5.5 Comparison with duopoly equilibrium

¿From Lemma 4, we reproduce below the duopoly equilibrium for the parameter range $\frac{2275}{1632}\tau \approx 1.39\tau \leq V < 1.76\tau$. For firm x

$$x = \frac{47\tau - 23V}{36\tau},\ p_x = \frac{13V + 11\tau}{36},\text{ and } \pi_x = \frac{(13V + 11\tau)^2}{1440\tau}.$$

For firm y, $y = 0$,

$$p_y = \frac{4V + x\tau + p_x}{10} = \frac{67V + 29\tau}{180},$$

and

$$\pi_y = \frac{(4V + x\tau + p_x)^2}{40\tau} = \frac{(67V + 29\tau)^2}{12\,960\tau}.$$

Compare three firm oligopoly and doupoly:

For the most specilised product: $p_z \geq p_y$ (Duopoly) if

$$\frac{619}{816}\tau \geq \frac{67V + 29\tau}{180} \text{ or if } V \leq \frac{7313}{4556}\tau \approx 1.61\tau.$$

p_x (Three firm) $\geq p_x$ (Duopoly) if

$$\frac{35}{34}\tau \geq \frac{13V + 11\tau}{36} \text{ or if } V \leq \frac{443}{221}\tau \approx 2\tau.$$

p_x (Three firm) $\geq p_y$ (Duopoly) if

$$\frac{35}{34}\tau \geq \frac{67V + 29\tau}{180} \text{ or if } V \leq \frac{2657}{1139}\tau \approx 2.33\tau.$$

Therefore, for the parameter range we focus on, the prices for more specilised products always increase when the number of firms increase from 2 to 3. The price for the most standardised produce increases if V is small and decreases if V is large. Note that for the parameter range we focus on, the standard product is not offered in the three firm oligopoly.

6 Welfare Analysis

With product differentiation, the welfare effect of a price incrase upon entry is in general ambigous. Although we have demonstrated price increase with entry, given the new product in the market and less travel cost for the consumers, total welfare or even consumer surplus may still increase. We explore in this section the welfare comparison between duopoly and three firm oligopoly with the restriction on parameters, $\frac{2275}{1632}\tau \approx 1.39\tau \leq V < 1.76\tau$.

6.1 Duopoly

Note that for the relevant equilibrium, $x > 0$, $y = 0$, $V(t_{xy}) \geq 0$, $\bar{t}_x = 1$. The consumer surplus for consumers who purchase from x is equal to

$$\begin{aligned} CS_x &= \frac{1}{2}(1-x)(V-p_x) + \frac{1}{2}(V - p_x + V(t_{xy}))(x - t_{xy}) \\ &= \frac{1}{2}(1-x)(V-p_x) + \frac{1}{2}(V - p_x + V - \tau(x - t_{xy}) - p_x)(x - t_{xy}). \end{aligned}$$

With $y = 0$,

$$t_{xy} = \frac{p_x - p_y + \tau x}{2\tau} = \frac{29\tau - 13V}{40\tau}.$$

This gives

$$\begin{aligned} CS_x &= \frac{1}{2}(1-x)(V-p_x) + \frac{1}{2}\left(2V - 2p_x - \tau\left(x - \frac{(29\tau - 13V)}{40\tau}\right)\right)\left(x - \frac{(29\tau - 13V)}{40\tau}\right) \\ &= \frac{1}{2}\left(1 - \frac{47\tau - 23V}{36\tau}\right)\left(V - \frac{13V + 11\tau}{36}\right) \\ &\quad + \frac{1}{2}\left(2V - 2\left(\frac{13V + 11\tau}{36}\right) - \tau\left(\frac{47\tau - 23V}{36\tau} - \frac{(29\tau - 13V)}{40\tau}\right)\right)\left(\frac{47\tau - 23V}{36\tau} - \frac{(29\tau - 13}{40\tau}\right. \\ &= \frac{(23V - 11\tau)^2}{2592\tau} + \frac{(191V - 143\tau)(209\tau - 113V)}{86\,400\tau}. \end{aligned}$$

For consumers who purchase from y, the consumer surplus is equal to

$$CS_y = \bar{t}_y(V - p_y) + \frac{1}{2}(V - p_y + V(t_{xy}))(t_{xy}).$$

With $y = 0$,

$$\bar{t}_y = \frac{V - p_y}{\tau} = \frac{113V - 29\tau}{180\tau}.$$

This gives

$$\begin{aligned} CS_y &= \frac{113V - 29\tau}{180\tau}\left(V - \frac{67V + 29\tau}{180}\right) \\ &\quad + \frac{1}{2}\left(2V - 2\left(\frac{67V + 29\tau}{180}\right) - \tau\left(\frac{29\tau - 13V}{40\tau}\right)\right)\left(\frac{29\tau - 13V}{40\tau}\right) \\ &= \frac{(113V - 29\tau)^2}{32\,400\tau} + \frac{(569V - 377\tau)(29\tau - 13V)}{28\,800\tau}. \end{aligned}$$

6.2 Three firm oligopoly

For consumers purchasing from firm x

$$\begin{aligned}
CS_x &= \frac{1}{2}(1-x)(V-p_x)+\frac{1}{2}(V-\tau(x-t_{xz})-p_x+V-p_x)(x-t_{xz}) \\
&= \frac{1}{2}\left(1-\left(\frac{69}{34}-\frac{V}{\tau}\right)\right)\left(V-\frac{35}{34}\tau\right) \\
&\quad +\frac{1}{2}\left(2V-\tau\left(\frac{69}{34}-\frac{V}{\tau}-\frac{13}{48}\right)-2\left(\frac{35}{34}\tau\right)\right)\left(\frac{69}{34}-\frac{V}{\tau}-\frac{13}{48}\right) \\
&= \frac{(34V-35\tau)^2}{2312\tau}+\frac{(2448V-3115\tau)(1435\tau-816V)}{1331\,712\tau}.
\end{aligned}$$

For consumers who purchase from firm y

$$\begin{aligned}
CS_y &= \frac{1}{2}(1-y)(V-p_y)+\frac{1}{2}(V-\tau(y-t_{yz})-p_y+V-p_y)(y-t_{yz}) \\
&= \frac{1}{2}\left(1-\left(\frac{409}{204}-\frac{V}{\tau}\right)\right)\left(V-\frac{205}{204}\tau\right) \\
&\quad +\frac{1}{2}\left(2V-\tau\left(\frac{409}{204}-\frac{V}{\tau}-\frac{67}{272}\right)-2\left(\frac{205}{204}\tau\right)\right)\left(\frac{409}{204}-\frac{V}{\tau}-\frac{67}{272}\right) \\
&= \frac{(204V-205\tau)^2}{83\,232\tau}+\frac{(816V-1025\tau)(1435\tau-816V)}{443\,904\tau}.
\end{aligned}$$

For consumers who purchase from firm z

$$\begin{aligned}
CS_z &= \frac{1}{2}(1-z)(V-p_z)+\frac{1}{2}(V-p_z+V-\tau(t_{xz}+z)-p_z)(z+t_{xz}) \\
&\quad +\frac{1}{2}(V-\tau z-p_z+V-\tau(z+t_{yz})-p_z)(t_{yz}) \\
&= \frac{1}{2}\left(1-\left(\frac{1435}{816}-\frac{V}{\tau}\right)\right)\left(V-\frac{619}{816}\tau\right) \\
&\quad +\frac{1}{2}\left(2V-2\left(\frac{619}{816}\tau\right)-\tau\left(\frac{13}{48}+\frac{1435}{816}-\frac{V}{\tau}\right)\right)\left(\frac{1435}{816}-\frac{V}{\tau}+\frac{13}{48}\right) \\
&\quad +\frac{1}{2}\left(2V-2\tau\left(\frac{1435}{816}-\frac{V}{\tau}\right)-2\left(\frac{619}{816}\tau\right)-\tau\left(\frac{67}{272}\right)\right)\left(\frac{67}{272}\right) \\
&= \frac{(816V-619\tau)^2}{1331\,712\tau}+\frac{(1224V-1447\tau)(69\tau-34V)}{27\,744\tau} \\
&\quad +\frac{67(3264V-4309\tau)}{443\,904}.
\end{aligned}$$

