added to that individual's account and serve to increase his or her assets. Hence contributions and benefits are basically balanced in a one-to-one relationship. However, the Japanese pension reform of 2004 did not go so far as to create a direct link between individual contributions and benefits.

Pension systems that use a concept such as notional defined contributions to create a link between individual contributions and benefits are thought to be effective in erasing economic differentials between generations. Because of that, moves to include DC elements in public pension systems are afoot in many countries. However, as we shall see in the next section, DC pension systems oblige the individual to shoulder the burden of risk associated with fluctuations in income and above-average longevity, raising serious questions as to whether they really are better than the traditional DB pension systems when viewed as instruments of social welfare. In the following sections I will compare the merits and demerits of these two types of public pension system, and use statistical simulations to address the question of how much of a DB element Japan needs to retain in her public pension system.

3. Two Types of Pension System

As we have seen, it is possible to divide pension systems into two broad types: those with defined benefits (DB systems), and those with defined contributions (DC systems). The former is the traditional approach employed in most public pension systems, but recently we have seen some countries, notably Sweden, start to move towards the DC approach. I now propose to look at the pros and cons of both systems.

3.1 Pros and Cons of Defined Benefit Pension Systems

In table 1 I have briefly summarized the pros and cons of DB and DC pension systems. Let us look first at the DB system, the mainstay of public pension systems around the world. Now an important function of public pensions is to guarantee security of income in old age. A DB system does that. Compared with the DC system, where one's income in old age depends on how profitably the pension fund is managed, the DB system greatly lessens the risk of changes in income during old age. The DB system also lessens the risks attendant on living longer than expected. Let us hypothetically assume a situation in which we can accurately predict when people will die. In such a situation we could predict roughly what scale of assets people would need to see them through their old age, and there would be little risk of people finding themselves with insufficient assets to pay for their living expenses between retirement and death. In the real world, however, we cannot accurately predict when people are going to die. Nor can we predict how long any particular person might live. Supposing somebody lives longer than expected, that person will also need more money than expected to cover post-retirement living expenses, and therefore risks suffering a shortage of assets. A DB pension system reduces that risk, since a person who lives longer than expected can still carry on drawing a pension at roughly the same level for however many years he or she may live.[5] These are the advantages of the DB model.

On the other side of the coin, the DB model is vulnerable to changes in the demographic profile. Public pensions are usually run on the tax-and-spend model, with contributions paid by the working generation being spent on benefits allotted to the retired generation. Since a pension fund using the DB system fixes the benefits paid to the latter, any increase in the relative size of the retired population necessitates extra income for the fund, which means raising the level of contributions paid by the working generation. Under a DB system, if the birthrate carries on falling and the population carries on aging, then the burden on the working generation will steadily rise, creating an unfair relationship between generations. If there is also an increase in life expectancy, then the period during which the average retiree receives his pension payments will lengthen, and this too will put upward pressure on contributions. These problems of widening intergenerational differentials and upward pressure on contributions have been identified by numerous observers as demerits of the DB model.

3.2 Pros and Cons of Defined Contribution Pension Systems

It has long been considered very difficult to apply the defined contribution model to public pension systems, but in Sweden they have succeeded in doing it by using the concept of Notional Defined Contribution (NDC).[6] Several other countries are also starting to introduce the Swedish model, and the number of countries using DC systems is gradually increasing. This model essentially entails an individual building up assets

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5 Diamond (1977) argues that it is difficult to offset the risk of longevity other than through the social security system.

6 The Swedish model of pensions based on notional defined contribution retains the tax-and-spend principle in which one generation's contributions pay for the previous generation's pensions. However, each individual's pension entitlement is limited to the amount s/he has contributed. The amount received by pensioners also varies with fluctuations in wage growth rate, economic growth rate, and demographic variables such as size of labor force, average life expectancy etc. In that sense the Swedish system is thought of as using the defined contribution model. For a detailed explanation of the new Swedish system, see National Social Insurance Board in Sweden (2002).

over his or her working career and then spending them to pay for retirement. Consequently it is more like a form of personal savings than a tax-and-spend system, and there is no need to raise contributions paid by the working generation even if the birthrate falls, the population ages and life expectancy increases. In other words, the merit of the DC system is that it is not affected by demographic changes and is neutral in its effect on intergenerational differentials.

