表4　年齢シフトモデルとリー・カーター・モデルによるIQR（女性）

e_0 Level	90	94	97
年齢シフトモデル	12.5	12.0	11.6
リー・カーター・モデル	12.1	11.4	10.8

する。

図13は死亡率曲線の将来推計値を比較したものである。これを見ると、リー・カーター・モデルによる試算値は、60~70歳代で年齢シフトモデルと比較して死亡率が低く推移した後、急速に増加し、80歳以降の高齢部分では逆に高いレベルとなっており、年齢シフトモデルによる試算値に比べて高齢部分での死亡率曲線の勾配が大きくなっていることが観察される。

図14は生存数曲線の将来推計値を比較したものであるが、死亡率曲線に見られた高齢での勾配の違いの影響により、リー・カーター・モデルによる生存数曲線は、死亡率の低下が生存数曲線の「矩形化」現象として現れる傾向がより強く出ている。一方、年齢シフトモデルにおける死亡率の低下は、生存数曲線の矩形化の動きというよりも、生存数曲線自体が右方向へシフトするという動きとして現れていることが観察できる、

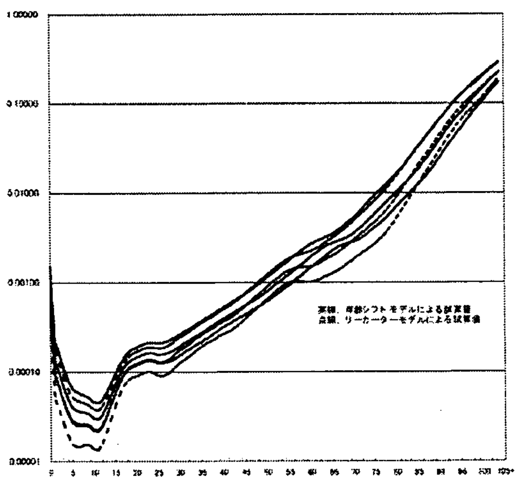

図13　死亡率曲線の比較（試算値・女性）

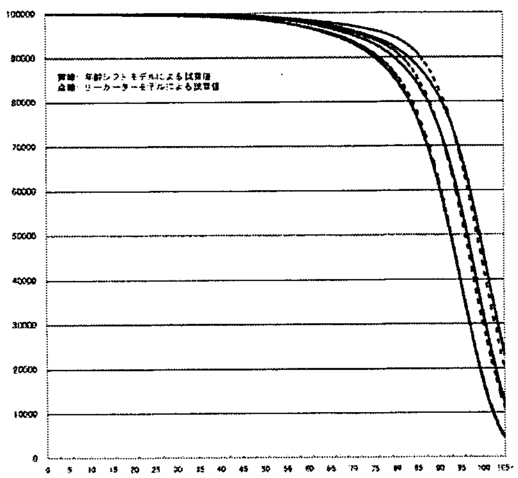

図14　生存数曲線の比較（試算値・女性）

そこで、これらについて、IQRを算定して示したものが表4である。これによれば、年齢シフトモデルによるIQRはリー・カーター・モデルによるIQRよりも高いレベルに留まっており、先に見た、リー・カーター・モデルでは死亡率の低下が生存数曲線の矩形化現象として現れる傾向がより強く出ている点を定量的にも確認することができる。

以上のような考察から、年齢シフトモデルは、わが国の近年の高齢死亡率改善が、死亡

が遅延している動きとして捉えられる点とも整合的なモデルであることが定量的にも示されたことになる。

おわりに

本研究では、Wilmoth and Horiuchi (1999) において検討された生存数曲線の矩形化に関する定量的な指標をわが国の実績生命表及び将来生命表に適用することにより、年齢シフトモデルによる将来生命関数の特性評価を行った。さらに、昨年度の年齢シフトモデルとリー・カーター・モデルの比較を一歩進めて、生存数曲線の矩形化に関する指標に基づき、より定量的に両モデルを比較した。これにより、年齢シフトモデルに基づく将来死亡年齢パターンは、同程度の平均寿命を表現するリー・カーター・モデルによるパターンと比較してその矩形化の度合は低く、従って、死亡の遅延による年齢シフトというメカニズムによって死亡率改善を表現するモデルであるという特性を定量的に評価することができた。

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Wilmoth, J. R. and S. Horiuchi (1999) "Rectangularization Revisited: Variability of Age at Death within Human Populations", *Demography*, Vol. 36, No. 4, pp. 475–495.

表５　各指標の推移と見通し（死亡中位・男性）

	FR	FD	SC	QP	PI	IQR
1891-1898	0.428	0.0172	0.00056	0.00119	0.0151	62.1
1899-1903	0.440	0.0183	0.00059	0.00120	0.0162	60.1
1909-1913	0.442	0.0182	0.00059	0.00121	0.0159	62.2
1921-1925	0.421	0.0184	0.00072	0.00128	0.0161	62.0
1926-1930	0.448	0.0196	0.00087	0.00133	0.0176	53.0
1935-1936	0.470	0.0203	0.00111	0.00141	0.0183	49.1
1947	0.501	0.0219	0.00110	0.00142	0.0195	43.2
1950-1952	0.596	0.0274	0.00205	0.00189	0.0236	25.6
1955	0.636	0.0308	0.00164	0.00210	0.0268	21.0
1960	0.653	0.0333	0.00151	0.00263	0.0289	18.9
1965	0.677	0.0352	0.00181	0.00293	0.0309	17.2
1970	0.693	0.0356	0.00168	0.00273	0.0309	16.8
1975	0.717	0.0368	0.00190	0.00305	0.0304	16.3
1980	0.733	0.0380	0.00201	0.00316	0.0319	15.9
1985	0.748	0.0380	0.00232	0.00321	0.0342	15.9
1990	0.759	0.0392	0.00201	0.00338	0.0356	15.9
1995	0.764	0.0383	0.00181	0.00358	0.0344	16.1
2000	0.777	0.0372	0.00185	0.00325	0.0336	16.5
2005	0.785	0.0378	0.00179	0.00339	0.0336	16.0
2010	0.795	0.0379	0.00171	0.00319	0.0331	16.1
2015	0.802	0.0380	0.00171	0.00322	0.0330	16.1
2020	0.808	0.0381	0.00171	0.00323	0.0326	16.1
2025	0.813	0.0382	0.00172	0.00325	0.0331	16.1
2030	0.818	0.0382	0.00172	0.00327	0.0326	16.0
2035	0.822	0.0383	0.00172	0.00328	0.0329	16.0
2040	0.826	0.0384	0.00172	0.00329	0.0325	16.0
2045	0.829	0.0384	0.00172	0.00331	0.0328	16.0
2050	0.833	0.0385	0.00172	0.00331	0.0330	16.0
2055	0.835	0.0385	0.00173	0.00331	0.0326	16.0

