2 A simple basic model

In this section, we present a basic model in which an agent reduces the number of children with the variety expansion of differentiated goods. In the following sections, we apply this basic model to the growth and international trade models.

We assume that agents consume differentiated goods and receive utility from children. Their utility function takes a CES form as follows:

$$U = \left[\left(\int_0^n x(i)^\rho di \right)^{\frac{\psi}{\rho}} + m^\psi \right]^{\frac{1}{\psi}}, \tag{1}$$

where n is the number of variety of differentiated goods, $x(i)$ is the consumption of differentiated goods indexed i, and m is the number of children. We assume that $\rho < 1$ and $\psi < 1$. $\sigma = \frac{1}{1-\rho}$ represents the elasticity of substitution among differentiated goods, and $\theta = \frac{1}{1-\psi}$ is the elasticity of substitution between the composite of differentiated goods and children.

We assume that agents have one amount of time that can be used for working or rearing children. Following Becker (1965) and others, we assume that, if they have a child, they have to use time τ to rear a child. Their budget constraint becomes

$$\begin{aligned} w(1-\tau m) &= \textstyle\int_0^n p(i)x(i)di, \\ w &= \textstyle\int_0^n p(i)x(i)di + w\tau m. \end{aligned} \tag{2}$$

We follow Dixit and Stiglitz (1977) and define $X = (\int_0^n x(i)^\rho di)^{\frac{1}{\rho}}$ and $P = (\int_0^n p(i)^{\frac{\rho}{\rho-1}} di)^{\frac{\rho-1}{\rho}}$. P is a price index of differentiated goods. From the first-order conditions of a consumer problem, we get

$$\int_0^n p(i)x(i)di = PX. \tag{3}$$

In our setting, the consumer has a preference for the variety of differentiated goods. We can derive (use Leibniz rule)[3]

$$\frac{\partial P}{\partial n} = \frac{\rho-1}{\rho} \left(\int_0^n p(i)^{\frac{\rho}{\rho-1}} di \right)^{\frac{-1}{\rho}} p(i)^{\frac{\rho}{\rho-1}} < 0. \tag{4}$$

We substitute $X = (\int_0^n x(i)^\rho di)^{\frac{1}{\rho}}$ and $P = (\int_0^n p(i)^{\frac{\rho}{\rho-1}} di)^{\frac{\rho-1}{\rho}}$ into the utility function and budget constraint, respectively, and solve a consumer's utility maximization problem. A first-order condition of this problem becomes

$$X = \left(\frac{P}{w\tau}\right)^{\frac{1}{\psi-1}} m. \tag{5}$$

We substitute (5) into the budget constraint: $w = PX + wtm$:

[3] In this section, we ignore the production side of the economy and assume that each price of differentiated goods is constant. In Sections 3 and 4, we consider the production side of the economy.

$$
\begin{aligned}
w &= (P^{\frac{\psi}{\psi-1}}(w\tau)^{\frac{1}{1-\psi}} + w\tau)m, \\
m &= \frac{w}{P^{\frac{\psi}{\psi-1}}(w\tau)^{\frac{1}{1-\psi}} + w\tau}.
\end{aligned} \tag{6}
$$

Thus,

$$
\frac{\partial m}{\partial P} = \frac{(\frac{\psi}{1-\psi})P^{\frac{1}{\psi-1}}(w\tau)^{\frac{1}{1-\psi}}w}{(P^{\frac{\psi}{\psi-1}}(w\tau)^{\frac{1}{1-\psi}} + w\tau)^2} > 0. \tag{7}
$$

From (4) and (7),

$$
\frac{\partial m}{\partial n} < 0. \tag{8}
$$

We then show that variety expansion reduces the fertility rate. In our setting, the consumer has a preference for a variety of differentiated goods. When the variety of differentiated goods increases, the price index of differentiated goods decreases (see (4)). Thus, the variety expansion lowers the relative price of the composite of differentiated goods and raises the expenditure share for differentiated goods. Therefore, the variety expansion raises the relative price of children and reduces the expenditure share for children. With this mechanism, variety expansion reduces the fertility rate.

The simple partial equilibrium model in this section shows that variety expansion reduces the number of children of a consumer. In Sections 3 and 4, we show that this simple mechanism can be used in the growth and international trade models. In those sections, we show that the fertility rate decreases with economic growth and openness of trade.

3 A growth model

In this section, we apply a basic model in the above section to a growth model. Our model in this section is an overlapping generation model in which agents live two periods, childhood and adulthood. In childhood, agents do not do any economic activities; they are reared exclusively by their parents. In adulthood, the agents consume differentiated goods and determine the number of children. Thus, the consumption behavior of agents is the same as in the model in Section 1. We assume that labor is the numeraire, $w = 1$. There is a continuum of population, L_t, at period t. The utility function of adult agents at period t is

$$
U_t = \left[(\int_0^{n_t} x_t(i)^\rho di)^{\frac{\psi}{\rho}} + m_t^\psi \right]^{\frac{1}{\psi}}, \tag{9}
$$

and their budget constraints are[4]

[4] In our model, we assume that there is no capital. As we show later, the firms obtain zero profits. Thus, the income of agents is composed only with their wages.

$$1 - \tau m_t = \int_0^{n_t} p_t(i) x_t(i) di.$$

The same procedure as that in Section 2 leads to the first-order condition:

$$m_t = (\frac{P_t}{\tau})^{\frac{1}{1-\psi}} X_t. \tag{10}$$

We substitute (10) into the budget constraint,$1 = P_t X_t + \tau m_t$, and get

$$\begin{aligned} 1 &= (P_t + P_t^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}}) X_t, \\ X_t &= \frac{1}{(P_t + P_t^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})}. \end{aligned} \tag{11}$$

