < p_x \leq 3V - 7\tau - 5\tau x; \\ \frac{3\tau + \tau x + V + p_x}{4} & 3V - 7\tau - 5\tau x \leq p_x \leq 3V - 7\tau - x\tau; \\ V - \tau & p_x \geq 3V - 7\tau - x\tau. \end{cases}$$

For $x \geq \frac{V-3\tau}{2\tau}$, the local best responses are

$$p_y = \begin{cases} \text{Not defined} & p_x \leq 3V - 7\tau - 5\tau x; \\ \frac{3\tau + \tau x + V + p_x}{4} & 3V - 7\tau - 5\tau x \leq p_x \leq 3V - 7\tau - x\tau; \\ V - \tau & p_x \geq 3V - 7\tau - x\tau. \end{cases}$$

Case 2 For $\tilde{p}_y \leq p_y \leq \underline{p}_y$ and $\tilde{p}_y \leq p_y \leq p_y^0$,

$$\pi_y = \left(\frac{4V + p_x - 5p_y + \tau(x - 3y)}{2\tau} \right) p_y.$$

Firm y's profit, π_y, decreases in y in this region and firm y would like to locate as close to the centre as possible. Given that the boundaries of this region is defined by $\tilde{p}_y \leq p_y \leq \underline{p}_y$ and $\tilde{p}_y \leq p_y \leq p_y^0$, the optimal location is y^* satisfies $p_y = \underline{p}_y$ if $p_y > 2V - \tau x - p_x$, $y^* = 0$ if $V - \tau \leq p_y \leq 2V - \tau x - p_x$ and y^* satisfies $p_y = \widetilde{p}_y$ if $p_y \leq V - \tau$. Note that the part where $p_y = \underline{p}_y$ is only relevant for $y \leq \frac{x\tau - V + p_x}{2\tau}$.

For $p_y \leq V - \tau$, the local optimum satisfies $p_y = \widetilde{p}_y$ with the local best response $p_y = \frac{3\tau + \tau x + V + p_x}{4}$ and $y^* = \frac{-7\tau + 3V - x\tau - p_x}{4\tau}$. With $y \geq 0$, the local best response is relevant for $p_x \leq 3V - 7\tau - x\tau$. For $p_x \leq 3V - 7\tau - x\tau$, $\frac{3\tau + \tau x + V + p_x}{4} \leq V - \tau$.

For $V - \tau \leq p_y \leq 2V - \tau x - p_x$, the local optimal location is $y = 0$ with

$$\pi_y = \left(\frac{4V + p_x - 5p_y + \tau x}{2\tau} \right) p_y.$$

The FOC gives the local best response

$$p_y = \frac{(4V + x\tau + p_x)}{10}.$$

$$\frac{(4V + x\tau + p_x)}{10} \geq V - \tau \text{ if } p_x \geq 6V - 10\tau - x\tau.$$

$$\frac{(4V+x\tau+p_x)}{10} \leq 2V - \tau x - p_x$$

if

$$p_x \leq \frac{16V - 11x\tau}{11}.$$

$$\frac{16V-11x\tau}{11} \geq 6V - 10\tau - x\tau \text{ if } V \leq \frac{11}{5}\tau \approx 2.2\tau.$$

For $p_y > 2V - \tau x - p_x$, the optimal y satisfies $p_y = \underline{p}_y$ or

$$y = \frac{x\tau - 2V + p_x + p_y}{\tau}.$$

This gives the profit

$$\pi_y = \frac{(5V - x\tau - p_x - 4p_y)\,p_y}{\tau}.$$

The FOC gives

$$p_y = \frac{5V - x\tau - p_x}{8}.$$

$$\frac{5V - x\tau - p_x}{8} \geq 2V - \tau x - p_x \text{ if } p_x \geq \frac{11V - 7x\tau}{7}.$$