表６　各指標の推移と見通し（死亡中位・女性）

	FR	FD	SC	QP	PI	IQR
1891-1898	0.443	0.0177	0.00056	0.00136	0.0149	60.1
1899-1903	0.449	0.0183	0.00067	0.00132	0.0158	58.2
1909-1913	0.447	0.0179	0.00067	0.00126	0.0160	60.3
1921-1925	0.432	0.0180	0.00072	0.00147	0.0162	61.1
1926-1930	0.466	0.0197	0.00085	0.00158	0.0170	56.1
1935-1936	0.497	0.0217	0.00087	0.00189	0.0181	52.7
1947	0.540	0.0236	0.00113	0.00189	0.0192	44.5
1950-1952	0.630	0.0291	0.00198	0.00239	0.0238	25.7
1955	0.677	0.0321	0.00195	0.00249	0.0284	20.3
1960	0.702	0.0366	0.00220	0.00276	0.0315	17.4
1965	0.729	0.0405	0.00228	0.00336	0.0356	15.4
1970	0.747	0.0401	0.00241	0.00359	0.0350	14.9
1975	0.769	0.0431	0.00259	0.00398	0.0383	14.1
1980	0.788	0.0440	0.00270	0.00417	0.0386	13.5
1985	0.805	0.0448	0.00286	0.00418	0.0374	13.2
1990	0.819	0.0457	0.00312	0.00443	0.0399	13.0
1995	0.828	0.0456	0.00275	0.00454	0.0408	13.3
2000	0.845	0.0448	0.00258	0.00433	0.0377	13.3
2005	0.854	0.0452	0.00294	0.00431	0.0381	13.2
2010	0.863	0.0457	0.00292	0.00451	0.0403	13.0
2015	0.869	0.0460	0.00296	0.00459	0.0407	12.9
2020	0.875	0.0463	0.00300	0.00469	0.0410	12.8
2025	0.879	0.0465	0.00303	0.00474	0.0412	12.8
2030	0.884	0.0468	0.00306	0.00481	0.0414	12.7
2035	0.887	0.0470	0.00309	0.00487	0.0415	12.6
2040	0.891	0.0471	0.00311	0.00490	0.0416	12.6
2045	0.894	0.0473	0.00314	0.00497	0.0419	12.5
2050	0.896	0.0474	0.00316	0.00500	0.0419	12.5
2055	0.899	0.0476	0.00317	0.00505	0.0420	12.5

表７　各指標の推移と見通し（死亡高位・男性）

	FR	FD	SC	QP	PI	IQR
1891-1898	0.428	0.0172	0.00056	0.00119	0.0151	62.1
1899-1903	0.440	0.0183	0.00059	0.00120	0.0162	60.1
1909-1913	0.442	0.0182	0.00059	0.00121	0.0159	62.2
1921-1925	0.421	0.0184	0.00072	0.00128	0.0161	62.0
1926-1930	0.448	0.0196	0.00087	0.00133	0.0176	53.0
1935-1936	0.470	0.0203	0.00111	0.00141	0.0183	49.1
1947	0.501	0.0219	0.00110	0.00142	0.0195	43.2
1950-1952	0.596	0.0274	0.00205	0.00189	0.0236	25.6
1955	0.636	0.0308	0.00164	0.00210	0.0268	21.0
1960	0.653	0.0333	0.00151	0.00263	0.0289	18.9
1965	0.677	0.0352	0.00181	0.00293	0.0309	17.2
1970	0.693	0.0356	0.00168	0.00273	0.0309	16.8
1975	0.717	0.0368	0.00190	0.00305	0.0304	16.3
1980	0.733	0.0380	0.00201	0.00316	0.0319	15.9
1985	0.748	0.0380	0.00232	0.00321	0.0342	15.9
1990	0.759	0.0392	0.00201	0.00338	0.0356	15.9
1995	0.764	0.0383	0.00181	0.00358	0.0344	16.1
2000	0.777	0.0372	0.00185	0.00325	0.0336	16.5
2005	0.785	0.0378	0.00179	0.00339	0.0336	16.0
2010	0.790	0.0378	0.00171	0.00319	0.0330	16.1
2015	0.797	0.0379	0.00171	0.00320	0.0332	16.1
2020	0.802	0.0380	0.00171	0.00322	0.0330	16.1
2025	0.806	0.0381	0.00172	0.00322	0.0333	16.1
2030	0.810	0.0381	0.00171	0.00324	0.0327	16.1
2035	0.813	0.0381	0.00172	0.00325	0.0330	16.1
2040	0.816	0.0382	0.00172	0.00325	0.0325	16.1
2045	0.819	0.0382	0.00172	0.00327	0.0327	16.0
2050	0.821	0.0383	0.00172	0.00328	0.0328	16.0
2055	0.823	0.0383	0.00172	0.00328	0.0330	16.0