From (11), we can derive a consumer's demand function for each differentiated good:

$$\frac{1}{(P_t + P_t^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})} = P_t^{\frac{-1}{\rho-1}} p_t(j)^{\frac{1}{1-\rho}} x_t(j), \tag{12}$$

$$x_t(j) = \frac{P_t^{\frac{1}{1-\rho}} p_t(j)^{\frac{1}{\rho-1}}}{(P_t + P_t^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})}.$$

In this growth model, we follow Grossman and Helpman (1991) and assume that the variety of differentiated goods increases with innovation activities. In our model, we assume that the innovation activity produces a patent. With this patent, a firm produces a variety of differentiated goods with a constant return to scale technology: $x(i)$ units of labor can produce $x(i)$ units of differentiated goods. To produce a patent in period t requires I_t units of labor. Here, we assume that $I_t = \frac{1}{n_{t-1}^{\gamma}}$, $\gamma \geq 1$. $I_t = \frac{1}{n_{t-1}^{\gamma}}$ means that there is knowledge accumulation in this economy. As in the standard endogenous growth models, such as those developed by Grossman and Helpman (1991) and Romer (1990), for each innovation firm, this knowledge accumulation is externality. The term of the validity of a patent is one period. After the term of the validity of a patent, differentiated goods are produced by perfectly competitive firms. The price of those goods is $p = 1$. At the validity term of the patent, firms set the standard mark-up price. Each firm sets the price $p_t(i) = p_t(j) = \frac{\sigma}{\sigma-1} = \frac{1}{\rho}$, which is not affected by n.

Here, $P_t = (\int_0^{n_t} p(i)^{\frac{\rho}{\rho-1}} di)^{\frac{\rho-1}{\rho}} = (n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-1}{\rho}}$, where $n_t = n_{t-1} + k_t$ and k_t represents the number of varieties that are created at period t. Using this and (12), the demand faced with a firm that is created at period t is

$$q_t = \frac{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{-1}{\rho}} p^{\frac{1}{\rho-1}}}{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-1}{\rho}} + (n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-1}{\rho(1-\psi)}} \tau^{\frac{-\psi}{1-\psi}}} L_t. \tag{13}$$

Then, instantaneous profits of a firm are

$$\pi_t = (\frac{1}{\sigma - 1})\frac{p^{\frac{1}{\rho-1}}}{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}}) + (n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-\psi}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}}L_t. \tag{14}$$

The free-entry condition in the innovation market means that $I_t = \pi_t$. From (13) and (14), the free-entry condition is written as

$$(\frac{1}{n_{t-1}})^{\gamma} = (\frac{1}{\sigma - 1})\frac{p^{\frac{1}{\rho-1}}}{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}}) + (n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-\psi}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}}L_t. \tag{15}$$

The right-hand-side (RHS) of (15) is the profit of a firm, and the left-hand-side (LHS) is the cost for an innovation to produce a patent. If $\rho \geq \psi$, the RHS is a decreasing function of n_t. This shows that intensive competition reduces the operating profits of a firm. In addition, the RHS is an increasing function of the population, L_t. This is because a large population results in a large market that brings a firm large profits. If

$$(\frac{1}{n_{t-1}})^{\gamma} < (\frac{1}{\sigma - 1})\frac{p^{\frac{1}{\rho-1}}}{n_{t-1} + n_{t-1}^{\frac{\rho-\psi}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}}L_t, \tag{16}$$

innovation occurs at period t. If $\rho \geq \psi$, the right-hand side of (16) is a decreasing function of n_{t-1}. If innovation occurs at period t, the fertility rate in period t becomes

$$m_t = \frac{1}{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\psi(\rho-1)}{\rho(\psi-1)}}\tau^{\frac{1}{1-\psi}} + \tau}. \tag{17}$$

(17) shows that the fertility rate decreases with innovation. Innovation activities raise the variety of differentiated goods, which lowers (raises) the relative price of the composite of differentiated goods (children). Therefore, when innovations progress, fertility rates decrease.

Proposition 1 *Fertility rates decrease with innovation.*

On the other hand, if

$$(\frac{1}{n_{t-1}})^{\gamma} \geq (\frac{1}{\sigma - 1})\frac{p^{\frac{1}{\rho-1}}}{n_{t-1} + n_{t-1}^{\frac{\rho-\psi}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}}L_t, \tag{18}$$

innovation does not occur in period t. If (18) is satisfied and

$$m_t = \frac{1}{n_{t-1}^{\frac{(\rho-1)\psi}{\rho(\psi-1)}}\tau^{\frac{1}{1-\psi}} + \tau} \leq 1, \tag{19}$$

innovation never occurs in period t and the following periods. In this case, the fertility rate is constant at $m_t = \frac{1}{n_{t-1}^{\frac{(\rho-1)\psi}{\rho(\psi-1)}}\tau^{\frac{1}{1-\psi}}+\tau}$ after period t.