On the other hand, the DC model has the demerit that it forces the individual to shoulder the burden of risk from changes in pension fund operating profits. In the DB model a certain level of post-retirement earnings is guaranteed, but in the DC model the amount paid out will change with fluctuations in operating profit. People who are successful in managing their pension fund will be rewarded with higher pension payments after retirement, while those who have failed to manage their pension well will see their pension payments fall correspondingly. Also the DC model offers no way of avoiding the risk of above-average longevity, since, in principle at least, no-one can get more out of the fund than they put in. So long as the saved-up assets are sufficient to cover post-retirement living expenses all will be well; but if the individual lives longer than expected, he will be unable to cover his living expenses and will run short of assets. To sum up, the DC model has the advantage that it is not influenced by changes in demographic structure, but also has the disadvantages that it forces the individual to shoulder the risk of changing pension-fund profitability and unexpected longevity – disadvantages that need to be very carefully considered.

<Table 1 about here>

4. Changes in the Socioeconomic Environment and their Impact on Public Pensions

4.1 Population of Elderly People

According to the National Institute for Research on Social Welfare and Population Problems, the population of Japan is starting to go into decline in 2007 – the year of this book's publication. The decline of the population will no doubt have various impacts on the economy, but when studying pension systems and population problems, a more important figure is the balance between the working population and the retired population. Since Japan has a DB system of public pensions, funded by taxing the working generation to pay pensions to the retired generation, any increase in the proportion of retirees in the population will lead to higher pension contributions for the working generation. Here I define that proportion as the number of people aged 65 and over, divided by the number of people aged from 15 to 64 (see figure 1).

<Figure 1 about here>

In the year 1930, there were 3.062 million people in Japan aged 65 and over (1.318 million men and 1.744 million women), while there were 37.804 million people aged between 15 and 64 (19.178 men and 18.626 million women). The elderly population index stood at just 8.1%. It remained under 10% through the 1950s and 1960s, but climbed above 10% in the 1970s, registering 10.2% in the year 1970. It continued to rise after that, reaching 15.2% in 1985, the year before the basic pension (*kiso nenkin*) was introduced. By 1995 it had reached 20.9% and by 2000 it stood at 25.5% (22.003 million elderly people against 86.223 million working-age people).[7] Estimates for future population trends suggest that the elderly index will reach 35% in 2010, 50% in 2030, and 67% in 2060.[8] Sometime around 2050 the population aging process should finally have run its course, but forecasts for subsequent decades are still in excess of 60%.

I now propose to briefly sum up the relationship between insurance contributions and population trends in a case where a DB-model pension system is being funded on tax-and-spend lines. Before doing that, however, I must first point out that the following mathematical relationship is among the preliminary assumptions of a pension system based on a pure tax-and-spend system of financing:

Total disbursement of pension payments = Total income from pension contributions (Formula 1)

Formula 1 signifies that all the money required to cover disbursements to pensioners, usually older people, must come from contributions paid by the generation still in employment, usually younger people. In other words, this formula encapsulates the tax-and-spend approach to pension financing, with its characteristic pattern wherein

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7 Data for years up to and including 2000 from the *Kokusei Chōsa Hōkoku* (National Census Report) compiled by Sōmushō (the Ministry of Internal Affairs and Communications).

8 Future estimates from *Nihon no Shōrai Suikei Jinkō* (Future Population Estimates for Japan) published by Kokuritsu Shakai Hoshō/Jinkō Mondai Kenkyūjo (National Institute of Population and Social Security Research).

wealth is transferred from younger to older generations. Formula 1 is the theoretical support for the description of public pensions as a system redistributing wealth between generations.