The total consumer welfare is higher with three firm oligopoly if

$$\begin{aligned}
& \frac{(34V-35\tau)^2}{2312\tau}+\frac{(2448V-3115\tau)(1435\tau-816V)}{1331\,712\tau} \\
& +\frac{(204V-205\tau)^2}{83\,232\tau}+\frac{(816V-1025\tau)(1435\tau-816V)}{443\,904\tau} \\
& +\frac{(816V-619\tau)^2}{1331\,712\tau}+\frac{(1224V-1447\tau)(69\tau-34V)}{27\,744\tau} \\
& +\frac{67(3264V-4309\tau)}{443\,904} \\
\geq\ & \frac{(23V-11\tau)^2}{2592\tau}+\frac{(191V-143\tau)(209\tau-113V)}{86\,400\tau} \\
& +\frac{(113V-29\tau)^2}{32\,400\tau}+\frac{(569V-377\tau)(29\tau-13V)}{28\,800\tau}.
\end{aligned}$$

Or if

$$30\,877\,916V^2-105\,782\,296\tau V+89\,329\,469\tau^2\leq 0.$$

This holds for

$$\frac{1555\,622-\sqrt{33\,880\,360\,425}}{908\,174}\tau\approx 1.51\tau\leq V\leq\frac{1555\,622+\sqrt{33\,880\,360\,425}}{908\,174}\tau\approx 1.92\tau$$

Therefore, for our parameter range, total consumer welfare increases for $1.51\tau\leq V<1.76\tau$.

7 Conclusion

We utilise a new product space specification to study firms' incentive to customise. At first glance, the product space looks similar to Chen and Riordan (2005). However, the interpretation is very different. In Chen and Riordan, they do not analyse firms' location choice. Each firm provides one variety and is located at the end point of each spoke. In our model, the product space gives a natural interpretation of standard versus customised products. Our results indicate that in a sequential move game, the first mover always offers the standard product. The follower customises. Competition among firms softens as consumers' travelling costs increase.

8 Appendix

Proof. of Lemma 3

Case (A) If $p_x\leq V-\tau$

Firm x solves

$$\max_{p_x} \left(\frac{4\tau + p_y - p_x + \tau y}{2\tau} \right) p_x = \frac{(11\tau - V - p_x)}{4\tau} p_x.$$

The FOC gives

$$p_x = \frac{11\tau - V}{2}.$$

Checking the boundary values:

$$\frac{11\tau - V}{2} \leq V - \tau \text{ if } V \geq \frac{13}{3}\tau \approx 4.33\tau.$$

For $y \geq 0$, we require

$$\frac{11\tau - V}{2} \geq 3V - 5\tau \text{ or } V \leq 3\tau.$$

Therefore, for $V \leq 3\tau$, $y > 0$ and the optimal $p_x^* = \widetilde{p}_x$. For $V \geq 3\tau$, the optimal $y = 0$ and $x = 0$ is never an equilibrium. At $p_x = V - \tau$,

$$\pi_y (y = x = 0) = 3 (V - \tau).$$

On the other hand, if y locates at $y > 0$, $\pi_y = \frac{V^2}{4\tau}$. Firm y has the incentive to locate at the centre if

$$\frac{V^2}{4\tau} \leq 3 (V - \tau).$$

Or if

$$V^2 - 12\tau V + 12\tau^2 \leq 0.$$

This holds for

$$\left(6 - \sqrt{24}\right) \tau \approx 1.1\tau \leq V \leq \left(6 + \sqrt{24}\right) \tau \approx 10.9\tau.$$

Therefore, for $\tau < V \leq (6 - \sqrt{24})\, \tau \approx 1.1\tau$, firm y does not have an incentive to locate at the centre and $p_x = V - \tau$ is a local maximum for $x = 0$. For $(6 - \sqrt{24})\, \tau \leq V \leq 3\tau$, firm y would have the incentive to locate at the centre and firm x faces the price under-cutting constraint. To make firm y indifferent between staying off the centre and locating at the centre, firm x needs to charge a price such that

$$3p_x = \frac{(V + \tau + p_x)^2}{16\tau}.$$

This holds for

$$p_x = 23\tau - V \pm \sqrt{48\tau (11\tau - V)}.$$

Note that $23\tau - V - \sqrt{48\tau(11\tau - V)} > 0$.

$$23\tau - V - \sqrt{48\tau(11\tau - V)} \leq V - \tau$$

if $V^2 - 12V\tau + 12\tau^2 \leq 0$. This holds for the parameter range. Also $23\tau - V + \sqrt{48\tau(11\tau - V)} > V - \tau$.

Therefore, the local equilibrium in this case is that for $V \leq (6 - \sqrt{24})\tau \approx 1.1\tau$, $p_x = V - \tau$ and for $(6 - \sqrt{24})\tau \approx 1.1\tau \leq V \leq 3\tau$, $p_x = 23\tau - V - \sqrt{48\tau(11\tau - V)}$.

The constraint $y \geq 0$ requires

$$23\tau - V - \sqrt{48\tau(11\tau - V)} \geq 3V - 5\tau.$$

Or

$$(7\tau - V) \geq \sqrt{3\tau(11\tau - V)}$$

$$7\tau - V \geq 0 \text{ if } V \leq 7\tau.$$

Therefore, this does not hold for $V \geq 7\tau$. For $V < 7\tau$, this holds for

$$V \leq \frac{11 - \sqrt{57}}{2}\tau \approx 1.73\tau.$$

At $p_x = 23\tau - V - \sqrt{48\tau(11\tau - V)}$, the resulting

$$\pi_x = \frac{(11\tau - V - p_x)}{4\tau}p_x = \frac{\left(V - 23\tau + 4\sqrt{\tau(33\tau - 3V)}\right)\left(3\tau - \sqrt{\tau(33\tau - 3V)}\right)}{\tau}.$$

Case (B) If $V - \tau \leq p_x \leq \frac{5}{3}V - \tau$.

Firm x solves

$$\max_{p_x} \pi_x = \frac{4V + y\tau - 5p_x + p_y}{2\tau}p_x = \frac{7Vp_x + 3\tau p_x - 9p_x^2}{4\tau}.$$

The FOC gives the local maximiser

$$p_x^* = \frac{7V + 3\tau}{18}.$$

Checking the boundary values:

$$\frac{7V + 3\tau}{18} \geq V - \tau \text{ if } V \leq \frac{21}{11}\tau \approx 1.91\tau.$$

$$\frac{7V + 3\tau}{18} \leq \frac{5}{3}V - \tau \text{ if } V \geq \frac{21}{23}\tau \approx 0.91\tau.$$

Therefore, for $V \leq \frac{21}{23}\tau$, the solution is a corner solution with the optimal $p_x^* = \frac{5}{3}V - \tau$. For $\frac{21}{23}\tau \leq V \leq \frac{21}{11}\tau$, the optimal $p_x = \frac{7V+3\tau}{18}$. For $\frac{21}{11}\tau \leq V$, the solution occurs at $p_x = V - \tau$. The analysis is performed in Case (A).