The above discussion demonstrates that there are two cases in the model: innovation and decline in fertility rates and no innovation and constant fertility rates. To see them explicitly, we study the dynamic property of the economy. There are two state variables L_t and n_t, which follow dynamic equations. Population, L_t, follows $L_{t+1} = m_t L_t$. We substitute (17) and $k_t = n_t - n_{t-1}$ into $L_{t+1} = m_t L_t$ and derive

$$L_{t+1} = \frac{1}{(n_{t-1} + (n_t - n_{t-1}) p^{\frac{\rho}{\rho-1}})^{\frac{\psi(\rho-1)}{\rho(\psi-1)}} \tau^{\frac{1}{1-\psi}} + \tau} L_t. \tag{20}$$

Next, we derive the dynamic equation of variety n_t. n_t increases with innovation activities. Thus, the dynamic equation of n_t is a free-entry condition of the innovation market. We substitute $k_t = n_t - n_{t-1}$ into (15) and obtain

$$(\frac{1}{n_{t-1}})^{\gamma} = (\frac{1}{\sigma - 1}) \frac{p^{\frac{1}{\rho-1}}}{(n_{t-1} + (n_t - n_{t-1}) p^{\frac{\rho}{\rho-1}}) + (n_{t-1} + (n_t - n_{t-1}) p^{\frac{\rho}{\rho-1}})^{\frac{\rho-\psi}{\rho(1-\psi)}} \tau^{\frac{-\psi}{1-\psi}}} L_t. \tag{21}$$

(20) and (21) express the dynamic property of the model. To analyze the dynamic property, we create a phase diagram.

The phase diagram is depicted in Figure 1. We show in Appendix 1 that $\frac{L_{t+1}}{L_t} = 1$ requires $n_t = (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$. In Appendix 2, we show that, with assumptions of $\rho \geq \psi$ and $\frac{\rho - \psi - \gamma\rho(1-\psi)}{\rho(1-\psi)} < 0$, we can depict a downward-sloping curve of $\frac{n_t}{n_{t-1}} = 1$ as in Figure 1.

In Figure 1, we can observe four regions. In Region 1, population increases, and innovation does not occur. In this region, the population is small which brings firms about small market. Thus, innovation does not occur in this region. In this region, the variety of differentiated goods is also small. Thus, the fertility rate is high, $m_t > 1$. Since the fertility rate is high, the population increases with time.

In Region 2, the population increases, while innovation activities progress. In Region 2, the population is large enough to induce innovation activities. Thus, the variety of differentiated goods increases with time. In addition, in this region, the variety of differentiated goods is relatively small. Thus, fertility rates are high, $m_t > 1$. Then, in Region 2, both the population and the variety of differentiated goods increase with time. However, fertility rates decrease with innovation. Therefore, in Region 2, fertility rates decrease with time.

In Region 3, the population decreases, and innovation occurs. In Region 3, the population is large enough. Since the market for differentiated goods is large, innovation progresses. Thus, the variety of differentiated goods increases with time in this region. However, since the variety of differentiated goods is large, fertility rates decrease, $m_t < 1$. Then, the population decreases with time in this region. In addition, the fertility rate decreases with innovation activities. Thus, the fertility rate decreases with time in Region 3.

In Region 4, the population decreases, and innovation stops. In Region 4, the population decreases. Since the market for differentiated goods decreases, it is impossible to obtain profits with innovation. Thus, innovation activities are not conducted, and the variety of differentiated goods is constant in this region. In Region 4, there is a large variety of differentiated goods, which lowers fertility rates, $m_t < 1$. Thus, the population decreases with time in this region. Since innovation does not occur, the fertility rate is constant in Region 4.

In Figure 1, we depict an example case of a dynamic path that starts from Region 1. In this path, in the early periods, the population increases as long as there are not any innovations (Region 1). An increase in population enlarges the market for differentiated goods. When the economy switches from Region 1 to 2, innovation activities start. In this region, the population increases ($m_t > 0$), while the fertility rates decrease with innovation (see Proposition 1). When the economy reaches Region 3, the population starts to decrease, and fertility rates continuously decrease with innovation ($m_t < 1$). Finally, the economy reaches Region 4, innovation activities stop, and population decreases at a constant rate. In this dynamic path, fertility rates decrease with economic growth (Regions 2 and 3). This result is consistent with experiences of economic development (See Galor (2004)). [5]

4 An international trade model

In this section, we apply a simple basic model in Section 2 to an international model. We study the effect of openness of trade on the fertility rate. To study the impact of trade, we compare the autarky economy with the open economy.

In this section, we study a static model, in which the utility function of agents is assumed to be (1). We assume that labor is the numeraire, $w = 1$. The budget constraint of agents is (2). On the production side, differentiated goods are produced by monopolistically competitive firms. Firms need fixed amounts of labor, i.e., F and $cx(i)$ amounts of variable labor to produce $x(i)$ amounts of differentiated goods. Each firm sets the price $p(i) = p(j) = \frac{\sigma}{\sigma-1} = \frac{1}{\rho}$, which is not affected by n.

Consider an autarky economy with a population L. The demand faced by a firm is derived by the same procedure as that used in Section 3:

$$q(j) = \frac{P_a^{\frac{1}{1-\rho}} p(j)^{\frac{1}{\rho-1}}}{(P_a + P_a^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})} L,$$

where $P_a = (\int_0^n p(i)^{\frac{\rho}{\rho-1}} di)^{\frac{\rho-1}{\rho}} = n_a^{\frac{\rho-1}{\rho}} p$ is a price index of differentiated goods in the autarky economy. In this section, n_a and n_I represent the variety of differentiated goods in a country in the case of autarky and international trade, respectively. The profits of a firm become

[5] In our model, there are many possibilities of dynamic paths. For example, if the economy starts from Region 4, the variety is constant, and population decreases with time.

$$\pi_a = (\frac{1}{\sigma - 1}) \frac{P_a^{\frac{1}{1-\rho}} p(j)^{\frac{1}{\rho-1}}}{(P_a + P_a^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})} L - F.$$

The free-entry condition requires that the profits of each firm become zero. Thus, the free-entry condition in the autarky economy becomes

$$F = (\frac{1}{\sigma - 1}) \frac{P_a^{\frac{1}{1-\rho}} p(j)^{\frac{1}{\rho-1}}}{(P_a + P_a^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})} L. \tag{22}$$

