$px + wn = w\bar{\ell}.$

Each consumer's consumption and number of children, given quality Q, is determined by the utility maximization given the budget constraint

$$x_\sigma^*(p, w; Q) = \frac{Q^\sigma \bar{\ell}}{(p/w)^\sigma \left(Q^\sigma (p/w)^{1-\sigma} + 1\right)}, \quad n_\sigma^*(p, w; Q) = \frac{\bar{\ell}}{Q^\sigma (p/w)^{1-\sigma} + 1},$$

(5.7)

where $\sigma \equiv 1/(1-\rho) > 1$.

Consumption is increasing and number of children is decreasing in quality, as in the previous section. The indirect utility is

$$v_\sigma(p, w; Q) = \bar{\ell}\left(Q^\sigma \left(\frac{w}{p}\right)^{\sigma-1} + 1\right)^{1/(\sigma-1)}.$$

The consumer must choose which quality to consume. If her marginal utility from more consumption is relatively large, she devotes fewer resources to children and has fewer children. If the quality is low and not as beneficial, she derives utility by having many children. She compares the utility levels from consuming each quality and buys whichever yields higher utility. We denote the prices of the goods with different qualities by p_H and p_L. Consumer will buy the high-quality good when

$$v_\sigma(p_H, w; Q) > v_\sigma(p_L, w; 1).$$

This condition is equivalent to

$$\sigma < \hat{\sigma} \equiv \frac{\ln(p_H/p_L)}{\ln(p_H/p_L) - \ln Q}. \tag{5.8}$$

Since $\sigma > 1$, there will be no demand for the low-quality good if $\ln(p_H/p_L) < \ln Q$. This occurs if low-quality product is more expensive ($p_L \geq p_H$), since $Q > 1$ and $p_H > p_L$, but the price premium for the high quality is small relative to difference in quality. It does not depend on the level of income.

Consumer's labor supply is the hours not devoted to raising children:

$$\ell_\sigma(p, w; Q) = \bar{\ell} - n_\sigma^*(p, w; Q) = \frac{Q^\sigma}{Q^\sigma + (p/w)^{\sigma-1}}. \tag{5.9}$$

<h3>Markets The labor each consumer supplies is either skilled, s, or unskilled, u. There are total of N consumers, and $\theta \in (0, 1)$ of the consumers are skilled. Labor endowment, $\bar{\ell}$, is the same for both types. We denote wages for skilled and unskilled by w_s and w_u. Production technology is constant returns to scale in labor:

one unit of skilled labor produces one unit of high-quality product and one unit of unskilled labor produces one unit of the standard product. Furthermore we assume that both products are supplied competitively. Thus we have $p_H = w_s$ and $p_L = w_u$.

Denoting relative wage by $\xi = (w_s/w_u) > 1$ and using (5.7), we obtain a skilled worker's demand for a high-quality product as

$$x_s^H(\xi) = x_\sigma^*(w_s, w_s; Q) = \frac{Q^\sigma \overline{\ell}}{Q^\sigma + 1}, \quad \sigma < \hat{\sigma} = \frac{\ln \xi}{\ln \xi - \ln Q},$$

and demand for low quality as

$$x_s^L(\xi) = x_\sigma^*(w_u, w_s; Q) = \frac{\overline{\ell}}{\xi^{-\sigma}\left(\xi^{\sigma-1} + 1\right)}, \qquad \sigma > \hat{\sigma}.$$

There will be positive demand for the low quality only if $\xi > 1$, since $\xi = p_H / p_L$.

We make the following observation:

<h4>**Claim 2** *High-skilled consumers consume more of both high and lowquality goods,* $x_s^H(\xi) > x_u^H(\xi)$ *and* $x_s^L(\xi) > x_u^L(\xi)$.

The total demand from all the skilled workers for high-quality products and low quality products is

$$x_s^H(\xi) = \theta N \int_1^{\hat{\sigma}} x_s^H(\xi) d\sigma, \quad x_s^L(\xi) = \theta N \int_{\hat{\sigma}} x_s^L d\sigma.$$

Similarly, for unskilled workers, we have the individual demands for high-quality goods,

$$x_u^H(\xi) = x_\sigma^*(w_s, w_u; Q) = \frac{Q^\sigma \overline{\ell}}{\xi^\sigma (Q^\sigma \xi^{1-\sigma} + 1)}, \quad \sigma < \hat{\sigma} = \frac{\ln \xi}{\ln \xi - \ln Q},$$

and demand for low-quality goods,

$$x_u^L(\xi) = x_\sigma^*(w_u, w_u; Q) = \frac{\overline{\ell}}{2}, \qquad \sigma > \hat{\sigma}.$$