表 8　各指標の推移と見通し（死亡高位・女性）

	FR	FD	SC	QP	PI	IQR
1891-1898	0.443	0.0177	0.00056	0.00136	0.0149	60.1
1899-1903	0.449	0.0183	0.00067	0.00132	0.0158	58.2
1909-1913	0.447	0.0179	0.00067	0.00126	0.0160	60.3
1921-1925	0.432	0.0180	0.00072	0.00147	0.0162	61.1
1926-1930	0.466	0.0197	0.00085	0.00158	0.0170	56.1
1935-1936	0.497	0.0217	0.00087	0.00189	0.0181	52.7
1947	0.540	0.0236	0.00113	0.00189	0.0192	44.5
1950-1952	0.630	0.0291	0.00198	0.00239	0.0238	25.7
1955	0.677	0.0321	0.00195	0.00249	0.0284	20.3
1960	0.702	0.0366	0.00220	0.00276	0.0315	17.4
1965	0.729	0.0405	0.00228	0.00336	0.0356	15.4
1970	0.747	0.0401	0.00241	0.00359	0.0350	14.9
1975	0.769	0.0431	0.00259	0.00398	0.0383	14.1
1980	0.788	0.0440	0.00270	0.00417	0.0386	13.5
1985	0.805	0.0448	0.00286	0.00418	0.0374	13.2
1990	0.819	0.0457	0.00312	0.00443	0.0399	13.0
1995	0.828	0.0456	0.00275	0.00454	0.0408	13.3
2000	0.845	0.0448	0.00258	0.00433	0.0377	13.3
2005	0.854	0.0452	0.00294	0.00431	0.0381	13.2
2010	0.859	0.0455	0.00289	0.00447	0.0403	13.1
2015	0.864	0.0458	0.00293	0.00454	0.0405	13.0
2020	0.869	0.0460	0.00296	0.00459	0.0407	12.9
2025	0.873	0.0462	0.00298	0.00465	0.0408	12.8
2030	0.876	0.0464	0.00301	0.00470	0.0411	12.8
2035	0.879	0.0465	0.00303	0.00476	0.0411	12.8
2040	0.882	0.0467	0.00305	0.00477	0.0412	12.7
2045	0.884	0.0468	0.00307	0.00483	0.0414	12.7
2050	0.886	0.0469	0.00308	0.00485	0.0415	12.7
2055	0.888	0.0470	0.00309	0.00489	0.0415	12.6

表9　各指標の推移と見通し（死亡低位・男性）

	FR	FD	SC	QP	PI	IQR
1891-1898	0.428	0.0172	0.00056	0.00119	0.0151	62.1
1899-1903	0.440	0.0183	0.00059	0.00120	0.0162	60.1
1909-1913	0.442	0.0182	0.00059	0.00121	0.0159	62.2
1921-1925	0.421	0.0184	0.00072	0.00128	0.0161	62.0
1926-1930	0.448	0.0196	0.00087	0.00133	0.0176	53.0
1935-1936	0.470	0.0203	0.00111	0.00141	0.0183	49.1
1947	0.501	0.0219	0.00110	0.00142	0.0195	43.2
1950-1952	0.596	0.0274	0.00205	0.00189	0.0236	25.6
1955	0.636	0.0308	0.00164	0.00210	0.0268	21.0
1960	0.653	0.0333	0.00151	0.00263	0.0289	18.9
1965	0.677	0.0352	0.00181	0.00293	0.0309	17.2
1970	0.693	0.0356	0.00168	0.00273	0.0309	16.8
1975	0.717	0.0368	0.00190	0.00305	0.0304	16.3
1980	0.733	0.0380	0.00201	0.00316	0.0319	15.9
1985	0.748	0.0380	0.00232	0.00321	0.0342	15.9
1990	0.759	0.0392	0.00201	0.00338	0.0356	15.9
1995	0.764	0.0383	0.00181	0.00358	0.0344	16.1
2000	0.777	0.0372	0.00185	0.00325	0.0336	16.5
2005	0.785	0.0378	0.00179	0.00339	0.0336	16.0
2010	0.799	0.0379	0.00171	0.00321	0.0331	16.1
2015	0.807	0.0381	0.00171	0.00323	0.0333	16.1
2020	0.814	0.0382	0.00172	0.00325	0.0332	16.1
2025	0.821	0.0383	0.00172	0.00328	0.0328	16.0
2030	0.826	0.0384	0.00172	0.00329	0.0326	16.0
2035	0.831	0.0384	0.00172	0.00332	0.0329	16.0
2040	0.836	0.0385	0.00173	0.00332	0.0326	16.0
2045	0.840	0.0386	0.00173	0.00335	0.0329	16.0
2050	0.844	0.0387	0.00173	0.00334	0.0331	16.0
2055	0.847	0.0387	0.00173	0.00338	0.0328	16.0