Check the boundary values:

$$6V - 10\tau - x\tau \geq 3V - 7\tau - x\tau \text{ if } V \geq \tau.$$

$$\frac{16V-11x\tau}{11} \geq 3V - 7\tau - x\tau \text{ if } V \leq \frac{77}{17}\tau \approx 4.53\tau.$$

$$\frac{11V-7x\tau}{7} \geq 3V - 7\tau - x\tau \text{ if } V \leq \frac{49}{10}\tau.$$

$$\frac{11V-7x\tau}{7} \geq 6V - 10\tau - x\tau \text{ if } V \leq \frac{70}{31}\tau \approx 2.2581\tau.$$

Note that

$$\frac{11V-7x\tau}{7} \geq \frac{16V-11x\tau}{11}.$$

For $V \leq \tau$, since the region $p_y \leq V - \tau$ is not relevant, the local best response is

$$p_y = \begin{cases} \frac{(4V+x\tau+p_x)}{10} & \text{if } 6V - 10\tau - x\tau \leq p_x \leq \frac{16V-11x\tau}{11} \\ 2V - \tau x - p_x & \text{if } \frac{16V-11x\tau}{11} \leq p_x \leq \frac{11V-7x\tau}{7} \\ \frac{5V-x\tau-p_x}{8} & \text{if } p_x \geq \frac{11V-7x\tau}{7} \end{cases}.$$

For $\tau \leq V \leq \frac{11}{5}\tau$, the local best response is

$$p_y = \begin{cases} \frac{3\tau+\tau x+V+p_x}{4} & \text{if } p_x \leq 3V - 7\tau - x\tau \\ V-\tau & \text{if } 3V-7\tau-x\tau \leq p_x \leq 6V - 10\tau - x\tau \\ \frac{(4V+x\tau+p_x)}{10} & \text{if } 6V-10\tau-x\tau \leq p_x \leq \frac{16V-11x\tau}{11} \\ 2V-\tau x - p_x & \text{if } \frac{16V-11x\tau}{11} \leq p_x \leq \frac{11V-7x\tau}{7} \\ \frac{5V-x\tau-p_x}{8} & \text{if } p_x \geq \frac{11V-7x\tau}{7} \end{cases}.$$

For $\frac{11}{5}\tau \leq V \leq \frac{77}{17}\tau$, the local best response is

$$p_y = \begin{cases} \frac{3\tau+\tau x+V+p_x}{4} & \text{if } p_x \leq 3V - 7\tau - x\tau \\ V-\tau & \text{if } 3V-7\tau-x\tau \leq p_x \leq V+\tau(1-x) \end{cases}.$$

For $V \geq \frac{77}{17}\tau$, the local best response is

$$p_y = \frac{3\tau+\tau x+V+p_x}{4}, y = \frac{-7\tau+3V-x\tau-p_x}{4\tau},$$

and

$$\pi_y = \frac{(V+3\tau+x\tau+p_x)^2}{16\tau}$$

for $p_x \leq V + \tau(1-x)$.

Case 3 Finally, for the case $\underline{p}_y \leq p_y \leq p_y^0$ the local best $y^* = 0$ with $p_y^* = \frac{V}{2}$. The solution is interior if $\frac{V}{2} \geq \underline{p}_y$. Or if

$$\frac{V}{2} \geq 2V - \tau(x-y) - p_x.$$

This holds for $p_x \geq \frac{3}{2}V - \tau x$. Note that this demand specification is only relevant for $p_x \geq V - \tau(x-2y)$.

Therefore, the local best response for $p_y \geq \underline{p}_y$ is

$$p_y = \begin{cases} 2V - \tau x - p_x & V - \tau x \leq p_x \leq \frac{3}{2}V - \tau x \\ \frac{V}{2} & p_x \geq \frac{3}{2}V - \tau x. \end{cases}$$

The optimal location is $y = 0$.

We are now ready to characterise the global best response for firm y. We compare the three different local best responses to get the global response. First, for $p_x \geq \frac{3}{2}V - \tau x$, the best response is $p_y = \frac{V}{2}$. This gives the unconstrained monopoly profit. Note that $\frac{16V-11x\tau}{11} \leq \frac{3}{2}V - \tau x < \frac{11V-7x\tau}{7}$.