Total demand for each quality from all unskilled workers is

$$X_u^H(\xi) = \int_1^{\hat{\sigma}} x_u^H(\xi) d\sigma, \quad X_u^L(\xi) = \int_{\hat{\sigma}} x_u^L(\xi) d\sigma.$$

Since the production of one unit of good requires one unit of labor, the demands for skilled and unskilled labor, L_s^D and L_u^D, are,

$$L_s^D(\xi) = \theta N X_s^H(\xi) + (1-\theta) N X_u^H(\xi), \tag{5.10}$$

$$L_u^D(\xi) = \theta N X_s^L(\xi) + (1-\theta) N X_u^L(\xi). \tag{5.11}$$

The labor supply is constructed in a similar way using the individual labor supplies. Individual labor supply as function of relative wage is, using (5.9),

$$\ell_s^H(\xi) = \ell_\sigma^*(w_s, w_s; Q) = \frac{Q^\sigma \overline{\ell}}{Q^\sigma + 1}, \qquad \sigma < \hat{\sigma},$$
$$\ell_s^L(\xi) = \ell_\sigma^*(w_u, w_s; 1) = \frac{\overline{\ell}}{\xi^{1-\sigma} + 1}, \qquad \sigma > \hat{\sigma}$$
$$\ell_u^H(\xi) = \ell_\sigma^*(w_s, w_u; Q) = \frac{Q^\sigma \overline{\ell}}{Q^\sigma + \xi^{\sigma-1}}, \qquad \sigma < \hat{\sigma},$$

$$\ell_u^L(\xi) = \ell_\sigma^*(w_u, w_u; 1) = \frac{\bar{\ell}}{2}, \qquad \sigma > \hat{\sigma}.$$

Aggregation yields the total labor supply of each type:

$$L_s^S = N\bar{\ell}\int_1^{\hat{\sigma}} \left\{ \theta \frac{Q^\sigma}{Q^\sigma + 1} + (1-\theta)\frac{Q^\sigma}{Q^\sigma + \xi^{\sigma-1}} \right\} d\sigma, \tag{5.12}$$

$$L_u^S = N\bar{\ell}\int_{\hat{\sigma}}^{\infty} \left\{ \theta \frac{Q^\sigma}{Q^\sigma + \xi^{1-\sigma}} + (1-\theta)\frac{1}{2} \right\} d\sigma. \tag{5.13}$$

It is easy to show, from (5.8), that $\hat{\sigma}$ is decreasing in ξ, that L_s^D and L_u^S are decreasing in $\xi = w_s / w_u$, and that L_s^S and L_u^D are increasing in ξ. The equilibrium relative wage for a given quality level, $\xi^*(Q)$, is determined by the skilled labor market-clearing condition

$$L_s^D(\xi) = L_s^S(\xi).$$

The unskilled labor market has cleared by Walrus's law.

<h2>5.3.2 Comparative Statics

We first see how the equilibrium labor supply and relative wage change with quality.

<h4>Claim 3 *(i)* L_s^S, L_u^S, *and* L_s^D *are increasing and* L_u^D *are decreasing in* Q.

(ii) Equilibrium relative wages and level of skilled labor are increasing in quality. That is, $\partial\xi^*(Q)/\partial Q > 0$ *and* $\partial L_s^*(Q)/\partial Q > 0$.

(These relationships are illustrated in figures 5.6 and 5.7; proof of this claim is in the appendix.) Higher quality products make consumption attractive for skilled workers and also increase the proportion of all workers that consume the high-quality products. Thus both demand and supply of skilled labor increasing in quality. The same effect decreases the supply of unskilled workers and demand is reduced for low-quality goods. Hence demand for unskilled workers decreases as product quality increases.

FIGURE 5.6 HERE

FIGURE 5.7 HERE

From (5.12), we have that skilled labor supply is increasing in population, $\partial L_s^S/\partial N > 0$, and from (5.10), we have that demand is also increasing in population, $\partial L_s^D/\partial N > 0$. (See the proof of claim 3 in the appendix.) So the implication is that:

<h4>**Claim 4** *Both equilibrium skilled and unskilled labor will increase when population increases,* $\partial L_s^* / \partial N > 0$ *and* $\partial L_u^* / \partial N > 0$.