Looking now at the relationship between contributions and population, the 'total income from pension contributions' specified in formula 1 is the product obtained by multiplying the number of workers by their average wage and then by the rate at which pension contributions are levied, i.e. :

Total income from pension contributions = Pension contribution rate (θ) x Wages (w) x Number of workers in labor force (L)

(Formula 2)

The other side of formula 1, 'total disbursement of pension payments' is the product obtained by multiplying the number of retirees by the average level of pension payments:

Total disbursement of pension payments = Income replacement rate (κ) x Wages (w) x Number of retirees (R)

(Formula 3)

By combining formulas 2 and 3, we can define the relationship between pension contributions and population thus:

Pension contribution rate (θ) = income replacement rate (κ) x (Number of retirees (R) / Number of workers in labor force (L))

(Formula 4)

It is clear from the above that if the value of R/L increases (meaning that the number of retirees increases relative to the number of workers), which is what happens in an aging society with a falling birthrate, then a pension system predicated on tax-and-spend principles will have to adjust either the income-replacement rate (κ) or the pension contribution rate (θ). In the traditional defined-benefit systems, the income-replacement rate (κ) is fixed; consequently if the number of retirees rises relative to the number of workers, the only option is to raise contributions. Let us see how that might work in practice, substituting the elderly population index figures introduced above for *R*/*L*, and defining the income-replacement rate as 50% of the wages of currently working generation. Using the formulae above, the contribution rates θ for 1970, 2000 and 2060 work out thus:

$$\theta_{1970} = 0.5\times10.2\% = 5.1\%$$
$$\theta_{2000} = 0.5\times25.5\% = 12.75\%$$
$$\theta_{2060} = 0.5\times66.9\% = 33.45\%$$

We can see very clearly how the DB system exerts upward pressure on contributions when the proportion of retirees in the population increases. Since that proportion is expected to rise very substantially in Japan over the coming decades, the problems associated with the DB system are only too evident.

4.2 Changes in operating profit rate

Very well: let us consider whether the defined-contribution model might be more suitable for the Japanese situation. With the proportion of retirees set to rise so sharply, the attractions of a pension system that is neutral in relation to population structure are undeniable. On the other hand, as we saw in section 2, the DC model exposes individual pensioners to the risks associated with changes in pension fund profitability. At this point we should add another important element to the picture: the relative profitability of safe assets and risky assets in Japan. Here I will use newly-issued ten-year government bonds to represent the former and the return on investment in stocks on Section 1 of the Tokyo Stock Exchange to represent the latter.[9] Figure 2 shows how the return on these two instruments has varied since 1970.

<Figure 2 about here>

In the 1970s, even a rock-solid investment like the ten-year government bond brought a return of 6% to 8%. That was maintained for the first half of the 1980s, but the return drifted down to around 5% in the second half of that decade. The return on the ten-year bond declined further in the 1990s and slipped below 1% in 1998, when it hit a low of 0.972%. Since then the return has continued to fluctuate below the 2% level.

[9] My data for interest on ten-year government bonds comes from the Bank of Japan's Financial and Economic Statistics (*Kin'yū Keizai Tōkei*), and the data on returns on investment in stocks on Section 1 of the TSE comes from the 'Stock Investment Returns' (*Kabushiki Tōshi Shū'eki-ritsu*) published by the Japan Securities Economic Research Institute (*Nihon Shōken Keizai Kenkyūjo*).

Now let us look at the return on stock investment, assuming the case where the investor sells stocks ten years after buying them. An investor buying in the first half of the 1970s and selling ten years after would have realized an average profit of between 9.8% and 16.4%. Buying in the second half of the 1970s would have produced even better profits ten years on, between 16% and 22.8%. After that, however, we enter the time-zone when investors would be selling after the collapse of the bubble economy, and returns plummet. From the second half of the 1980s, we enter negative territory. For instance buying in 1987 and selling in 1997 would have generated a loss of 3.5%. Table 2 displays the mean return and standard deviation for both investment instruments. The government bond shows a mean return of 5.22% over the period 1972-2003, with a standard deviation of 2.53 points, while the stock investment shows a mean return of 9.40% over the period 1970-1992, but, as one would expect with a riskier investment, it also has a much higher standard deviation, at 8.62 points. The big picture: holding a higher proportion of risky assets will tend to increase return, but will also generate higher risk of fluctuation.