To study the effect of international trade, we follow Melitz (2003) and assume that there are $1+h$ identical countries and start to trade. International trade incurs transportation costs. We follow Fujita, Krugman, and Venables (1999) and assume that international trade incurs iceberg transportation costs. Hence, $T > 1$ units of a good must be shipped in order for one unit to arrive at its destination. Here, we consider a symmetric equilibrium in which all countries have the same number of varieties and the same price index. In this open economy, the price index of differentiated goods becomes

$$P_I = \left[n_I \left(1 + hT^{\frac{\rho}{\rho-1}} \right) \right]^{\frac{\rho-1}{\rho}} p. \tag{23}$$

In the case of openness of trade, each consumer in a country consumes $n_I(1+h)$ variety of differentiated goods. Since agents consume differentiated goods that are produced in foreign countries, the total variety of goods may increase in comparison to that in an autarky economy. However, the variety produced in country n_I may be lowered by international trade. This is because international trade manifests the competition among firms, which lowers profits.

The free-entry condition in the open economy becomes

$$F = (\frac{1}{\sigma - 1}) \left[\frac{P_I^{\frac{1}{1-\rho}} p(j)^{\frac{1}{\rho-1}}}{(P_I + P_I^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})} (1 + hT^{\frac{\rho}{\rho-1}}) \right] L. \tag{24}$$

We substitute the price-index of an autarky economy into (22) and (23) into (24) to derive the free-entry conditions in the case of autarky and open economies, as follows:

$$F = (\frac{1}{\sigma - 1}) \frac{L}{n_a p + n_a^{\frac{\rho-\psi}{\rho(1-\psi)}} p^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}}}, \tag{25}$$

$$F = (\frac{1}{\sigma - 1}) \frac{L(1 + hT^{\frac{\rho}{\rho-1}})}{n_I \left(1 + hT^{\frac{\rho}{\rho-1}}\right) p + \left[n_I \left(1 + hT^{\frac{\rho}{\rho-1}}\right) \right]^{\frac{\rho-\psi}{\rho(1-\psi)}} p^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}}}. \tag{26}$$

From (25) and (26), we can observe two effects of international trade on a firm's profits. In the numerator of RHS of (26), there is a term $(1 + hT^{\frac{\rho}{\rho-1}})$, which raises the firms' profits. This is the market expansion effect of international trade. In the case of openness of trade, firms can access the markets of foreign countries. Thus, the market faced by a firm is enlarged with open trade. However, there are terms $n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right)$ in the denominator of RHS of (26). These terms express the intensive competition effect of international trade. In autarky, firms compete exclusively with domestic firms. On the other hand, in an open economy, firms compete with foreign firms in addition to domestic firms. International trade then manifests competition among firms.

To investigate the impact of trade on fertility, we compare the effects of international trade on the price index. Thus, we compare P_a with P_I. Consumer behavior is the same as that in Section 2. Thus, if $P_I < P_a$, open trade lowers the fertility rate. If $P_I < P_a$, the next condition must be satisfied:

$$n_a < n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right). \tag{27}$$

Thus, if the total variety of differentiated goods increases significantly with open trade, the fertility rate will decrease. Here, from (25) and (26),

$$\frac{1}{n_a p + n_a^{\frac{\rho-\psi}{\rho(1-\psi)}} p^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}}} = \frac{1+hT^{\frac{\rho}{\rho-1}}}{n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right)p + \left[n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right)\right]^{\frac{\rho-\psi}{\rho(1-\psi)}} p^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}}}.$$

We rewrite the equation above as follows:

$$(n_a p + n_a^{\frac{\rho-\psi}{\rho(1-\psi)}} p^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})(1+hT^{\frac{\rho}{\rho-1}}) = n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right)p + \left[n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right)\right]^{\frac{\rho-\psi}{\rho(1-\psi)}} p^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}}. \tag{28}$$

(28) leads to

$$n_a p + n_a^{\frac{\rho-\psi}{\rho(1-\psi)}} p^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}} < n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right)p + \left[n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right)\right]^{\frac{\rho-\psi}{\rho(1-\psi)}} p^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}}. \tag{29}$$

If $\rho \geq \psi$, (29) means that $n_a < n_I\left(1+hT^{\frac{\rho}{\rho-1}}\right)$. Thus, if $\rho \geq \psi$, open trade lowers the fertility rate. In addition, we observe that a decline in transportation costs, T, lowers the price index in the case of an open economy. Thus, a decline in transportation costs reduces the fertility rate.

Proposition 2 ·*If $\rho \geq \psi$, openness of trade lowers fertility rates.*
·*A decline in transportation costs reduces fertility rates in an open economy.*

If $\rho \geq \psi$, the total variety increases with international trade. The market expansion effect overcomes the intensive competition with international trade. $\rho \geq \psi$ means that $\sigma \geq \theta$: the elasticity of substitution among differentiated goods is larger than the elasticity of substitution between the composite of differentiated goods and children. This is the realistic case.

In the real world, globalization progresses. Our international model shows that globalization among developed countries results in a decline in fertility rates in each country.

5 Conclusion

We presented a simple model in which variety expansion reduces fertility rates. We showed that this simple model can be applied to the model of economic growth and international trade. In the model of economic growth, fertility rates decrease with innovation activities that are the engine of economic development. In the international trade model, we showed that open trade reduces the fertility rate and a decline in transportation costs lowers fertility rates. Thus, our model presented a new mechanism of fertility decline.