Again, as demonstrated by the proof of claim 3 in the appendix, both demand and supply of skilled labor are increasing due to the rising number of skilled consumers: $\partial L_s^S / \partial \theta > 0$, by (5.12), and $\partial L_s^D / \partial \theta > 0$, by (5.10).

<h4>**Claim 5** *Equilibrium skilled labor and equilibrium relative wage are increasing in the proportion of skilled consumers,* $\partial L_s^* / \partial \theta > 0$ *and* $\partial \xi^* / \partial \theta > 0$.

<h3>**Birthrate The** number of children per working woman is obtained as follows:

$$n_s^H(\xi) = n_\sigma^*(w_s, w_s; Q) = \frac{\bar{\ell}}{Q^\sigma + 1}, \quad \sigma < \hat{\sigma},$$
$$n_s^L(\xi) = n_\sigma^*(w_u, w_s; 1) = \frac{\bar{\ell}}{\xi^{\sigma-1} + 1}, \quad \sigma > \hat{\sigma}$$
$$n_u^H(\xi) = n_\sigma^*(w_s, w_u; Q) = \frac{\bar{\ell}}{Q^\sigma \xi^{1-\sigma} + 1}, \quad \sigma < \hat{\sigma},$$
$$n_u^L(\xi) = n_\sigma^*(w_u, w_u; 1) = \frac{\bar{\ell}}{2}, \quad \sigma > \hat{\sigma}.$$

For a given wage level, women who consume high-quality goods devote ever more resources to consumption, and so the number of children drops when quality improves. Hence, due to the increasing equilibrium relative wage, we can say the following:

<h4>**Claim 6** *(i) Skilled workers who consume high-quality products have fewer children. That is,* $n_s^H < n_u^H$ *for* $\sigma < \hat{\sigma}$ *and* $n_s^L < n_u^L$ *for* $\sigma > \hat{\sigma}$. *(ii) Skilled workers have fewer children particularly as quality of products improves. That is,* $dn_s^H / dQ < 0$ *for* $\sigma < \hat{\sigma}$ *and* $dn_s^L / dQ < 0$ *for* $\sigma > \hat{\sigma}$. *(iii) Unskilled workers who consume low-quality products have the same number of children even as quality of products improves. That is,* $dn_u^L / dQ = 0$ *for* $\sigma > \hat{\sigma}$.

Although there is an income effect, the substitution effect dominates, and skilled workers that consumer low quality reduce the number of children. For unskilled consumers that buy high-quality goods, improvement in quality makes consumption more attractive (lowers number of children) but as the relative wages become lower and the substitution effect works in the opposite direction. The total effect is not so clear.

<h2>5.3.3 Endogenous Quality and Easterlin Hypothesis

Let us assume that quality level of production is increasing because of the increasing size of the skilled labor. That is, Q = $Q_T(L_s)$ is an increasing function of Q. Subscript T refers to "technology," which is what this relationship reflects. We will denote the inverse relationship between the market equilibrium supply of skilled labor and quality of $L_s^*(Q)$ by $Q = Q_M(L_s)$, which is an increasing function from claim 3. The equilibrium level of labor L_s^* and equilibrium quality level of production, $Q^* = Q_M(L_s^*) = Q_T(L_s^*)$, is where the two curves intersect.

When marginal increase in quality from labor is very large, then the equilibrium is unstable. Graphically this would mean slope of Q_T is steeper than $Q_M\left(Q_T' > Q_M'\right)$. This is the equilibrium case E_1 in figure 5.8. A perturbation away from E_1 results in either a spiral increase in quality and skilled labor supply or a decrease in quality and skilled labor supply. When technology is mature so that marginal product quality improvement is very small, then equilibrium is stable ($Q_T' < Q_M'$). This is equilibrium E_2 in figure 5.8. There may be multiple equilibria, some stable and others unstable. A slight perturbation from low product quality with small skilled labor force will start a spiral of labor and quality improvement until E_2 is reached.

FIGURE 5.8 HERE

Now using claim 4, we analyze the effect of declining population. The claim implies that the $Q_M(L_s)$ function will shift upward in the $L_s - Q$ space (figure 5.9).