表 10　各指標の推移と見通し（死亡低位・女性）

	FR	FD	SC	QP	PI	IQR
1891-1898	0.443	0.0177	0.00056	0.00136	0.0149	60.1
1899-1903	0.449	0.0183	0.00067	0.00132	0.0158	58.2
1909-1913	0.447	0.0179	0.00067	0.00126	0.0160	60.3
1921-1925	0.432	0.0180	0.00072	0.00147	0.0162	61.1
1926-1930	0.466	0.0197	0.00085	0.00158	0.0170	56.1
1935-1936	0.497	0.0217	0.00087	0.00189	0.0181	52.7
1947	0.540	0.0236	0.00113	0.00189	0.0192	44.5
1950-1952	0.630	0.0291	0.00198	0.00239	0.0238	25.7
1955	0.677	0.0321	0.00195	0.00249	0.0284	20.3
1960	0.702	0.0366	0.00220	0.00276	0.0315	17.4
1965	0.729	0.0405	0.00228	0.00336	0.0356	15.4
1970	0.747	0.0401	0.00241	0.00359	0.0350	14.9
1975	0.769	0.0431	0.00259	0.00398	0.0383	14.1
1980	0.788	0.0440	0.00270	0.00417	0.0386	13.5
1985	0.805	0.0448	0.00286	0.00418	0.0374	13.2
1990	0.819	0.0457	0.00312	0.00443	0.0399	13.0
1995	0.828	0.0456	0.00275	0.00454	0.0408	13.3
2000	0.845	0.0448	0.00258	0.00433	0.0377	13.3
2005	0.854	0.0452	0.00294	0.00431	0.0381	13.2
2010	0.866	0.0459	0.00294	0.00457	0.0406	12.9
2015	0.874	0.0463	0.00299	0.00466	0.0409	12.8
2020	0.880	0.0466	0.00303	0.00476	0.0412	12.7
2025	0.886	0.0469	0.00308	0.00487	0.0414	12.7
2030	0.891	0.0472	0.00311	0.00490	0.0416	12.6
2035	0.895	0.0474	0.00315	0.00500	0.0418	12.5
2040	0.899	0.0476	0.00317	0.00502	0.0420	12.5
2045	0.903	0.0478	0.00321	0.00510	0.0422	12.4
2050	0.906	0.0480	0.00323	0.00520	0.0423	12.4
2055	0.909	0.0482	0.00325	0.00519	0.0424	12.3

10 Mortality Projection Model for Japan with Age-Shifting Structure

Futoshi Ishii*

Introduction

The official population projection for Japan is prepared by the National Institute of Population and Social Security Research (NIPSSR) based on the results of the Population Census every five years. The most recent projection was released in Dec. 2006 based on the 2005 Census (NIPSSR 2007). The cohort component method was used in the projection. To make an assumption for the survivorship ratio in the projection, use of the mortality projection model to obtain future life tables is necessary.

The mortality projection model should be selected in terms of detailed observations for trends of life expectancy and the available data set. Japanese life expectancy has increased rapidly over time and is still increasing with top class values in the world. This unique trend is one of the factors that makes it difficult to project future mortality situations for Japan.

In the latest projection, a new mortality projection model was studied and developed. The model is basically based on the Lee-Carter model, which is regarded as the standard method internationally, but was modified by adding new features to suit the characteristics of mortality trends in Japan.

In this paper, first we observe recent trends in Japanese mortality. Next, we review the Lee-Carter model and its application to Japan, and explore aspects to improve the mortality projection model for Japan. Then we discuss the new model, the age-shifting model, and compare with the Lee-Carter model.

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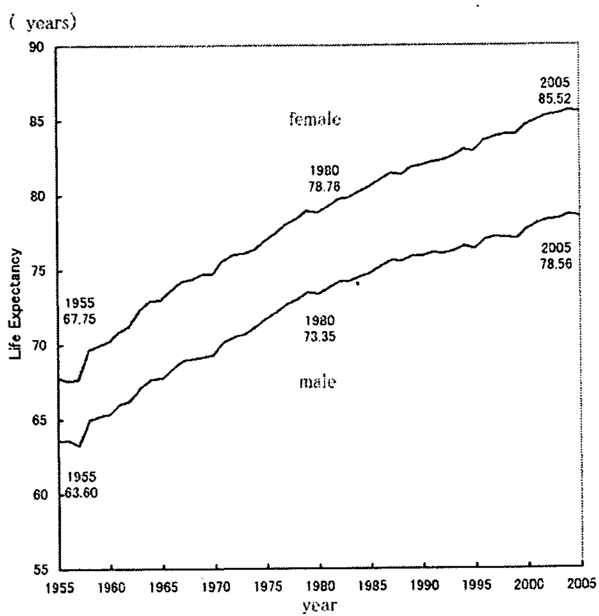
1 Recent Trends in Japanese Mortality

In this section, we observe recent trends in Japanese mortality.

Figure 1 shows the Japanese life expectancy from 1955 to 2005. The life expectancy values in 1955 are 63.60 for male and 67.75 for female, and 78.56 for male 85.52 for female in 2005. During this period, they are prolonged 14.96 for male and 17.77 for female. Figure 1 shows that the prolonging rate has been tapering recently. However, increases since 1980 are 5.21 for male and 6.76 for female which shows life expectancy has still been increasing in recent years. It is characteristic for the Japanese mortality situation that life expectancy is increasing steadily, maintaining top levels in the world.

Figure 1 Life Expectancy for Japan from 1955 to 2005

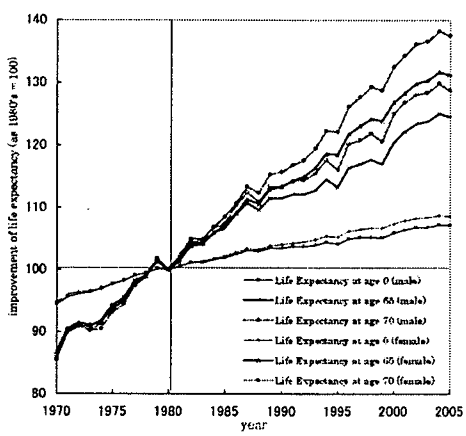


Source: MHLW (Ministry of Health, Labour and Welfare), "Life Tables"

It is also an important feature in the recent Japanese mortality trend that the improvements in older age had a larger impact on the prolonging in life expectancy in both sexes. Figure 2 shows the indices of life expectancy at age 0, 65 and 70 compared with the 1980's. We can observe that the increase in life expectancy at age 65 and 70 are much greater than those at age 0. This shows that improvements are remarkable in older age.