<Table 2 about here>

I stated earlier that one of the demerits of the defined-contribution model is that the individual has to shoulder the risk of fluctuating returns. Let us now look at a simple numerical example to show the differing effect on the individual of returns that do and do not fluctuate.

Let us suppose that an individual invests in a non-fluctuating instrument, such that he can definitely have ¥1 million to consume at the end of the investment period. Let us compare that with the case of a similar scale of initial investment in an instrument that fluctuates such that there is a 50% chance that the investor will have ¥0.5 million and a 50% chance that he will have ¥1.5 million at the end of the investment period. If we define the utility function as $u=\log(c)$, then the expected utility (EU) of the non-fluctuating investment will be $E[u]=\log(c)$, and that of the fluctuating investment will be$E[u]=0.5\times\log(c-\alpha)+0.5\times\log(c+\alpha)$. Here, c represents consumption of ¥1 million and the value of α is ¥0.5 million. Substituting these values into the equations produces EU values of 6 for the non-fluctuating investment, higher than the 5.9375 generated by the fluctuating investment. Measuring the two investments in terms of equivalent variation, the non-fluctuating investment comes out 15.5% higher. The DB-type pension guarantees a certain level of pension receipt after retirement, whereas the benefit received from a DC-type pension is influenced by individual asset management outcomes. Assuming that the desired level of post-retirement income is the same in both cases, people will tend to choose the DB type rather than the DC type.[10] It follows that in order for the DC type to become more advantageous than the DB type, the merit of not having to raise contributions in an era of falling birthrate and aging population will have to outweigh the demerits of risk attendant on fluctuating returns.

4.3 Lengthening Life Expectancy, Increasing Survival Probability

Let us next consider trends in life expectancy. In 1947 the average Japanese man could expect to live to the age of 50.06, and the average Japanese woman, 53.96. In the period 1950-1955, life expectancy averaged 61.6 for men and 65.6 for women, and international comparisons showed Japan ranking 29th in the world for male life expectancy and 35th in the world for female life expectancy. After that there was a spectacular improvement, to the point where life expectancy in Japan was 78.36 for men and 85.33 for women in 2003 (see figure 3). In the period 1995-2000, Japanese life expectancy averaged 77.1 for men and 83.8 for women, putting Japan at Number One in the world for both sexes. Authoritative estimates predict that in this field at least, Japan will still be Number One in the period 2045-2050. It is predicted that by that time male life expectancy will have reached 80.95 and female life expectancy 89.22.[11]

Increasing life expectancy means that people have a higher probability of surviving as they get older. That in turn means that the length of time they live after retirement will tend to lengthen.[12] As I explained in section 2, a DB-model public pension guarantees a certain level of pension income even if the retiree lives longer than expected. On the other hand, the DC model works on the principle that the individual cannot receive more from the pension fund than he or she puts into it, and as such faces the risk of running out of money to pay for living expenses if he or she lives longer than expected. As we have seen, in the case of Japan life expectancy has greatly increased in the last 50 years and is expected to increase further still. Figure 4 plots projected survival probabilities for Japanese people (the average for men and women) born in

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[10] See Matsuura and Shiraishi (2004) for a detailed empirical analysis of asset selection by Japanese households and corporations.

[11] These estimates are taken from the lifespan tables in 'Future Population Estimates for Japan' (*Nihon Shōrai Suikei Jinkō*), published by Kokuritsu Shakai Hoshō/Jinkō Mondai Kenkyūjo (National Institute of Population and Social Security Research). I have used estimates made in January 2002.

[12] Of course, if the retirement age is raised in pace with increases in average life expectancy, then longer life expectancy need not necessarily lead to a lengthening of the post-retirement period. As things stand in the real world today, however, actual retirement ages are not directly linked to mean life expectancy.

2000 and those born in 2050. This figure graphically illustrates the fact that longevity risk is going to increase in the coming decades. It follows that any discussion on pension reform will have to take particularly careful account of the merit of the DB system that it reduces the risk attendant on long life.