For $\frac{21}{23}\tau \approx 0.91\tau \leq V \leq \frac{21}{11}\tau$, $p_x = \frac{7V+3\tau}{18}$. If $y > 0$,

$$\pi_y = \frac{(V + \tau + p_x)^2}{16\tau} = \frac{(25V + 21\tau)^2}{5184\tau}.$$

¿From Lemma 2, if $y = x = 0$,

$$\pi_y \left(p_y = \frac{7V + 3\tau}{18} \right) = 3\frac{V - p_y}{\tau} (p_y) = \frac{(7V + 3\tau)(11V - 3\tau)}{108\tau}.$$

Firm y would choose to locate at the centre and undercut p_x if

$$\frac{(25V + 21\tau)^2}{5184\tau} \leq \frac{(7V + 3\tau)(11V - 3\tau)}{108\tau}.$$

This holds for

$$V \geq \frac{237 + \sqrt{2737\,152}}{3071}\tau \approx 0.62\tau.$$

Therefore, when the solution is interior, for the relevant parameter range, firm y always has the incentive to locate at $y = x = 0$ and undercut p_x.To eliminate y's incentive to undercut, x needs to price such that

$$3\frac{V - p_x}{\tau} (p_x) = \frac{(V + \tau + p_x)^2}{16\tau}.$$

This gives the constrained price

$$p_x = \frac{23V - \tau - \sqrt{48(2V - \tau)(5V + \tau)}}{49}.$$

Note that $\left. \frac{\partial(3\frac{V-p_x}{\tau}p_x)}{\partial p_x} \right|_{p_x = \frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49}} > 0$ and indeed firm y would not have the incentive to undercut p_x at this range..This solution falls into the relevant parameter range if

$$\frac{23V - \tau - \sqrt{48(2V - \tau)(5V + \tau)}}{49} \leq \frac{5}{3}V - \tau.$$

This is true for the relevant parameter range.

$$\frac{23V - \tau - \sqrt{48(2V - \tau)(5V + \tau)}}{49} \geq V - \tau$$

if

$$24\tau - 13V \geq \sqrt{12(2V - \tau)(5V + \tau)}.$$

$$24\tau - 13V \geq 0 \text{ if } V \leq \frac{24}{13}\tau \approx 1.85\tau.$$

Therefore, the inequality does not hold for $V \geq \frac{24}{13}\tau \approx 1.85\tau$. For $V \leq \frac{24}{13}\tau \approx 1.85\tau$, the inequality holds if

$$V \leq \left(6 - \sqrt{24}\right)\tau \approx 1.1\tau.$$

Therefore, for $\frac{21}{23}\tau \leq V \leq \left(6 - \sqrt{24}\right)\tau$, the local optimal price for firm x is $p_x = \frac{23V - \tau - \sqrt{48(2V-\tau)(5V+\tau)}}{49}$. The resulting profit for firm x is

$$\Pi_x = \frac{\left(9\sqrt{48\,(2V-\tau)\,(5V+\tau)} + 136V + 156\tau\right)\left(23V - \sqrt{48\,(2V-\tau)\,(5V+\tau)} - \tau\right)}{9604\tau}.$$

Case (C) If $\frac{5}{3}V - \tau < p_x < 2V - \tau$

In this case, given p_x, if $y > 0$, firm y chooses (y, p_y) such that $\bar{t}_x = \underline{t}_y$. If $y = 0$, y chooses $p_y = p_x - \varepsilon$. In equilibrium $\varepsilon \to 0$.

$$\frac{5}{3}V - \tau \leq \frac{V}{2} \text{ if } V \leq \frac{6}{7}\tau \approx 0.86\tau.$$

For $\frac{6}{7}\tau \leq V \leq \frac{21}{23}\tau$: both local maximisers fall outside of the relevant regions and the global maximum occurs at $p_x^* = \frac{5}{3}V - \tau$ with the resulting profit

$$\pi_x = 3\frac{V - p_x}{\tau}p_x = \frac{(5V - 3\tau)(3\tau - 2V)}{3\tau}.$$

For $p_x = \frac{5}{3}V - \tau$ and $y > 0$,

$$\pi_y = \frac{(\tau - V + p_x)(3V - \tau - p_x)}{2\tau} = \frac{4V^2}{9\tau}.$$

Firm y would have the incentive to locate at the centre for the relevant parameter range. Therefore, firm x needs to price below $p_x = \frac{5}{3}V - \tau$ and eliminate firm y's incentive to locate at the centre. For $p_x \leq \frac{5}{3}V - \tau$, p_y's best response is analysed in Case (B) above.

For $\frac{2}{3}\tau \leq V \leq \frac{6}{7}\tau$, $\frac{5}{3}V - \tau \leq \frac{V}{2} \leq 2V - \tau$ and the local optimal p_x without the price undercutting constraint is $p_x = \frac{V}{2}$. Firm y's best response when $y > 0$ is to price such that $\underline{t}_y = \bar{t}_x$. Since firm x gets effectively the unconstrained monopoly profit, firm y would always get less profit locating off the centre and would always has the incentive to locate at $y = 0$. To eliminate firm y's incentive to undercut, firm x needs to price such that

$$3\frac{V - p_x}{\tau}p_x = \frac{(\tau - V + p_x)(3V - \tau - p_x)}{2\tau}.$$

This gives the constrained p_x

$$p_x = \frac{V + \tau - \sqrt{2\left(8V^2 - 9V\tau + 3\tau^2\right)}}{5}.$$

Note that $\frac{V+\tau-\sqrt{2(8V^2-9V\tau+3\tau^2)}}{5} \geq 0$ in the relevant parameter range.

$$\frac{V + \tau - \sqrt{2\left(8V^2 - 9V\tau + 3\tau^2\right)}}{5} \geq \frac{5}{3}V - \tau$$

if

$$6\tau - \frac{22}{3}V \geq \sqrt{2\left(8V^2 - 9V\tau + 3\tau^2\right)}.$$

This does not hold if

$$6\tau - \frac{22}{3}V \leq 0 \text{ or } V \geq \frac{9}{11}\tau \approx 0.82\tau.$$

For $V \leq \frac{9}{11}\tau \approx 0.82\tau$, this holds for

$$V \leq \frac{63 - \sqrt{297}}{68}\tau \approx 0.673\tau.$$

$$\frac{V + \tau - \sqrt{2\left(8V^2 - 9V\tau + 3\tau^2\right)}}{5} \leq 2V - \tau$$

if

$$6\tau - 9V \leq \sqrt{2\left(8V^2 - 9V\tau + 3\tau^2\right)}.$$

This holds in the relevant parameter range.

Therefore, for $\frac{63-\sqrt{297}}{68}\tau \approx 0.67\tau \leq V \leq \frac{6}{7}\tau$, the equilibirum falls into the first part of firm y's best response. The analysis is presented in Case (B). For $\frac{2}{3}\tau \leq \tau \leq \frac{63-\sqrt{297}}{68}\tau \approx 0.67\tau$, the optimal p_x is $p_x = \frac{V+\tau-\sqrt{2(8V^2-9V\tau+3\tau^2)}}{5}$. The resulting π_x is

$$\pi_x = \frac{3\left(\sqrt{2\left(8V^2 - 9V\tau + 3\tau^2\right)} + 4V - \tau\right)\left(V - \sqrt{2\left(8V^2 - 9V\tau + 3\tau^2\right)} + \tau\right)}{25\tau}.$$

Case (D) If $2V - \tau \leq p_x$

When $p_x \geq 2V - \tau$, both firms act as local monopolist. The relevant demand is $D^x = 3\frac{V-p_x}{\tau}$. The local maximiser is interior if

$$\frac{V}{2} \geq 2V - \tau \text{ or } V \leq \frac{2}{3}\tau.$$

For $p_x \geq 2V - \tau$, to make firm y has no incentive to choose $y = 0$, firm x has to choose a price such that

$$3\frac{V - p_x}{\tau} p_x = \frac{V^2}{2\tau}.$$

This gives the constrained p_x:

$$p_x = \frac{3 - \sqrt{3}}{6} V.$$

Note that $\left.\frac{\partial(3\frac{V-p_x}{\tau}p_x)}{\partial p_x}\right|_{p_x = \frac{3-\sqrt{3}}{6}V} > 0$ and firm y has no incentive to undercut at this price.

$$\frac{3 - \sqrt{3}}{6} V \geq 2V - \tau$$

if

$$V \leq \frac{6}{9 + \sqrt{3}} \tau \approx 0.56\tau.$$

Therefore, for $V \leq \frac{6}{9+\sqrt{3}}\tau \approx 0.56\tau$, the local optimal $p_x = \frac{3-\sqrt{3}}{6}V$. For $\frac{6}{9+\sqrt{3}}\tau \approx 0.56\tau \leq V \leq \frac{2}{3}\tau$, the price necessary for firm y not to have the incentive to locate at $y = 0$ is less than $2V - \tau$. Therefore, the equilibrium falls into the second part of firm y's best response with $\underline{t}_y = \bar{t}_x$. The equilibrium price is $p_x = \frac{V+\tau-\sqrt{2(8V^2-9V\tau+3\tau^2)}}{5}$. Note that for $\frac{6}{9+\sqrt{3}}\tau \approx 0.56\tau \leq V \leq \frac{2}{3}\tau$, $\frac{V+\tau-\sqrt{2(8V^2-9V\tau+3\tau^2)}}{5} \leq 2V - \tau$. ■

Proof. of Lemma 4

We proceed according to y's global best response. The analysis here focuses on y's best response for $p_x \geq 6V - 10\tau - x\tau$. For $p_x < 6V - 10\tau - x\tau$, the solution is always a corner solution with $p_x^* \geq 6V - 10\tau - x\tau$.