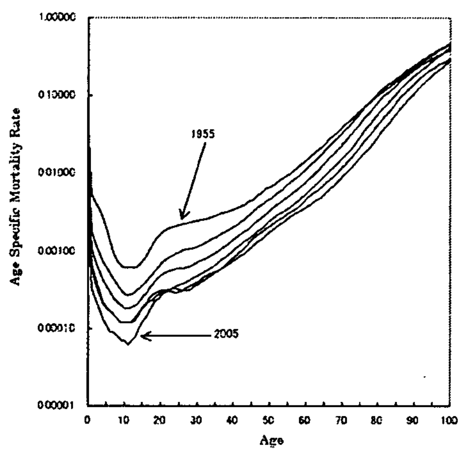
Moreover, we can also observe these facts from the change in the shape of the mortality curves. Figure 3 shows that Japanese mortality improved first in the young age bracket, and then recently in the old age bracket. In addition, we can also recognize these changes as the shifting of curves in the direction of older people, that is, delays in the timing of death. Therefore, the feature where the improvement in the older age bracket is remarkable is related to the age-shifting of mortality curve, which illustrates an important point in construction of mortality projection models for Japan.

Figure 2 Improvement of Life Expectancy at Age 0, 65 and 70 (as 1980's = 100), from 1970 to 2005



Source: MHLW(Ministry of Health, Labor and Welfare), "Life Tables"

Figure 3 Age Specific Mortality Rate for Japanese female, from 1955 to 2005



Source: MHLW(Ministry of Health, Labor and Welfare), "Life Tables"

2 The Lee-Carter Model and its Application to Japan

There are various models for mortality projection. Among them, the Lee-Carter model(Lee and Carter 1992) is now regarded as the standard method internationally. Tuljapurkar et al. (2000) applied this model to the mortality in G7 countries and demonstrated its effectivness.

We first review this model here. Let $\ln(m_{x,t})$ be the natural logarithm of central death rates. Then, the Lee-Carter model is expressed as follows.

$$\ln(m_{x,t}) = a_x + k_t b_x + \epsilon_{x,t}$$

where a_x is the average mortality age pattern and $\epsilon_{x,t}$ represents error terms. To estimate the parameters b_x, k_t, applying singular value decomposition(SVD) to the matrix $\ln(m_{x,t}) - a_x$

$$\ln(m_{x,t}) - a_x = \sum_i u_{xi} q_i v_{ti} \qquad (q_1 \geq q_2 \geq \cdots)$$

Then we observe the term relating q_1 (the first singular value), and set

$$k_t = q_1 v_{t1}$$

$$b_x = u_{x1}$$

The future values of k_t are projected using time series analysis, and then the future mortality rates are projected using the projected k_t values.

There are also many studies that applied the Lee-Carter model to Japanese mortality. Wilmoth (1996) applied this model to Japanese total mortality(Method I), and compared the projection by forcing its future trend to match the projected Swedish trend (Method II) and the projections by cause-specific mortality (Methods III and IV).

Komatsu (2002) studied and developed the projection model applying the Lee-Carter method for the previous Japanese official population projection in 2002 (NIPSSR 2002), which will be described later (we call this "the Komatsu model" in this paper). Moreover, Ogawa et al. (2002), Nanjo and Yoshinaga (2003), Kogure and Hasegawa (2005), Ozeki (2005), Oikawa (2006) also studied the application of the Lee-Carter model to Japanese mortality.

The Komatsu model, which was used in the previous projection in 2002, applied a Lee-Carter model that is slightly modified to suit Japanese mortality projection. The main differences are: (1) a_x is the average of the most recent two years in the Komatsu model, which is the average of the whole term in the original Lee-Carter model, (2) k_t is projected by non-linear curve fitting in the Komatsu model[*1], not by time series analysis as in the original Lee-Carter model.

However, from the observation comparing the projected mortality rates in the previous projection to the actual ones, the projected mortality rates for older ages have turned out to be higher than actual ones.

In Lee and Miller (2001), the performance of the Lee-Carter model in mortality projections was evaluated using the data in the U.S., Canada, Sweden, France and Japan. The study showed that the projected life expectancy using the Lee-Carter model tends to be lower, especially if the projected period is getting longer. It also pointed out that this tendency would be related to

[*1] The fitted function is the average of the exponential and the logarithm function, which is supposed to fit well with the recent trend in Japanese mortality.

the changing age pattern of decline in some way. Relating to this point, we have seen in the previous section that we can recognize the recent changes in the Japanese mortality age pattern as a shifting of curves in the direction of older people. Therefore, seeking a model that could express an age-shifting structure would improve the mortality projection.

As such a mortality model that has an age-shifting structure, Bongaarts (2005) proposed "the shifting logistic model," noticing that the slope parameter in the three parameter logistic curve, which is fitted to the mortality data in each country, is almost constant over time.

Ishii (2006) studied the Lee-Carter model with an age-shifting structure, and discussed that the model has an advantage in fitting with Japanese old-age mortality. In this study, we moved further and developed a new mortality projection model with an age-shifting structure used in the 2006 official projection, which is a Lee-Carter model applied with the shift amount in the shifting logistic model.

3 Mortality Projection Model with Age-shifting Structure

In this section, we describe the mortality projection model with age-shifting structure (we call this "the age-shifting model" in this paper). The age-shifting model is constructed as follows.