In the modern world, fertility rates in developed countries are low and decrease with time. Our model made it clear that low fertility rates in developed countries are to be attributed to a large variety of consumption goods. Developed countries have experienced variety expansion by innovation activities, which reduce fertility rates. In addition, from the 1980s, globalization among developed countries has progressed. This globalization raises the total variety of differentiated goods that consumers in developed countries can access. With this variety of expansion, open trade reduces fertility rates in developed countries.

Our model is very simple. Thus, we can extend the model in some directions. For example, we can study the effects of public policy, such as subsidies for rearing children. We can extend the growth model and study human capital accumulation, as in Becker, Murphy, and Tamura (1991). This extension analyzes the quantity-quality trade-off that is an important mechanism of the fertility decline in developed countries. Finally, we can think of the relationship between the fertility rate and urbanization with the mechanism used in this model. Urbanization is an important aspect of economic development. Sato and Yamamoto (2005) showed that demographic transition and urbanization are closely related with each other. These extensions are important for future studies.

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Appendix 1

From (20), when innovation activities are done, $\frac{L_{t+1}}{L_t} = 1$ requires

$$n^{*} = \frac{(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}} - n_{t-1}(1 - p^{\frac{\rho}{\rho-1}})}{p^{\frac{\rho}{\rho-1}}}. \tag{30}$$

On the other hand, when innovation activities are not carried out, $\frac{L_{t+1}}{L_t} = 1$ requires

$$n^{**} = (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}. \tag{31}$$

From (30), $n_{t-1} \lesseqgtr (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}} \to n^{*} \gtreqless (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$. Thus, when $n^{*} \gtreqless (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$, $\frac{L_{t+1}}{L_t} \lesseqgtr 1$. Therefore, $\frac{L_{t+1}}{L_t} = 1$ requires $n = (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$ ..

Appendix 2

From (21), $\frac{n_t}{n_{t-1}} = 1$ requires that

$$(\frac{1}{n})^{\gamma} = (\frac{1}{\sigma-1})\frac{p^{\frac{1}{\rho-1}}}{n + n^{\frac{\rho-\psi}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}}L_t. \tag{32}$$

(32) is written as

$$L_t = (\frac{\sigma-1}{p^{\frac{1}{\rho-1}}})(n^{1-\gamma} + n^{\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}). \tag{33}$$

$\frac{\partial(n^{1-\gamma}+n^{\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}})}{\partial n} = (\frac{\psi-\rho}{(\psi-1)\rho\tau^{\frac{\psi}{1-\psi}}}n^{\frac{-\psi(1-\rho)}{\rho(1-\psi)}})\frac{1}{n^{\gamma}} - (\gamma-1)\frac{1}{n^{\gamma}} - (\frac{1}{\tau^{\frac{\psi}{1-\psi}}}n^{\frac{\rho-\psi}{\rho(1-\psi)}})\frac{\gamma}{n^{\gamma+1}} < 0$, since we assume that $\rho \geq \psi$ and $\gamma \geq 1$. Thus, the line $\frac{n_t}{n_{t-1}} = 1$ is a downward-sloping curve as in Figure 1. In addition, we assume that $\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)} < 0$. We define $R(n) = (\frac{\sigma-1}{p^{\frac{1}{\rho-1}}})(n^{1-\gamma} + n^{\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}})$. With the assumption of $\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)} < 0$, $\lim_{n\to 0} R(n) = \infty$ and $\lim_{n\to\infty} R(n) = 0$. Thus, the phase diagram can be depicted as in Figure 1.

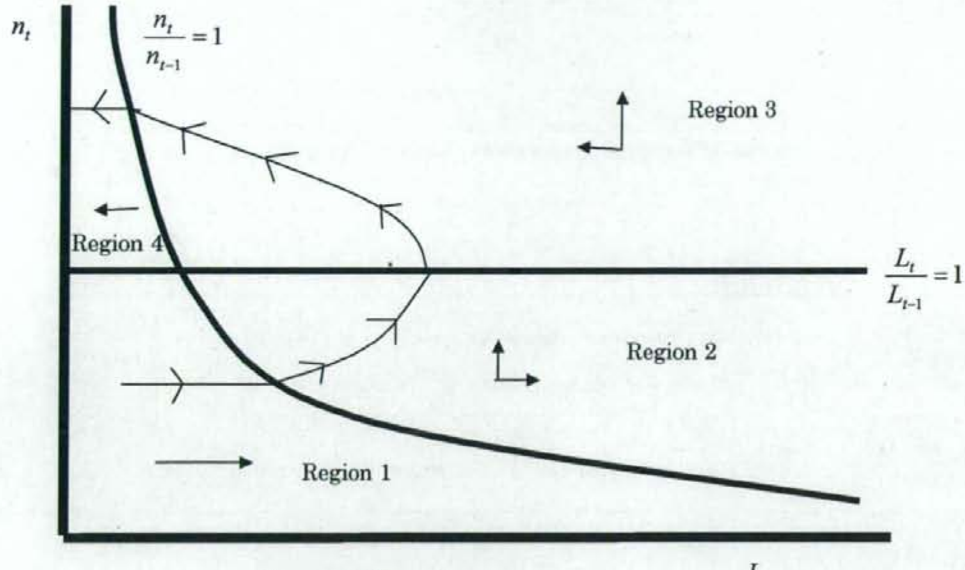

Figure 1: Phase diagram

Low Fertility and Population Aging in Germany and Japan:

Prospects and Policies

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1. Introduction

Almost all developed countries currently have below replacement fertility (United Nations (2007)), a situation in which the average woman of reproductive age does produce enough offspring to replace herself with one daughter of reproductive age in the next generation. Indeed, a number of these countries are in a group characterized as having the lowest-low fertility. This normally means that, on average, 100 woman of reproductive age replace themselves with less than 75 daughters. In the long-run, when age structure effects of baby booms and the like are washed out, that rate of reproduction would lead to population shrinkage of around 0.95 percent per year. This pace of population shrinkage would cause a population of 100 million people to be reduced to 38 million over the course of a century. If, on average, 100 women of reproductive age produced 70 daughters, in the long run a population of 100 million would have 30 million descendents a century later. If they had 65 daughters, the population would shrink to only 24 million after 100 years. All of these figures assume that there are no changes in mortality conditions and no immigration. These rates of reproduction are not hypothetical. Countries with the lowest-low fertility are experiencing them now and are taking actions to raise their fertility rates. But what, if anything, should be done about those low fertility rates and why should it be done? These are the questions addressed in this paper.