Case (A) $p_x \leq \frac{16V - 11x\tau}{11}$

For this case, $\bar{t}_y \geq \underline{t}_x$. This gives firm x's profit maximisation problem

$$\max_{p_x} \pi_x = (1 - t_{xy})\, p_x = \left(1 - \frac{\tau x + p_x - p_y}{2\tau}\right) p_x.$$

Substitute in the condition that $\bar{t}_x = 1$ into firm y's best response gives

$$p_y = \frac{4V + \tau x + p_x}{10} = \frac{(3V + \tau + 2p_x)}{10}.$$

The maximisation problem is then

$$\max_{p_x} \pi_x = \frac{(13V + 11\tau - 18p_x)\, p_x}{20\tau}.$$

The FOC gives

$$p_x = \frac{13V + 11\tau}{36} \text{ and } x = \frac{47\tau - 23V}{36\tau}.$$

The resulting profit is

$$\pi_x = \frac{(13V + 11\tau)^2}{1440\tau}.$$

After substituting in the best location, $\bar{t}_x = 1$, or $x = 1 - \frac{V - p_x}{\tau}$, the boundary for this case is

$$p_x \leq \frac{27V - 11\tau}{22}.$$

The solution is interior if

$$\frac{13V + 11\tau}{36} \leq \frac{27V - 11\tau}{22} \text{ or } V \geq \frac{319}{343}\tau \approx 0.93\tau.$$

Case (B) $\frac{16V - 11x\tau}{11} \leq p_x \leq \frac{3}{2}V - x\tau$

For this case, y prices such that $\bar{t}_y = \underline{t}_x$. The optimal (x, p_x) combination should still satisfy the condition that $\bar{t}_x = 1$. This gives the boundary of this region

$$\frac{27V - 11\tau}{22} \leq p_x \leq \frac{5V - 2\tau}{4}.$$

Firm x's beset pricing is to charge $\frac{V}{2}$ if possible.

$$\frac{V}{2} \geq \frac{27V - 11\tau}{22} \text{ if } V \leq \frac{11}{16}\tau \approx 0.69\tau.$$

For $V \leq \frac{11}{16}\tau \approx 0.69\tau$, firm x charges $p_x = \frac{V}{2}$ with $\pi_x = \frac{V^2}{2\tau}$. For $V > \frac{11}{16}\tau$, the constrained optimisation is to charge $p_x = \frac{27V - 11\tau}{22}$.

$$\pi_x = \frac{(13V + 11\tau - 18p_x)\, p_x}{20\tau} = \frac{(27V - 11\tau)\,(11\tau - 5V)}{242\tau}.$$

In this case,

$$x = 1 - \frac{V - p_x}{\tau} = \frac{(5V + 11\tau)}{22\tau}.$$

Case (C) $p_x \geq \frac{3}{2}V - x\tau$

The optimal pricing for firm x when it is a local monopolist is $\frac{V}{2}$. Therefore, firm y also acts as a local monopolist if

$$\frac{V}{2} \geq \frac{3}{2}V - x\tau \text{ or if } x \geq \frac{V}{\tau}.$$

When there is enough space for firm x to act as a local monopolist, there is no unique sulution for the optimal location, x^*. All locations satisfying

$\underline{t}_x \geq \bar{t}_y$ and $\bar{t}_x \leq 1$ are optimal. As noted above, the first condition, $\underline{t}_x \geq \bar{t}_y$, gives $x \geq \frac{V}{\tau}$. The second condition, $\bar{t}_x \leq 1$, gives

$$\frac{V - p_x}{\tau} + x \leq 1 \text{ or } x \leq \frac{2\tau - V}{2\tau}.$$

The two constraints can be satisfied simultaneously if

$$\frac{2\tau - V}{2\tau} \geq \frac{V}{\tau} \text{ or } V \leq \frac{2}{3}\tau.$$

For$V \leq \frac{2}{3}\tau$., both firms can act as local monopolist. For $\frac{2}{3}\tau \leq V \leq \frac{11}{16}\tau$, firm x acts effectively like a local monopolist while firm y prices such that $\bar{t}_y = \underline{t}_x$.

Therefore, the optimal (x, p_x) is

$$(x, p_x) = \begin{cases} \left(x \in \left[\frac{V}{\tau}, \frac{2\tau - V}{2\tau}\right], \frac{V}{2}\right) & V \leq \frac{2}{3}\tau \\ \left(\frac{2\tau - V}{2\tau}, \frac{V}{2}\right) & \frac{2}{3}\tau \leq V \leq \frac{11}{16}\tau \\ \left(\frac{5V+11\tau}{22\tau}, \frac{27V-11\tau}{22}\right) & \frac{11}{16}\tau \leq V \leq \frac{319}{343}\tau \\ \left(\frac{47\tau-23V}{36\tau}, \frac{13V+11\tau}{36}\right) & \text{if } \frac{319}{343}\tau \leq V \leq \frac{47}{23}\tau \end{cases}.$$

$$\pi_x = \begin{cases} \frac{V^2}{2\tau} & V \leq \frac{11}{16}\tau \\ \frac{(27V-11\tau)(11\tau-5V)}{242\tau} & \frac{11}{16}\tau \leq V \leq \frac{319}{343}\tau \\ \frac{(13V+11\tau)^2}{1440\tau} & \text{if } \frac{319}{343}\tau \leq V \leq \frac{47}{23}\tau \end{cases}.$$

For $V \geq \frac{47}{23}\tau$, the solution would not occur at $x > y \geq 0$ and $r_x = r_y$. This completes the analysis for the case $x > y \geq 0$ and $r_x = r_y$. ■

Proof. of Proposition 2

$$p_x^* = \begin{cases} \frac{3-\sqrt{3}}{6}V & V \leq \frac{6}{9+\sqrt{3}}\tau \\ \frac{V+\tau-\sqrt{2(8V^2-9V\tau+3\tau^2)}}{5} & \frac{6}{9+\sqrt{3}}\tau \approx 0.56\tau \leq V \leq \frac{63-\sqrt{297}}{68}\tau \\ \frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49} & \frac{63-\sqrt{297}}{68}\tau \approx 0.673\tau \leq V \leq (6-\sqrt{24})\,\tau \\ 23\tau - V - \sqrt{48\tau\,(11\tau - V)} & (6-\sqrt{24})\,\tau \approx 1.1\tau \leq V \leq \frac{11-\sqrt{57}}{2}\tau \approx 1.73\tau \end{cases};$$

$$\pi_x = \begin{cases} \frac{V^2}{2\tau} \\ \frac{3\left(\sqrt{2(8V^2-9V\tau+3\tau^2)}+4V-\tau\right)\left(V-\sqrt{2(8V^2-9V\tau+3\tau^2)}+\tau\right)}{25\tau} \\ \frac{\left(9\sqrt{48(2V-\tau)(5V+\tau)}+136V+156\tau\right)\left(23V-\sqrt{48(2V-\tau)(5V+\tau)}-\tau\right)}{9604\tau} \\ \frac{\left(V-23\tau+4\sqrt{\tau(33\tau-3V)}\right)\left(3\tau-\sqrt{\tau(33\tau-3V)}\right)}{\tau} \end{cases}.$$

$$\pi_x = \begin{cases} \frac{V^2}{2\tau} & V \leq \frac{11}{16}\tau \approx 0.69\tau \\ \frac{(27V-11\tau)(11\tau-5V)}{242\tau} & \frac{11}{16}\tau \leq V \leq \frac{319}{343}\tau \approx 0.93\tau \\ \frac{(13V+11\tau)^2}{1440\tau} & \frac{319}{343}\tau \leq V \leq \frac{47}{23}\tau \approx 2.04\tau \end{cases}.$$

For $V \leq \frac{6}{9+\sqrt{3}}\tau$, x gets $\pi_x = \frac{V^2}{2\tau}$ in either cases and is indifferent between offering the standard or a customised product. Firm y however, gets higher profit if x locates off the centre. For $\frac{6}{9+\sqrt{3}}\tau \leq V \leq \frac{11}{16}\tau$, firm x gets higher profit locating off the centre. We discuss the remaining of the cases in turn.