Figure 4 Estimated S_t and β_t, from 1970 to 2005

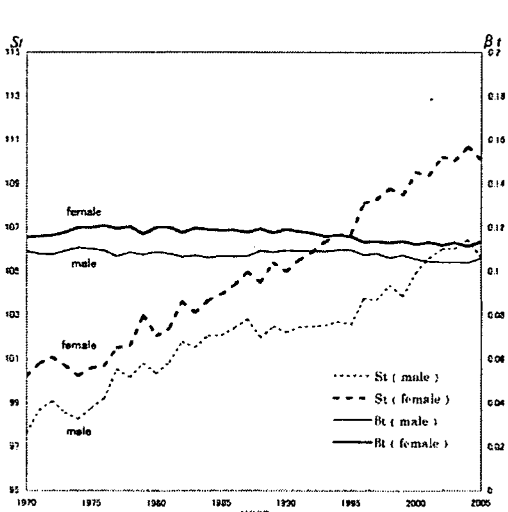
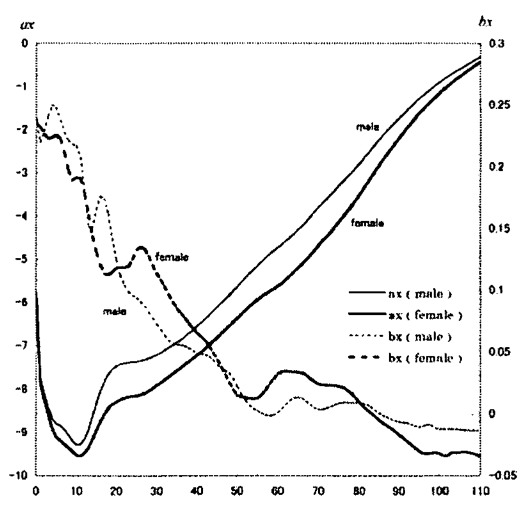


Figure 5 Estimated a_x and b_x



First, we fit the three parameter logistic curve

$$\mu_{x,t} = \frac{\alpha_t \exp(\beta_t x)}{1 + \alpha_t \exp(\beta_t x)} + \gamma_t$$

to the actual Japanese life tables[*2]. Then we obtain a parameter $S_t = -\frac{\ln(\alpha_t)}{\beta_t}$, which is used to express the shift amount in the shifting logistic model (Bongaarts 2005), and another parameter β_t, which express the slope of the curve (Figure 4) .

We now consider the linear transformation setting the intercept (amount of age-shifting) as the difference $S_{t_0} - S_t$, where $t = t_0 (= 2005)$ is the base point year, and the slope as the ratio of β_t to the base point. We apply this transformation to the central death rate $m_{x,t}$ to basically obtain the age-shifted mortality set. However, in detail we didn't apply this transformation to the age bracket less than age $x = 25 (= B_1)$ at the base point year or completely apply more than $x = 50 (= B_2)$. We used linear interpolation for the other age bracket.

The transformation is precisely accomplished by the following formula. Let x be the original age and y be the transformed one, and define the relation $x = f(y)$ as follows.

$$f(y) \underset{\text{def}}{=} \begin{cases} y & (y \leq B_1) \\ \left\{ \frac{\beta_{t_0}}{\beta_t} (B_2 - S_{t_0}) + S_t - B_1 \right\} \frac{y - B_1}{B_2 - B_1} + B_1 & (B_1 \leq y \leq B_2) \\ \frac{\beta_{t_0}}{\beta_t} (y - S_{t_0}) + S_t & (B_2 \leq y \leq S_{t_0}) \\ y - S_{t_0} + S_t & (S_{t_0} \leq y) \end{cases}$$

Then set

$$\hat{m}_{y,t} \underset{\text{def}}{=} m_{f(y),t}$$

and apply the Lee-Carter method to the natural logarithm of $\hat{m}_{y,t}$, which represents the age-shifted mortality rates. We used a_x in the Lee-Carter method as the average in the most recent five years and estimated the parameters b_x and k_t applying the SVD to the matrix $\ln(\hat{m}_{y,t}) - a_x$ (Figure 5) .

Projecting the parameter k_t, we applied nonlinear curve fitting (Figure 6), using the same function as the Komatsu model. We projected the parameter S_t using linear regression with k_t and fixed the parameter β_t with the averages in recent figures (Figure 7).

[*2] We recalculated the actual Japanese life tables from population census, population estimates (Bureau of Statistics) and vital statistics (MHLW). However, the life expectancies by the recalculated tables are almost the same values as the official life tables for Japan (MHLW)

Figure 6 Actual and Projected k_t, from 1970 to 2055

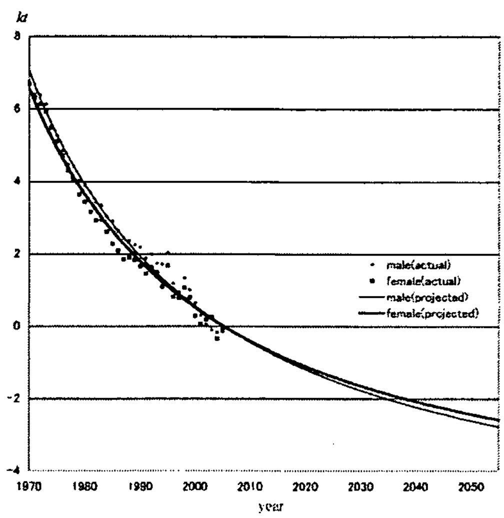
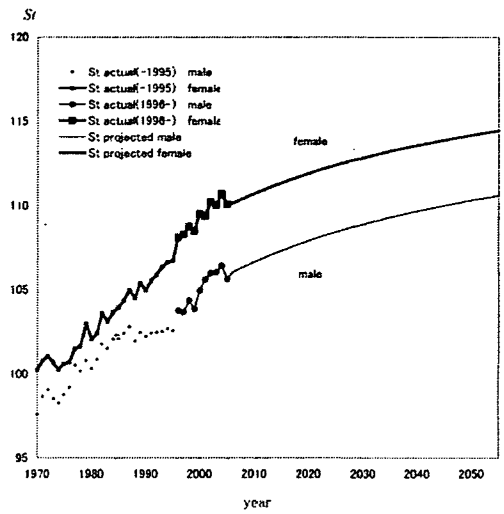


Figure 7 Actual and Projected S_t, from 1970 to 2055



We made three variants of mortality levels. For this purpose, we estimated 99% confidence intervals for the parameter k_t using the bootstrap method. Then we set "high mortality variant" as the upper limit of the confidence intervals and "low mortality variant" as the lower (Figure 8).