It only takes a moment's thought to realize that the lowest-low fertility is a very peculiar phenomenon. No other species of animal, with ample food and the other essentials of life, fails to reproduce itself. In long-run, this is a strategy that is likely to lead to extinction. Part of the visceral dislike that some people feel about population shrinkage comes from the association of population shrinkage with wars, famine, disease, and massive out-migration. When we think about population shrinkage, these ingrained associations color our attitudes. Today, we are dealing with a new situation, one of voluntary population shrinkage in an environment of high living standards. People are choosing to have few children not because of deprivation, but even as they have wealth that was hardly imaginable a century earlier. While the aggregation of individual actions does not always lead to a socially preferred outcome, economists like to understand why it does not before they recommend policies designed to make people change their behavior. Indeed, it is argued here that such an understanding is crucial to the formulation of appropriate future policies regarding fertility.

The very peculiarity of voluntary population shrinkage in an environment of high living standards suggests a useful perspective on policy formation. The voluntary shrinkage of rich national populations is a very recent phenomenon. For all of its existence humans had two strong forces that sustained childbearing, the sex drive and the need to have children to support their aging parents. With modern contraceptives and modern finance, these two props have disappeared. Given the variety of cultures, policies and socio-economic institutions among rich countries today, it is not surprising that, in some places, no socio-economic arrangements have emerged yet to sustain fertility.

From a longer-term perspective, the voluntary population shrinkage of wealthy countries could be a transitory phenomenon. Humans respond to their living conditions and a transitory period of shrinkage could be just what is needed to alter older ways of doing business to newer ones that support fertility. We can learn some lessons here from the earlier debate on family planning. Advocates of family planning scared many people with their computations of how many people would be on Earth if past growth rates continued for centuries. Of course, these growth rates did not continue. Advocates of pro-fertility policies can now scare people with computations of how few people would live in certain countries if low fertility persisted for centuries. But these low fertility rates may also not persist. We discuss reasons why current low levels of fertility may not continue in Section 6 below.

In nature, species that cannot adjust to changes in their environments die out. In our own genus, Homo erectus, Homo habilis, and Homo ergaster, among others, are no longer with us. Closer to home, many cultures, such as those of the ancient Egyptians and the ancient Mayans, have died out. But we have no evidence, at least as of yet, of rich populations simply breeding themselves out of existence.

Of course, if we believed strongly that fertility would rebound on its own, the case for policies to increase fertility would weaken. On this score, we can learn another lesson from the earlier debate on family planning. We can also learn that adjustments to the environment are imperfect. Economic development and family planning programs helped reduce fertility in many countries around the world. Still, there are a number of countries, especially in Sub-Saharan Africa, where rapid population growth is one cause of lowering standards of living and in which fertility remains high. There is no reason to argue that countervailing forces will automatically emerge that will raise fertility back to replacement level in all countries. In some countries, weaker versions of those forces could be present and good policies would identify and strengthen them.

Thus, there are two intellectual traps for us to avoid, the Scylla and Charybdis of these sorts of population discussions. The first is that because fertility is low now, it must remain low forever in the absence of new policies. The second is that no new policies are needed because of automatic homeostatic forces. We must steer clear of both of these if we are to arrive safely at some understanding of appropriate responses to the challenge of very low fertility.

Once we allow ourselves to consider the possibility, but not the necessity, of new policies two observations immediately constrain our choices. First, in general, fertility policies are costly. Policies of providing subsidized childcare or mandating minimum paid parental childbearing leaves cost money. Sometimes the money comes from the government and has to be raised through taxes. Sometimes it is a levy on businesses. In addition to monetary costs, there are also political adjustment costs. The costs and benefits of fertility policies are not borne uniformly. There are always gainers and losers. The distributional consequences of these policies always have to be considered. Some fertility policies would benefit members of younger generations at the expense of older populations. These policies may become more difficult to enact as electorates age. Fertility policies always have opportunity costs.

The second observation is that higher fertility is not a goal in itself. Economists normally assume that improving human well-being is the goal toward which policies should be directed. Perhaps many of the goals of fertility policies would be better met by improving the skills and capabilities of the people who would be in the country in any case. Perhaps spending more money on schooling would have more of a positive impact on the well-being of citizens than spending the money on having more people. Fertility policies need to be evaluated not just on whether they would produce more births, but in the broader context of whether they improve well-being more than alternative policies.

Policy formulation and evaluation need to be dynamic. Fertility policies need to be designed to be evaluated and adjusted. Here we can run into two sorts of problems. The first is a sort of policy hubris. Policies are put in place and their effects on fertility are measured in their first years of operation. But many other things could have influenced fertility in those years. To evaluate and adjust a policy requires some knowledge of how it works. There is a literature in fertility analysis on the effects of social interactions (see Kohler (2001), for example). The literature suggests that fertility behavior is influenced, in part, by the actions of other people. A policy may appear to have little effect in its first few years of implementation, but could have a much larger effect over time because of its indirect effects through social interactions. In order to understand how fertility policies work, we need to establish a much deeper scientific grasp of fertility determinants than we currently have. A major hindrance to a better scientific understanding of how fertility policies work is the lack of appropriate data. We need, at a minimum, data on birth rates by age, birth order, and year and preferably age- and birth order-specific birth probabilities. Supplementing these with information about the education level of the mother and her location would be a great boon. Without the needed information, the development and evaluation of fertility policies will remain a shot in the dark.