Case (A) $\frac{11}{16}\tau \leq V \leq \frac{319}{343}\tau$

Firm x gets higer profit locating off the centre if

$$\frac{(27V - 11\tau)(11\tau - 5V)}{242\tau}$$
$$\geq \frac{\left(9\sqrt{48(2V-\tau)(5V+\tau)} + 136V + 156\tau\right)\left(23V - \sqrt{48(2V-\tau)(5V+\tau)} - \tau\right)}{9604\tau}.$$

It is difficult to sign this inequality directly. However, we can utilise a p_x, $\frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49} \leq p_x \leq \frac{V}{2}$, such that

$$3\frac{V-p_x}{\tau}(p_x)\bigg|_{p_x} \geq 3\frac{V-p_x}{\tau}(p_x)\bigg|_{p_x=\frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49}}$$
$$= \frac{\left(9\sqrt{48(2V-\tau)(5V+\tau)} + 136V + 156\tau\right)\left(23V - \sqrt{48(2V-\tau)(5V+\tau)} - \tau\right)}{9604\tau}$$

for the comparison. Consider $p_x' = \frac{20V-11\tau}{49} \leq \frac{V}{2}$.

$$\frac{20V - 11\tau}{49} \geq \frac{23V - \tau - \sqrt{48(2V-\tau)(5V+\tau)}}{49}$$

if

$$148\tau^2 + 204V\tau - 471V^2 \leq 0$$

This holds for

$$\frac{-51-\sqrt{20028}}{74}V \approx -2.6 \leq \tau \leq \frac{-51+\sqrt{20028}}{74}V \approx 1.223V.$$

Thus $\frac{20V-11\tau}{49} \geq \frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49}$ for the relevant parameter range.

$$3\frac{V-p_x}{\tau}(p_x)\bigg|_{p_x=\frac{20V-11\tau}{49}} = \frac{(29V+11\tau)(20V-11\tau)}{2401\tau}.$$

$$\frac{(27V-11\tau)(11\tau-5V)}{242\tau} \geq \frac{(29V+11\tau)(20V-11\tau)}{2401\tau}$$

if

$$261\,239\tau^2 - 869\,110V\tau + 464\,495V^2 \leq 0.$$

This holds for

$$\frac{39\,505-\sqrt{557\,800\,320}}{23\,749}V \approx 0.67V \le \tau \le \frac{39\,505+\sqrt{557\,800\,320}}{23\,749}V \approx 2.66V$$

This holds in the relevant parameter range. Therefore

$$\begin{aligned}\frac{(27V-11\tau)(11\tau-5V)}{242\tau} &\geq 3\frac{V-p_x}{\tau}(p_x)\bigg|_{p_x=\frac{20V-11\tau}{49}} \\ &> 3\frac{V-p_x}{\tau}(p_x)\bigg|_{p_x=\frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49}}.\end{aligned}$$

and $\pi_x(x>0) > \pi_x(x=0)$.

Case (B) $\frac{319}{343}\tau \le V \le (6-\sqrt{24})\tau$

Firm x prefers locating off the centre if

$$\begin{aligned}&\frac{(13V+11\tau)^2}{1440\tau} \\ \geq\ & \frac{\left(9\sqrt{48(2V-\tau)(5V+\tau)}+136V+156\tau\right)\left(23V-\sqrt{48(2V-\tau)(5V+\tau)}-\tau\right)}{9604\tau}.\end{aligned}$$

Given the complicated expression of the equilibrium price imposed by the price undercutting constraint when $x=0$, we cannot sign this inequality directly. However, given the shape of the profit function, we can sign this inequality by working with a p_x close enough to the equilibirum price, $\frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49}$.

Recall that $p_x = \frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49}$ solves

$$3\frac{V-p_x}{\tau}(p_x) = \frac{(V+\tau+p_x)^2}{16\tau}.$$

The profit function $3\frac{V-p_x}{\tau}(p_x)$ attains its maximum at $p_x=\frac{V}{2}$. Therefore, for $\frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49} \le p_x \le \frac{V}{2}$, the profit level increases in p_x. Consider $p_x' = \frac{23V-\tau}{49}$. Since $\frac{23V-\tau-\sqrt{48(2V-\tau)(5V+\tau)}}{49} \le \frac{23V-\tau}{49} \le \frac{V}{2}$,

$$3\frac{V-p_x}{\tau}(p_x)\bigg|_{p_x=\frac{23V-\tau}{49}} = \frac{3(26V+\tau)(23V-\tau)}{2401\tau}$$

$$\frac{(13V+11\tau)^2}{1440\tau} \geq \frac{3(26V+\tau)(23V-\tau)}{2401\tau}$$

if

$$294\,841\tau^2+699\,646V\tau-2177\,591V^2 \geq 0.$$

This holds for

$$\frac{-349\,823-\sqrt{764\,419\,239\,360}}{294\,841}V \approx -4.15V \leq \tau \leq \frac{-349\,823+\sqrt{764\,419\,239\,360}}{294\,841}V \approx 1.78V$$

This holds in this parameter range and therefore $\pi_x(x>0) \geq \pi_x(x=0)$.

Case (C) $\left(6-\sqrt{24}\right)\tau \leq V \leq \frac{11-\sqrt{57}}{2}\tau \approx 1.73\tau$

Firm x prefers locating off the centre if

$$\frac{(13V+11\tau)^2}{1440\tau} \geq \frac{\left(V-23\tau+4\sqrt{\tau(33\tau-3V)}\right)\left(3\tau-\sqrt{\tau(33\tau-3V)}\right)}{\tau}.$$

Recall that the $p_x(x=0) = 23\tau - V - \sqrt{48\tau(11\tau-V)}$ solves

$$3p_x = \frac{(V+\tau+p_x)^2}{16\tau}.$$

We can pick a higher price for comparision. Let $p_x = 2\tau$.

$$2\tau \geq 23\tau - V - \sqrt{48\tau(11\tau-V)}$$

if

$$V \leq \left(-3+\sqrt{96}\right)\tau \approx 6.8\tau.$$

$$\frac{(13V+11\tau)^2}{1440\tau} \geq 3p_x|_{p_x=2\tau} = 6\tau$$

if

$$169V^2 + 286\tau V - 8519\tau^2 \geq 0.$$

This holds for

$$V \leq \frac{-11+\sqrt{8640}}{13}\tau \approx 6.3\tau.$$

This holds. Therefore, x would always prefer to locate off the centre. ■

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Pre-grant patent publication and cumulative innovation[☆]

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ABSTRACT

We examine the implications of pre-grant publication (PP) of patent applications in the context of a cumulative innovation model. We show that PP leads to fewer applications and fewer inventions, but it may raise the probability that new technologies will reach the product market and thereby enhances consumer surplus and possibly total welfare as well.