<Figure 3 about here>

<Figure 4 about here>

5. A Simple Simulation Analysis

Since the DB and DC pensions both have merits and demerits, it follows that using one or the other of the two systems will carry the possibility of lowering the level of utility to the people covered by it. In this section I will consider what kind of public pension system would be most appropriate to an aging society with a falling birthrate, using a simplified version of a simulation model I devised for an earlier study (Miyazato 2004). I will leave a detailed account of the underlying assumptions of this model to an analytical appendix in section 7, but for now let me give a brief account of the essence of the model. Its main features are as follows:

1. Assuming that expected values are of the same amount, then people will prefer stable income and consumption to fluctuating income and consumption.
2. People's income is generated by one or more of the following three sources: labor income, profits on asset management, and pension receipts.
3. The return on asset investment is variable and will fluctuate from one period to the next. Labor income, however, will tend to rise at a certain rate but will not show fluctuations in rate of growth from one period to the next.
4. People face the risk of survival (or the risk of death).

These are the four main features of the simulation model. Item 1 relates to utility for people, and signifies that where people have the same expectations for the value of consumption they will be able to engage in after retirement, then a stable level of consumption will offer a higher level of utility than a fluctuating level of consumption. This is a hypothesis often used in simulation analysis. Item 2 relates to people's budgets. In this paper I have opted to avoid analytical complexity by assuming that people have only three kinds of income source: income earned by their labor; interest (or operating profit, such as dividends etc.) earned on savings and other assets; and pension payments received after retirement. Item 3 is included to account for fluctuations in pension fund operating income, which we have seen is a feature of the DC-type system. Item 4 factors in the probability of a person living or dying during each of two periods in their lives, in order to evaluate the risk of longevity in the risk simulation.[13] These, then, were the hypothetical premises on which I based my simulation.

<Figure 5 about here>

The results of the simulation are shown in figures 5 and 6. Figure 5 displays results when we assume wage growth at 2% p.a., the return on investment income averaging 2% p.a. with a standard deviation of 5 percentage points, and a survival probability of 100% in the first period and 80% in the second period. With these settings in place, I then added in two different models for population growth: one at the standard rate of 2% p.a., the other with a much lower rate of 0.5% p.a.. In this analysis I used equivalent variation rather than direct comparison to calculate utility levels.[14] In figure 5 the horizontal axis shows income replacement rate, so that we may view it as displaying the scale of a DB pension. The vertical axis shows the degree of welfare gain accruing at each level of income replacement. I have carried out the analysis using 0.05 intervals for income replacement rate. Looking at the standard case for population growth, we find that welfare gain continues for some time to rise in a linear relation to income replacement rate. In other words if we start from a position of zero income replacement and gradually increase the rate, then for quite a long interval increases in the rate will generate corresponding increases in utility. Zero income replacement signifies the complete absence of a DB pension system, leaving people to depend on their own accumulated savings to pay for living expenses in old age. This can also be thought of as a situation where social security in old age is conducted entirely through DC-type pensions. The complete absence of a DB component leaves people fully exposed to the risks of operating income fluctuation and unpredicted longevity.

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[13] Oshio (2000) also carries out a simulation analysis of public pensions that accounts for uncertainty, but the risk of longevity is not accounted for in the Oshio model.

[14] For example, if we are comparing an income replacement rate of 0.2 with one of 0.0, then the equivalent variable (*tōka henbun*) is the amount of money we would have to add to the 0.0 case in order to generate the same level of utility as the 0.2 case. The figure for equivalent variation can then be used to calculate welfare gain (*kōsei ritoku*).

Avoiding those risks has the effect of raising utility for the people, so it is clearly desirable to raise the income replacement rate. On the other hand, raising the income replacement rate on a DB-type pension also carries the demerit of raising the level of contributions that working people must pay into the fund. If people are forced to pay excessively high contributions, then lifetime disposable income will fall substantially, and the level of utility will actually fall for most people. The results of the simulation also indicate that raising the income replacement level too high results in lowering the welfare gain. In the standard case, an income replacement rate of 0.45 will generate the highest welfare gain. Raising it beyond that level will actually result in lowering the welfare gain.