In order to consider fertility policy in a more concrete setting, we look at two countries here, Germany and Japan. Both are very wealthy and both are experiencing very low fertility. In 2006, 100 German women, on average, had a number of babies that would result in 65 daughters replacing them in the next generation. In Japan, the comparable number was 64 daughters. They are hardly the only rich countries to have the lowest-low fertility, but they provide good representative cases for study.

2. The Quantitative Effect of Fertility Policies

Before we begin discussing fertility policies it is useful to have in mind what such policies are likely to achieve. The easiest way to do this is to look at some of the population projection variants produced by the United Nations (United Nations (2007)). These variants differ only in terms of their assumptions concerning fertility and so they are exactly what we need for our purposes. Figures 1a and 1b show the UN fertility assumptions for four of their variants for Germany and Japan respectively. They are graphed in terms of the average number of daughters born to 100 women of reproductive age in the indicated period, who would survive to become reproductive age women in the next generation. A figure of 100 would indicate that women were exactly reproducing themselves. For short, we refer to these numbers as "daughters per 100 women".

The four variants are the medium variant, which is the best guess as to what the future is likely to be, a constant fertility variant, and two variants, a high one and a low one, that bracket the medium variant. Fertility in the medium variant, for both Germany and Japan is expected to rise. In Germany it increases by 29 percent over the period 2000-05 to 2045-50, from 65 daughters per 100 women to 84 daughters per 100 women. In Japan the increase over the same period is 24 percent, from 62 daughters per 100 women to 77 daughters per 100 women. The high and low variants are quite unlikely. The high variant for Germany has the number of daughters per 100 women rising from 65 in 2000-05 to 108 in 2045-50. In Japan, the rise during that interval is from 62 daughters per 100 women to 102. In the high variant, fertility is above replacement level in both countries by mid-century. The low variants show precipitous falls in fertility in both countries after 2000-05 followed by an increase that leaves fertility even lower in 2045-50 than it was in 2000-05.

There are two main reasons for the UN's assumption that fertility would increase in its medium variant. The first is a technical issue. The figures on the number of daughters per 100 women are influenced both by the average number of children women have over their reproductive lifetimes and the ages at which they have those children. Increases in the average age at childbearing, which have been occurring in both Germany and Japan, depress the figures on the number of daughters per 100 women relative to what they would be if that average age were constant. Because the average age at childbearing cannot increase indefinitely, if the average number of children women have over their reproductive lifetimes remains constant, the number of daughters per 100 women must rise. The second reason is that the UN assumes that Germany and Japan will introduce additional policy measures that will increase fertility.

Thus, comparing population sizes assuming constant fertility and the UN medium variant provides a rough estimate of the combined effects of the two causes. Tables 1a and 1b show those combined effects. If fertility remains constant, the UN forecasts the German population to be 69.7 million in 2050, down from 82.7 million in 2005. If fertility follows the path of the medium variant, German population would be 74.1 million in 2050, or 4.4 million people higher. In Japan, the situation is similar. With constant fertility, the UN forecasts that Japan's population would be 99.3 million in 2050, down from 127.9 million in 2005. If fertility followed the medium variant Japan's population in 2050 would be 102.5 million, 3.2 million people more than it would have had under the constant fertility scenario.

These are strikingly small effects especially when we remember that not all of them can be attributed to expected new population policies. Would the welfare of Germans in 2050 really be that much higher if the German population were 4.4 million people larger? Would the addition of 3.2 million Japanese in 2050 markedly improve the welfare of the Japanese people?

Under the constant fertility scenario, Germany's 2050 population would be roughly what it was in 1954 and Japan's 2050 population would be roughly what it was in 1966. If Germany's population size was acceptable in 1954 and Japan's was acceptable in 1966, then, to make an argument in favor of increasing fertility, we would need to know why they would be unacceptable in 2050.

Another way of looking at the data in Figures 1a and b and Tables 1a and b, is to look at the difference between the medium variant and the high variant. We can do a thought experiment in which the medium variant is what would happen in the absence of additional policies that increased fertility and the high scenario is what would happen if we had wildly successful fertility policies. We characterize the high scenario as being wildly successful because it assumes that by 2015-20, fertility is increased by around 24 daughters per 100 women compared to what it would be in the medium scenario and that this difference remains through 2045-50. There is no suggestion in the literature that such a large effect of is even possible. So by comparing the UN high variant with its medium variant, we get an upper bound on the effects of any plausible fertility policy. In Germany, this upper bound is an increase in the population in

2050 by 11.4 million people, from 74.1 million to 85.6 million. Instead of decreasing by 8.6 million people, as it would under the medium variant, it would increase by 2.9 million. In Japan, the upper bound on the population increase would be 15.8 million people. Instead of decreasing by 25.4 million people, Japan's population would decrease by 9.6 million. Of course, the likely effects of a fertility policy would be much smaller.

An objection to this analysis could be that we should not stop at 2050. This is the Scylla of population size discussions and it is dangerous to go there. Things do not stay the same. Making long term forecasts on the basis of constant fertility is misleading. This essay was written in 2008. Think back to 1958. A discussion that forecasted the fertility situation that we now observe would have been considered ludicrous. Nevertheless, we are now living in it. Fertility in 2050 is not likely to be easier to forecast than today's fertility was half a century ago.