1. Introduction

The two main objectives of patent systems are to encourage investments in R&D by granting inventors a temporary monopoly over the use of their inventions and to facilitate the dissemination of R&D knowledge. One aspect of patent systems that reflects the desire to balance these conflicting objectives is the requirement to publicly disclose pre-grant patent applications after 18 months from the date of application. This requirement, which is in place in practically every industrialized country (see Ragusa, 1992), implies that inventors may face the risk that their knowledge will be made public even if eventually their patent applications are rejected. Not surprisingly, opponents of this requirement argue that this risk may discourage innovations, especially by small independent inventors who lack the means to vigorously protect their intellectual property. A notable exception to the 18 months rule is the current U.S. patent system which allows applicants to keep their patent applications confidential until an actual patent is issued, provided that they do not seek patent protection in another country in which the 18 months rule applies.[1]

In this paper we examine the implications of pre-grant publication of patent applications in the context of a cumulative innovation model. In this model, two firms engage in an R&D process aimed at developing a new commercial technology. Our analysis begins when one of the two firms has managed to accumulate enough interim R&D knowledge to file for a patent.[2] We then examine what are the effects of pre-grant patent publication (PP) on the incentives of the leading firm to apply for a patent on its interim R&D knowledge, and on the R&D investments of the two firms which determine their likelihood to successfully develop the new commercial technology.

In principle, pre-grant patent publication (PP) may have two main effects: first, it creates a technical spillover because the lagging firm gets access to the leading firm's interim R&D knowledge when the patent application is made public even if the application is eventually rejected. Second, PP may credibly reveal to the lagging firm that the

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[1] The Patent Reform Act of 2007 (H.R. 1908 and S.1145 of the 110th Congress) proposes to eliminate the exemption. Until the passage of the American Inventors Protection Act (AIPA) in 1999, all patent applications in the U.S. were kept confidential until a patent was actually granted. Since 1999, approximately 10% of all applicants opt-out of publication (FTC, 2005, p. 11).

[2] For instance, in the context of biotechnology, the interim R&D knowledge could represent a research tool like a cell line, chemical reagent, or antibody which is used in research but need not have an independent commercial value.

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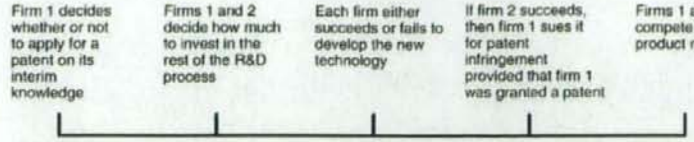

Fig. 1. The sequence of events.

leading firm is indeed leading and may also affect its beliefs about the extent of this lead. In this paper we focus on the first, technological spillover, effect of PP. This effect figures prominently in the public debate in the U.S. about PP.

We show that the implications of PP depend on the strength of patent protection, which depends in our model on two factors: (i) the likelihood that the patent office will grant the leading firm a patent on its interim R&D knowledge, and (ii) the likelihood that the patent will be upheld in court. PP matters however only if patent protection is strong or intermediate because under weak protection, the leading firm does not file for a patent even when patent applications are kept confidential. On the other hand, when patent protection is strong, the leading firm files for a patent even when a PP is in place. But since PP creates a technological spillover, it induces the leading firm to cut its R&D investment while inducing the lagging firm to invest more. When the cost of R&D is quadratic, PP raises the overall likelihood that the new technology will reach the product market, and hence it benefits consumers. If in addition the marginal cost of R&D is sufficiently large, then PP also raises social welfare (measured as the sum of the expected consumers' surplus and expected profits). On the other hand, PP hurts the leading firm and hence, weakens its incentives to accumulate interim R&D knowledge in the first place.

Things are more subtle when patent protection is intermediate. Now the leading firm files for a patent when patent applications are confidential but not when they are made public. Moreover, the effect of PP on the R&D investments depends on the likelihood that patents will be upheld in court: when this likelihood is large, PP induces the leading firm to cut its R&D investment while inducing the lagging firm to invest more. When the likelihood that patents will be upheld in court is small, PP has an ambiguous effect on the R&D investments. Nonetheless, when the cost of R&D is quadratic, PP still benefits consumers regardless of the likelihood that patents will be upheld in court. And, when the marginal cost of R&D is sufficiently large, PP enhances social welfare if patents are likely to be upheld in court, but it decreases social welfare otherwise.

The economic literature has already studied various aspects of patent laws, including the optimal length and breadth of patents (e.g., Nordhaus, 1969; Gilbert and Shapiro, 1990; Klemperer, 1990; Gallini, 1992; Chang, 1995; Green and Scotchmer, 1995; Matutes et al., 1996; O'Donoghue et al., 1998), priority rules such as "first to file" versus "first to invent" (e.g., Scotchmer and Green, 1990), novelty requirements (e.g., Scotchmer, 1996; Eswaran and Gallini, 1996; O'Donoghue, 1998), the optimal renewal of patents (Cornelli and Schankerman, 1999), and the optimal length of protection given to the first firm to discover interim R&D knowledge (Bloch and Markowitz, 1996). However, pre-grant patent publication has received very little attention in the economic literature. Given the continuing debate in the U.S. about the 18 months rule, it seems that a formal economic analysis of this issue is badly needed.

We are aware of only two papers that examine the implication of PP. Aoki and Prusa (1996) assume that PP reveals information about the quality choice of the first filer. They show that this information allows firms to coordinate their R&D investments and achieve a more collusive outcome. Unlike the current paper though, the decision to patent is not endogenous, filing for a patent does not create a technological spillover, and patenting does not allow the first filer to exclude its rival from the product market. Johnson and Popp (2003) examine citation analysis on all U.S. domestic patents from 1976 to 1996 and find that more "significant" patents (those that are subsequently cited more often) tend to take longer through the application process and hence are more likely to be affected by PP. Moreover, their analysis suggests that earlier disclosure should lead to faster diffusion of R&D knowledge. While faster diffusion benefits future inventors, it hurts the filing inventors and may therefore make them more reluctant to file for patents.

The rest of the paper is organized as follows: in Section 2 we describe the model and in Sections 3 and 4 we study the equilibrium under the PP and CF systems. In Section 5 we compare the two filing systems in terms of the equilibrium patenting and investment behavior of the two firms and use the results to examine the implications of PP for consumers' surplus and social welfare. We then consider the possibility that the two firms will engage in licensing in Section 6, and in Section 7 we examine the implications of PP for the firms' incentives to accumulate interim R&D knowledge. We conclude in Section 8. All proofs are in the Appendix A.

2. The model

Two firms engage in an R&D process aimed at developing a new commercial technology. Suppose that the R&D process has reached a critical point where one of the two firms, firm 1, has accumulated enough interim knowledge to apply for a patent. This knowledge represents, say, a research tool or some basic technology which lowers the cost of R&D in the rest of the R&D process. Although the patent (if granted) covers only the interim knowledge of firm 1, it nonetheless allows it to sue firm 2 for patent infringement if firm 2 eventually manages to develop the new technology. In most of the paper, we shall assume that when firm 1 holds a patent, it always sues firm 2 when the latter develops the new technology; this assumption can be justified on the grounds that firm 1 wishes to develop reputation for vigorously protecting its intellectual property. In Section 6 we shall relax this assumption and consider ex post licensing which takes place when firm 1 fails to develop the new technology while firm 2 succeeds.[3] The cost of applying for a patent is that some of firm 1's interim knowledge is spilled over to firm 2 either through the patent application (if it is made public), or through an actual patent (if and when it is granted).[4]

Given firm 1's patenting decision, but before the patent office makes a decision, the two firms decide how much to invest in the rest of the R&D process. The investment of each firm determines its eventual probability of succeed. We assume that the outcome of the R&D process is binary: each firm either succeeds to develop the new technology or it fails and develops nothing. Once the R&D process ends, the two firms compete in the product market. The sequence of events is summarized in Fig. 1.

[3] Another possibility is that firm 1 will license its interim R&D knowledge to firm 2 ex ante, before the outcome of the R&D process is decided. For analysis of this kind of licensing, see Spiegel (2008).

[4] This tradeoff is reminiscent of the tradeoff in Horstman et al. (1985), although the technological spillover in their model arises because patenting reveals to the lagging firm how profitable it would be to imitate the leading firm. For a related tradeoff, see Erkal (2005).

2.1. The filing system

We consider two filing systems: under a pre-grant patent publication system (PP system), the contents of patent applications are automatically published after a certain period of time from the application date (typically 18 months). Under a confidential filing system (CF system), patent applications are kept confidential until a patent is granted; if an application is rejected, then no information is revealed.