FIGURE 5.9 HERE

<h4>Claim 7 *(i) If the technology is in its infancy, then equilibrium product quality and skilled labor supply increase as population declines. That is,*

$$Q'_T > Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial N} < 0, \quad \frac{\partial L_s^*}{\partial N} < 0.$$

(ii) If the technology is mature, then equilibrium product quality and skilled labor supply decrease as the population decliness. That is,

$$Q'_T < Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial N} > 0, \quad \frac{\partial L_s^*}{\partial N} > 0.$$

When the technology is maturedeclining population results ins "contraction" of the economy. That is, product quality and supply of skilled labor are reduced. By claim 6, t lower product quality will increase birthrate by all but the the unskilled workers who consume the high-quality product.. This situation is consistent with the cohort effect.

The situation is different when the increasing marginal returns of technology have not yet been exhausted . The new equilibrium results in more skilled labor and higher quality products. But the products are more polarized in quality, skilled labor has higher relative wages and works more. Utility is derived from more consumption, so there are fewer children. The cohort effect does not hold because the economy adjusts to the lower population level in accord with the available technology.

Now we consider the effect of more skilled workers, using claim 5. Recall that the claim is that the $Q_M(L_s)$ function will shift downward in the $L_s - Q$ space (figure 5.10). Immediately we have the following:

<h4>**Claim 8** *(i) If the technology is in its infancy, then equilibrium quality and skilled labor supply decrease as the number of skilled workers increase. That is,*

$$Q_T' > Q_M' \quad \Rightarrow \quad \frac{\partial Q^*}{\partial \theta} < 0, \quad \frac{\partial L_s^*}{\partial \theta} < 0.$$

(ii) If the technology is mature, then equilibrium quality and skilled labor supply increase as the number of skilled workers increase. That is,

$$Q_T' < Q_M' \quad \Rightarrow \quad \frac{\partial Q^*}{\partial \theta} > 0, \quad \frac{\partial L_s^*}{\partial \theta} > 0.$$

FIGURE 5.10 HERE

Equilibrium quality will decrease (increase) when technology is in its infancy (maturity). When the number of skilled workers increase, each skilled worker needs to supply less labor to maintain the same quality. When the marginal quality of labor is very large, the product quality must be lower to accommodate it. Lower quality labor (and lower wages) is likely to entail higher birthrates. Thus, when technology is sufficiently productive, the increasing proportion of skilled workers will increase the birthrate. Then again, when the marginal product of labor is low, higher labor entails higher quality. As a result the birthrate may fall.

Claims 7 and 8 suggest that increasing the proportion of skilled labor can be effective in reversing the decline in the birthrate whenever the cohort effect does not hold. This is the case when the marginal return from increasing the skilled labor is large. Then again, as the technology matures, the Esterlin hypothesis is likely to hold, and the same policy will prevent the feedback mechanism that would otherwise function.

<h1>5.4 Concluding Remarks

We have presented an alternative explanation of the positive relationship between total fertility rate (TFR) and female labor participation rate (FLPR) observed in a cross section of OECD countries in recent years.

We first argued that when consumption and child rearing require both time and goods, there will always be a negative relationship between consumption and number of children. However, the relationship between children and labor participation is not clear. To test the theory, we employed Japanese cross-sectional data from eight different points in time (every five years from 1970 to 2005) and found that also a positive correlation has existed in recent years between TFR and FLPR. Still, we found that FLPR has a significantly negative effect on TFR after we accounted for the unobservable heterogeneity, simultaneity, or endogeneity problem and the measurement error problem by fixed effect IV estimation. We observed that the use of fixed effects instrumental variables model (FE-IV model) guarantees a consistent estimator, and furthermore that consumption variables are statistically significant and have negative impact on TFR. These results are consistent with our new model as well as traditional economic models on the relation between TFR and FLPR.

Using a general equilibrium model with vertical quality differentiation and heterogeneous labor, we showed how low fertility is associated with consumption of higher quality products. Higher product qualityt has two effects: it makes consumption relatively more attractive to having children but also increases wage for skilled workers. The second effect makes working more attractive reducing the birthrate. There will be a negative relationship between birthrate and consumption.

General equilibrium analysis suggests that if the technology is productive enough, the economy will adjust to smaller population, and the cohort effect does not reverse the trend of a declining population. We also showed that increasing the proportion of skilled consumers (potential workers) can increase birthrate and reverse the trend precisely when the cohort effect does not hold. We note that the same relationship between population size and number of skilled consumers means that changing the number employed can prevent the natural feedback mechanism from functioning as it should have functioned.

The two situations are further characterized by technology, where high marginal return comes from skilled labor at the introduction of a new technology and is exhausted with mature technology. The economy may correct itself when technology matures, where we observed the equilibrium to be stable. Therefore a possible policy solution is to push technology quickly to maturity.[1]

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