Figure 9 shows the projected life expectancy at birth with this model. In the medium mortality variant, which is considered as the standard case, the life expectancy at birth will reach 83.67 for males and 90.34 for females in 2055, which were 78.53 and 85.49 in 2005. In the high variant, the life expectancy will reach 82.41 and 89.17 in 2055, and 84.93 and 91.51 in the low mortality variant.

4 Comparison the age-shifting model with the Lee-Carter model

In this section, we compare the age-shifting model with the Lee-Carter model (we simply call the Lee-Carter model without age-shifting structure the "Lee-Carter model" here).

Figure 10 - 13 show the relative level of age specific mortality rates (natural logarithm) for females, that is, the difference of mortality rates in each year and the average rates from 2001 to 2005.

Figure 8 Actual and Projected k_t (high, medium and low variant), from 1970 to 2055

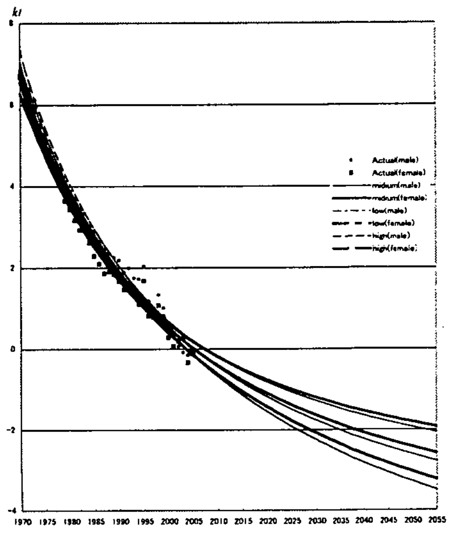


Figure 9 Actual and Projected Life expectancy at Birth, from 1955 to 2055

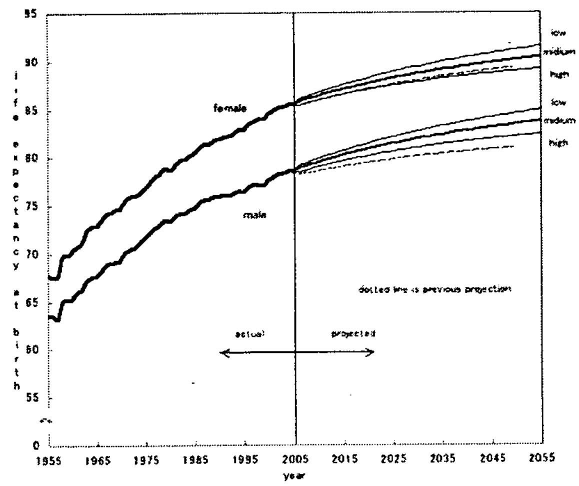


Figure 10 shows the actual data. This figure presents the relative change in the mortality pattern to the average mortality pattern. The mortality rates have decreased during this period, therefore the curves are moving downward over time.

Figure 11 shows the relative change by the Lee-Carter model for the estimated and projected values. We can observe that this model expresses the motion of the mortality curve only in the vertical direction.

The original Lee-Carter model uses the first singular value only. If we use the first and second singular values, we can obtain better estimates as in Figure 12. We can observe from this plot that the relative change of the mortality pattern shows some age-shifting effect. On the other hand, Figure 13 shows the estimated and projected mortality by age-shifting model. We can see a similar pattern as in the estimated part in Figure 12. This observation suggests that the age-shifting model could have a higher performance in ability to express mortality pattern change.

Next, we will compare q_x and l_x functions in both models. Here, we compared life table functions in 2050, whose e_0s are around 90 years, and another two cases assuming unrealistically low k_t values on condition that both models have almost the same level of life expectancy (about 94 and 97 years), only for the purpose of evaluating the impact of model selection.

Figure 10 Relative Change of Age Specific Mortality Rates (actual, female), from 1970 to 2005

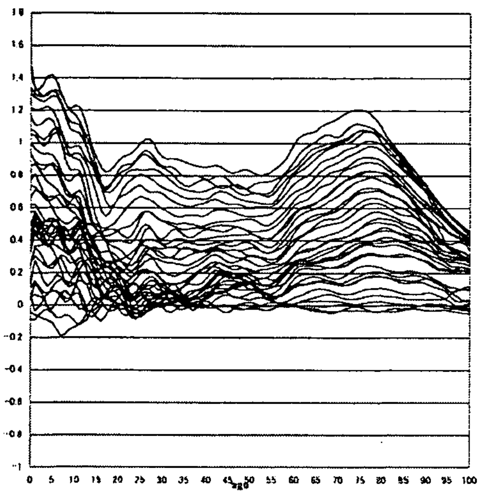


Figure 11 Relative Change of Age Specific Mortality Rates (Lee-Carter model, female), from 1970 to 2055

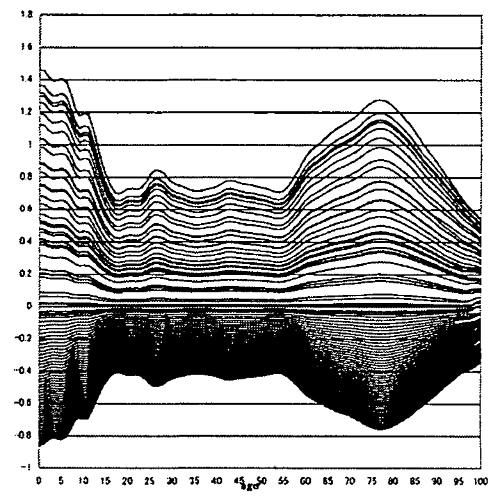


Figure 14 shows the projected q_x functions. The q_x curves by the Lee-Carter model are lower than those by the age-shifting model in ages from 60 to 80, and then they increase rapidly and are higher in ages over 80. This means that the slopes of the q_x curves by the Lee-Carter model in older-age brackets are higher than those by the age-shifting model.