In practice, patent protection is imperfect both because patent applications are sometimes rejected by the patent office if they are not deemed sufficiently novel, useful, or non-obvious, and because actual patents are not always upheld in court.[5] We capture these imperfections by assuming that firm 1's patent application is approved with probability $\theta \in [0, 1]$, and if firm 1 sues firm 2 for patent infringement, then it wins in court with probability $\gamma \in [0, 1]$.[6] Throughout we treat θ and γ as exogenous parameters.[7]

2.2. The cost of R&D

Given firm 1's filing decision, but before the patent office decides whether to grant firm 1 a patent, firms 1 and 2 simultaneously choose how much to invest in the rest of the R&D process.[8] For analytical convenience, we shall assume that the two firms directly choose their probabilities of success, q^1 and q^2, and these choices determine their respective R&D cost functions, which are given by $C(q^1)$ and $\beta C(q^2)$, where $\beta > 1$ because firm 2 does not have full access to firm 1's interim knowledge. We assume that $C(\cdot)$ is twice continuously differentiable, increasing, and strictly convex, with $C(0) = 0$.[9] The value of β depends on the degree of technological spillover which in turn depends on whether firm 1 applies for a patent and on which filing system is in place. We assume that the value of β is lowest and equals β_L if firm 1 applies for a patent and a PP system is in place; in that case, firm 2 gets access to firm 1's interim knowledge through firm 1's patent application. The value of β is intermediate and equals β_M if a patent is granted and a CF system is in place; firm 2 then gets access to firm 1's only through the patent itself. Finally, the value of β is largest and equals β_H if either firm 1 does not apply for a patent, or if it does but its patent application is rejected and a CF system is in place. In both cases, there is no technological spillover.[10]

We assume that the fact that firm 1's cost of R&D is lower is common knowledge. As mentioned in the Introduction, without this assumption, PP would not only create a technological spillover, but would also reveal to firm 2 that firm 1's cost is $C(q)$ and not higher. This will affect firm 1's incentive to file for a patent under the PP system. In the current paper, however, we wish to focus on the technological spillover effect and hence eliminate the effect of PP on firm 2's beliefs by adopting the common knowledge assumption.[11]

2.3. Competition in the product market

Once the R&D process ends, the two firms compete in the product market. Instead of assuming a specific type of product market competition, we simply assume that if only one firm succeeds to develop the new technology (this firm can be either firm 1 or 2), then the net present value of its profits is π_{yn} and the net present value of its rival's profits is π_{ny}. If both firms succeed to develop the new technology, then the net present value of their profits is π_{yy}, and if neither firm succeeds, the net present value of their profits is π_{nn}.[12] Throughout, we make the following assumptions:

A1. $\pi_{yn} > \pi_{yy} \geq \pi_{nn} \geq \pi_{ny}$.

A2. $C'(1) > \max\{\pi_{yn} - \pi_{nn}, \pi_{yy} - \pi_{ny}\}$, and $C''(q) > \Pi$, where $\Pi \equiv \pi_{yn} + \pi_{ny} - \pi_{yy} - \pi_{nn}$ for all $q \in [0, 1]$.

Assumption A1 holds whenever the products of firms 1 and 2 are substitutes. Assumption A2 ensures that the best-response functions of firms 1 and 2 are well behaved. Moreover, the first part of Assumption A2 ensures that it is too costly to invest up to the point where developing the new technology becomes a sure thing, irrespective of whether the rival firm does or does not develop the new technology.

3. The pre-grant patent publication (PP) system

When firm 1 files for a patent under the PP system, it can prevent firm 2 from bringing the new technology to the product market (if firm 2 develops it) with probability $\gamma\theta$, which is the probability that a patent is granted and is upheld in court. Hence, $\gamma\theta$ reflects the effective patent protection that firm 1 enjoys. Recalling that the success probabilities of firms 1 and 2 are q^1 and q^2, the expected payoffs of the two firms are

$$\begin{aligned}\pi^1(q^1, q^2|F) = {} & q^1\left[q^2(1-\gamma\theta)\pi_{yy} + (1-q^2(1-\gamma\theta))\pi_{yn}\right] \\ & + (1-q^1)\left[q^2(1-\gamma\theta)\pi_{ny} + (1-q^2(1-\gamma\theta))\pi_{nn}\right] - C(q^1).\end{aligned} \tag{1}$$

and

$$\begin{aligned}\pi^2(q^1, q^2|F) = {} & q^1\left[q^2(1-\gamma\theta)\pi_{yy} + (1-q^2(1-\gamma\theta))\pi_{ny}\right] \\ & + (1-q^1)\left[q^2(1-\gamma\theta)\pi_{yn} + (1-q^2(1-\gamma\theta))\pi_{nn}\right] - \beta_L C(q^2).\end{aligned} \tag{2}$$

The first bracketed term in Eq. (1) is firm 1's payoff when it succeeds to develop the new technology. With probability $q^2(1-\gamma\theta)$, firm 2 also succeeds and is free to use the new technology in the

[5] In 2003, the grant rates were 59.9% at the EPO, 49.9% at the JPO, and 64% at the USPTO (USPTO, 2004, Table 4). Allison and Lemley (1998) find that out of the 300 final patent validity decisions by U.S. courts during the period 1989–1996, only 162 patents (54%) were held valid. In Japan, the original patent was upheld in only 23 out of the 51 patent infringement suits studied between April 2000 and January 2003 (45.1%) (Material prepared for 4th meeting of Subcommittee on Intellectual Property Disputes, Committee for Legal System Reform Headquarters for Promotion of Judicial Reform, Prime Minister's Office (January 31, 2003)).

[6] The assumption that patent protection is imperfect has also been made elsewhere.
Q1 Meurer (1989), Anton and Yao (2003, 2004), and Choi (1998) assume that patents can be challenged in court and may be ruled as invalid, but the possibility that patent applications may be rejected plays no role in these papers. Kabla (1996) assumes that patent applications may be rejected, but does not consider the possibility that patents may not be upheld in court. Waterson (1990) and Crampes and Langinier (2002) assume that suing for patent infringement is costly so patentholders do not always sue imitators. Finally, Crampes and Langinier (1998) show that under certain conditions, firms may choose not renew their patents in order to conceal favorable market information from potential entrants.

[7] According to the enablement doctrine of patent law, "claims ought to be bounded to a significant degree by what the disclosure enables, over and beyond prior art" (Merges and Nelson, 1994, p.101). Thus, in a more general model where firm 1 can choose the scope of its disclosure, the likelihood that a court will uphold firm 1's patent would be an endogenous variable.

[8] This timing reflects the fact that patent examination is typically a lengthy process: pendency time at USPTO was 26.7 months in 2003. Pendency times at EPO and JPO were 37.7 and 31.1 months respectively. (See USPTO, 2004 for details, including definition.)

[9] Given that $C(\cdot)$ is increasing, there is a 1:1 relationship between the probability of success and the cost of achieving it so it is equally possible to assume that the two firms choose how much to spend on R&D and these choices determine their respective probabilities of success.

[10] The assumption that $\beta_H > \beta_M > \beta_L$ is consistent with Mansfield et al. (1981) who estimate that the average ratio between the cost of imitating an existing technology ($\beta_L C(q)$ or $\beta_M C(q)$ in our model) and the cost of innovating it from scratch ($\beta_H C(q)$ in our model) is 0.65.

[11] For papers that study the effect of voluntary disclosure of R&D knowledge on the beliefs of rival firms, see for example Lichtman et al. (2000), Gordon (2004), Jansen (2008), and Gill (2008).

[12] To economize on notation we assume that the product market profits are symmetric: π_{yy}, π_{yn}, π_{ny} and π_{nn} are the same for both firms. This assumption is not important however and none of our results depends on it.