The effects on population size of any plausible fertility policy are not expected to be large by 2050. In order to understand if the magnitudes of the gains will offset the costs, we need a clearer picture of the goals that fertility policies are supposed to achieve.

3. Why Do We Need Any Fertility Policy?

In this section, we deal with the major arguments in favor of pronatalist policies.

3.1 Arguments based on population size

Here we discuss arguments in favor of policies to increase fertility based on population size alone. We defer consideration of arguments based on age structure effects to a later sub-section. Two types of arguments occur often. The first is based on ethnic identity. There is sometimes a fear that if the numbers of people with a certain culture and heritage falls too low, then that culture and heritage will get lost. This argument is surely true, but it hardly applies to either Germany or Japan. Sweden's population was 9.0 million in 2005 (United Nations (2007)). It still has a viable culture and is able to maintain its traditions and historical heritage. The argument that we need fertility policies in Germany and Japan to maintain culture, traditions, and heritage must, therefore, be rejected.

Another argument for policies to increase fertility is that international power and prestige are related to population size. This may be true, but international power and prestige is based on many other factors as well. Nevertheless, having a few million more people by 2050 is hardly likely, by itself, to have much influence on a country's international power and prestige. Even if it did, people may not wish to pay much for it. After all, would the citizens of Switzerland or

Sweden, for example, wish to double their population sizes simply because of the additional prestige that could possibly come along with it? Probably not.

If our view only goes up to 2050, it is difficult to imagine that fertility policies would have anything other than a trivial effect on the ability of Germany and Japan to maintain their cultural traditions or on their relative international power and prestige. Recall again, that here we are addressing only population size not population age structure. Arguments concerning population size have the most power when they are applied to the long-run. But here we have to be careful about assuming that current conditions will remain constant into the future. Arguments concerning population size suggest that appropriate policies need to be designed for the long-haul and that they need to be flexible enough to take changing conditions into account. They do not suggest that we need to implement strong fertility-increasing policies right away.

3.2 Arguments based on population growth

It could be argued that we need more babies because population shrinkage reduces the rate of per capita income growth. This argument is somewhat ironic given the large literature arguing the rapid population growth decreases the rate of per capita income growth, but nevertheless deserves mention here. Empirical evidence suggests that changes in the rate of population growth, holding age structure constant, has only a very weak effect on the rate of per capita economic growth (Kelley and Schmidt (2005)). Most of the ill-effects of low fertility on the economy, such as difficulties in sustaining national pension commitments arise because of the age structure effects that we discuss below.

Population growth is not free, even if population policies to promote fertility were costless. Children are an investment. They cost money to raise, to educate, and to keep healthy before they become productive in the labor force. There is no evidence to suggest that investing in more children is more efficient than in investing more in the children that would be there in any case.

Arguments based on population size and growth do not, in themselves, provide a strong basis for advocating polices to increase fertility immediately. In the long-run, countries need to find a way to maintain their populations. Whether this could happen automatically, even in the absence of fertility policies, is addressed in Section 4below.

3.3 Arguments based on fertility levels

When we study complex dynamic processes like population growth, it is often useful to distinguish conditions that lead to negative feedbacks from those that lead to positive feedbacks. Negative feedbacks are stabilizing and positive ones are destabilizing. If the lowest-low fertility was part of a negative feedback system, then it would generate forces within the system to bring fertility back up. If it were part of a positive feedback system, then the lowest-low fertility would generate forces that tend to keep fertility low or perhaps even drive it lower.

Recently, Lutz and Skibekk (2005) proposed a hypothesis that is a variant of the positive feedback conceptualization. They suggested the possibility that when fertility fell below 75 daughters per 100 women social changes would begin that would make increases in fertility more difficult to achieve. The low level fertility trap hypothesis is plausible enough, but its application to Germany and Japan is not immediately clear. Both countries are below the trap threshold and have been there for some years now. A potential elaboration of the low level fertility trap hypothesis might be that the difficulty of escaping from the trap increases with the number of years in which fertility is below the threshold level. It is important to emphasize here that the feedback mechanism envisioned in the low level fertility trap hypothesis works off of the level of fertility itself, not on population size or population growth. Fertility is treated as a social phenomenon with past levels of fertility positively influencing current levels through changes in the socio-economic environment in which people live. Long periods of the lowest-low fertility could allow adaptations to take place that make the lowest-low fertility the normal state of affairs and make changing it harder. If this were the case, then it argues for strong fertility-increasing policies now, because to achieve the same effects later would be more costly and more difficult.

One intriguing feature of the low level fertility trap hypothesis is that it links the long-run and the short-run. It suggests the possibility that we might not be able to solve the long-term problem of population shrinkage if we do not take actions soon, not because population size will quickly get too small, but rather because of the cost of raising fertility will become so large that even in the long-run it would be extremely difficult to do it.

The implications of the low level fertility trap hypothesis for the development of fertility policies are as unique as the hypothesis itself. Policies that are just addressed to the issue of numbers would emphasize long-run policies that increased the average number of children ever born to women. Demographers call these "quantum" policies. If we were basing policies on correctness of the low level fertility trap hypothesis, we could, in addition, employ policies that had only transitory effects. Policies that affect only the timing of births and not the average number of births are called "tempo" policies. Their effects are inherently temporary, but even temporary effects could be very powerful according to the low level fertility trap hypothesis. A temporary increase in fertility would enhance the effects of other policies, thus potentially propelling a country out of the trap. An interesting tempo policy suggested in Lutz and Skibekk (2005) is to