Figure 15 shows the projected l_x functions. Due to the differences of the slopes of the q_x functions in older-age brackets, the l_x curves by the Lee-Carter model seem to be more "rectangularized," corresponding to mortality decline, than those by the age-shifting model. In contrast, we can observe from the graph that mortality decline is expressed as a right-hand shifting of l_x curve in the age-shifting model.

We confirm this point in more quantitative analysis. Wilmoth and Horiuchi (1999) discussed rectangularization of human survival curves, and compared some measures of the variability of the age distribution of deaths or the rectangularity of the survival curve. In the article, they chose the interquartile range (IQR) to compare survival curves for Sweden, Japan and the United States. IQR is defined as follows:

$$IQR = x_2 - x_1$$

where x_1 and x_2 are ages such that $l_{x_1} = 0.75$ and $l_{x_2} = 0.25$.

IQR is the distance between the lower and upper quartiles of the distribution of ages at death.

Figure 12 Relative Change of Age Specific mortality rates (using second singular value, female), from 1970 to 2005

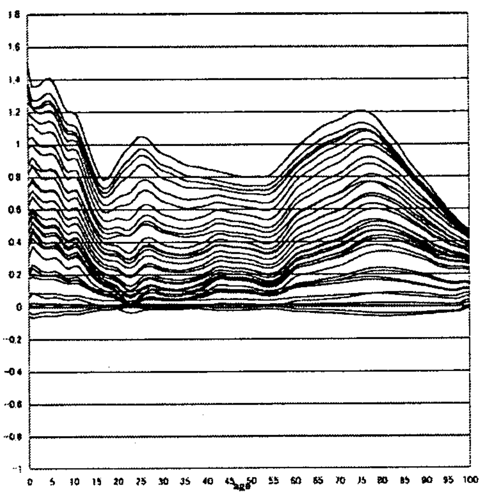


Figure 13 relative change of age specific mortality rates (age shifting model, female)

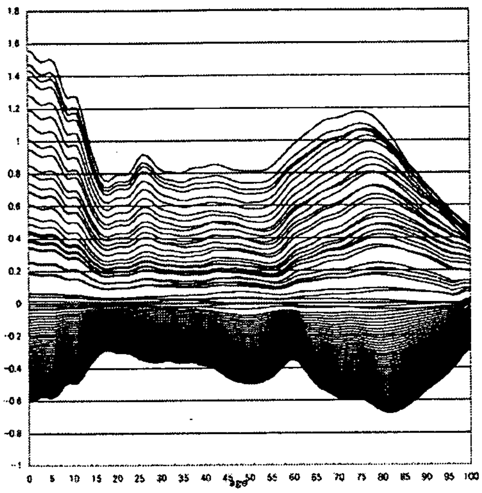


Figure 14 Comparison of q_x, female

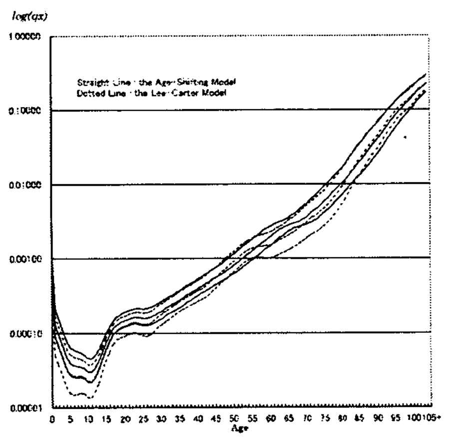
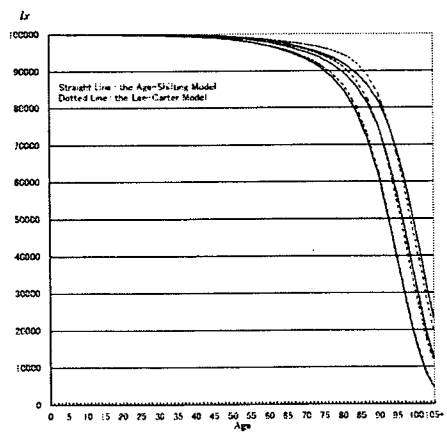


Figure 15 Comparison of l_x, female



IQR decreases as the survival curve becomes more rectangular.

Table 1 shows the IQR for both models. From this table, we can see that the level of IQR in the age-shifting model is greater than the Lee-Carter model, and the difference of the IQR between e_0 level 90 and 97 in the age-shifting model is smaller.

These observations show that the age-shifting model corresponds to the fact that the recent

Table 1 IQR for Age-shifting Model and Lee-Carter Model

e_0 Level	90	94	97
Age-shifting Model	12.5	12.0	11.6
Lee-Carter Model	12.1	11.4	10.8

mortality decline in Japan is explained as delaying the timing of death.

Thus, we could confirm that the projected age pattern of mortality by the age-shifting model is more plausible for the Japanese mortality situation, comparing q_x and l_x functions and evaluating IQR values. This would be another advantage of this model.

Concluding Remarks

In this paper, we studied and developed the age-shifting model for Japanese mortality projection. The main findings are summarized as follows.

- The improvements in older age had a larger impact on the prolonging in life expectancy in the recent Japanese mortality trend. We can recognize this as the shifting of curves in the direction of older people, that is, delays in the timing of death.
- Applying the shift amount and the slope parameters in the shifting logistic model, we developed the age-shifting model, which is a Lee-Carter model with an age-shifting structure.
- In comparison with the Lee-Carter model, the age-shifting model could produce a more plausible age pattern of mortality based on the observation of q_x and l_x functions and IQR values.

As we have seen, the age-shifting model had succeeded in improving the Japanese mortality projection to some extent. However,there would still remain some points that could be improved in this model. First, we should further explore the parameters and the functions used in this model. For example, we used the parameter S_t in the shifting logistic model as the shift amount. However, there might be a better function to evaluate the amount. Second, we should further examine the universality of this model. This includes the application to other countries' data and/or other time periods. Further study would be needed on these points.

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