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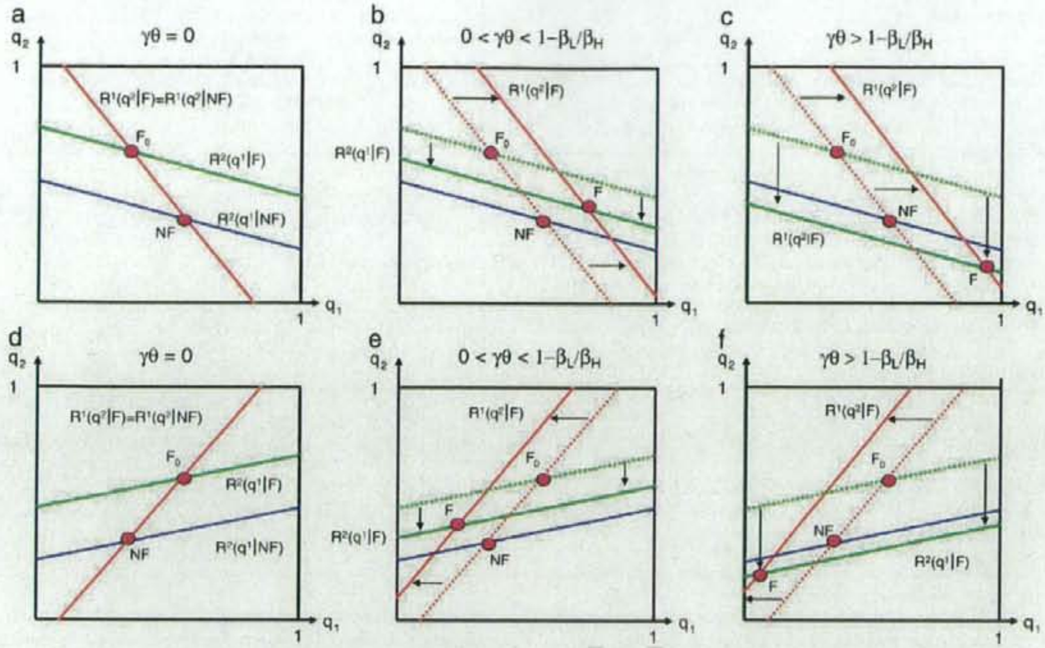

Fig. 2. The Nash equilibrium in the filling and the no-filling subgames and how it changes with increases in $\gamma\theta$.

product market, so firm 1's payoff is π_{yy}; with probability $1-q^2(1-\gamma\theta)$, firm 2 either fails or else it succeeds but it is prevented from using the new technology, so firm 1's payoff is π_{yn}. The second bracketed term in Eq. (1) represents the corresponding expressions when firm 1 fails to develop the new technology. The interpretation of Eq. (2) is similar. Firm 2's cost is $\beta_L C(q^2)$ because firm 2 gets access to firm 1's interim knowledge through firm 1's patent application.

Absent filing, firm 1 cannot prevent firm 2 from using the new technology if firm 2 develops it. Hence, the expected payoffs of the two firms are

$$\pi^1(q^1,q^2|NF) = q^1\left[q^2\pi_{yy} + (1-q^2)\pi_{yn}\right] + (1-q^1)\left[q^2\pi_{ny} + (1-q^2)\pi_{nn}\right] - C(q^1) \tag{3}$$

and

$$\pi^2(q^1,q^2|NF) = q^1\left[q^2\pi_{yy} + (1-q^2)\pi_{ny}\right] + (1-q^1)\left[q^2\pi_{yn} + (1-q^2)\pi_{nn}\right] - \beta_H C(q^2) \tag{4}$$

These expressions differ from the corresponding expressions in the filing subgame in two ways: first, the probability that firm 2 uses the new technology in the product market is now q^2 instead of $q^2(1-\gamma\theta)$. Second, absent filing, there is no technological spillover, so firm 2's cost of R&D is $\beta_H C(q^2)$ instead of $\beta_L C(q^2)$, where $\beta_H > \beta_L$.

Let $R^1(q^2|F)$ and $R^2(q^1|F)$ be the best-response functions in the filing subgame; these functions are defined implicitly by $\frac{\partial\pi^1(q^1,q^2|F)}{\partial q^1} = 0$ and $\frac{\partial\pi^2(q^1,q^2|F)}{\partial q^2} = 0$. Similarly, the best-response functions in the no-filing subgame, $R^1(q^2|NF)$ and $R^2(q^1|NF)$, are defined implicitly by $\frac{\partial\pi^1(q^1,q^2|NF)}{\partial q^1} = 0$ and $\frac{\partial\pi^2(q^1,q^2|NF)}{\partial q^2} = 0$. Assumptions A1 and A2 ensure that the best-response functions in both subgames are well-defined and single-valued. The best-response functions are downward sloping in the (q^1, q^2) space (q^1 and q^2 are strategic substitutes) if $\Pi \equiv \pi_{yn} + \pi_{ny} - \pi_{yy} - \pi_{nn} > 0$ and are upward sloping (q^1 and q^2 are strategic complements) if $\Pi < 0$. To interpret Π, note that it can be written as $(\pi_{yn}-\pi_{nn}) - (\pi_{yy}-\pi_{ny})$, where $\pi_{yn}-\pi_{nn}$ is the extra profits generated by the new technology when the rival fails to develop it, and $\pi_{yy}-\pi_{ny}$ is the corresponding extra profit when the rival succeeds. When $\Pi > 0$, having the new technology is more profitable when the rival does not have it and conversely when $\Pi < 0$.

The Nash equilibrium in the filing subgame, (q^1_F, q^2_F), is determined by the intersection of $R^1(q^2|F)$ and $R^2(q^1|F)$, while the Nash equilibrium in the no-filing subgame, (q^1_{NF}, q^2_{NF}), is determined by the intersection of $R^1(q^2|NF)$ and $R^2(q^1|NF)$. Assumptions A1 and A2 ensure that (q^1_F, q^2_F) and (q^1_{NF}, q^2_{NF}) are unique and lie inside the unit square (recall that q^1 and q^2 are probabilities and hence must be between 0 and 1).[13]

To see how the effective patent protection, $\gamma\theta$, affects the R&D investments, note that $\frac{\partial^2\pi^1(q^1,q^2|F)}{\partial q^1\partial(\gamma\theta)} = q^2\Pi$ and $\frac{\partial^2\pi^2(q^1,q^2|F)}{\partial q^2\partial(\gamma\theta)} = -\left[q^1(\pi_{yy}-\pi_{ny}) + (1-q^1)(\pi_{yn}-\pi_{nn})\right]$. Hence, when $\gamma\theta$ increases, $R^1(q^2|F)$ shifts outward if $\Pi > 0$ (q^1 and q^2 are strategic substitutes) and inwards if $\Pi < 0$ (q^1 and q^2 are strategic complements); by contrast, Assumption A1 ensures that $R^2(q^1|F)$ always shifts inward. As a result, q^1_F increases with $\gamma\theta$ if $\Pi > 0$ and decreases with $\gamma\theta$ if $\Pi < 0$, while q^2_F always decreases with $\gamma\theta$ irrespective of Π. Intuitively, as $\gamma\theta$ increases, firm 2 is less likely to bring the new technology to the product market and hence its marginal benefit from R&D falls; as a result firm 2 invests less. As for firm 1, note that its marginal benefit from R&D is a weighted average of $\pi_{yn}-\pi_{nn}$ and $\pi_{yy}-\pi_{ny}$. When $\gamma\theta$ increases, firm 1 is more likely to block firm 2 from using the new technology and hence its extra profits is more likely to be $\pi_{yn}-\pi_{nn}$ rather than $\pi_{yy}-\pi_{ny}$. This in turn boosts firm 1's incentive to invest if and only if $\pi_{yn}-\pi_{nn} > \pi_{yy}-\pi_{ny}$, i.e., if and only if $\Pi > 0$.

Fig. 2 illustrates the equilibria in the filing and the no-filing subgames and shows how they are affected by $\gamma\theta$.[14] Panels a–c show

[13] The proof appears in a technical appendix which is available at www.tau.ac.il/~spiegel.

[14] For simplicity, we draw the best-response functions as straight lines even though in general this need not be the case. This however does not affect any of our conclusions.

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