In figure 5 I have also displayed results of an analysis for the case of a decline in the population growth rate. The standard model assumes 2% population growth, so that in 30 years the population will grow by about 81% ($1.02^{30} \fallingdotseq 1.811$) , but if we assume a lower growth rate of 0.5%, then the population will only grow by about 16% in 30 years ($(1.005)^{30} \fallingdotseq 1.161$). In that case, the optimal income replacement rate will come at 0.4, some 5 points earlier than in the standard case. A decline in population growth forces the smaller working generation to pay higher contributions to finance the pensions of the relatively larger retired generation, and this in turn reduces lifetime disposable income. Maintaining a high level of income replacement at a time of declining population growth forces people to pay very high contributions, exacerbating the main demerit of the DB system. That is why an excessive rate of income replacement in a society with a declining rate of population growth will lower utility for the people.

Next I considered the effect on optimal income replacement rate of an increase in survival probability. The results of this analysis are shown in figure 6.

<Figure 6 about here>

In figure 6 the standard case assumes survival probability of 100% in the first period, and survival probability of 50% in the second period. The case with elevated survival probability assumes 100% in the first period and 80% in the second. The values ascribed to other variables in the two simulations were as follows: wage growth at 2% p.a., the return on investment income averaging 2% p.a. with a standard deviation of 5 percentage points, and population growth at 2%. In figure 6 the standard case generated an optimal income-replacement rate of 0.35. Here too, an excessive increase in the income-replacement rate results in lowering the welfare gain for the people in general.

Looking now at the case with an elevated survival probability, the optimal level of income replacement comes in at 0.45, considerably higher than in the standard case. One of the merits of the DB-type pension is that it avoids the risk associated with longevity; when survival probability rises, the influence of that merit pushes up the optimal income-replacement rate. Of course, if an increase in survival probability is not matched by an increase in the birthrate, then the proportion of retired people in the overall population will rise and that will apply upward pressure on pension contributions. That will cancel out some of the merit of the DB system; even so, the advantage of avoiding longevity risk is a very considerable merit of the DB system in a society where people's life-spans are lengthening.

To fully draw out the policy implications of these different types of pension one would have to conduct a more refined simulation. However, even with the rough-and-ready methods used here, it still seems fair to conclude that a hybrid pension scheme, combining elements of defined benefit and defined contribution, would probably be more effective at raising utility for the general public than simply abandoning the former model in favor of the latter. To some degree at least, the people should be given the peace of mind that comes from having a guaranteed minimum level of post-retirement income.

6. Conclusion

In this chapter we have considered to what degree it remains necessary to maintain the traditional defined-benefit model for public pensions, featuring as it does the function of sharing out risk more evenly among members of society. We have asked to what degree it might be better for society if we switched to a defined-contribution model, in order to erase differentials between generations. We have seen that in an aging society with a falling birthrate, the DB type of pension forces higher levels of contribution and creates intergenerational disparities. At the same time, the DB system still has the important merit of enabling people to avoid the risks associated with longevity and fluctuations in fund operating income. The simulation analysis used here has shown that totally abolishing the DB system of public pensions in favor of the DC model, in other words switching completely to individual accounting, would reduce utility to the people. The reason for that is simple: totally abolishing the basic guarantee represented by 'defined benefit' would also mean abolishing the risk-sharing function of pensions and the social security system more generally. That said, the intergenerational differentials that have opened up in Japan's aging/low birthrate society are now serious enough to make it a necessity that we sacrifice a degree of risk-sharing by introducing defined contribution elements into social security systems. That naturally leads us to a crucial question: how should we balance the need to keep DB elements with the need to introduce DC elements? I would argue that the simulation analysis discussed here, and similar exercises using values as close as possible to reality, indicate that the optimal rate of income replacement in a public pension system is around 0.4, or 40%.[15] We should therefore retain sufficient elements of the DB system to support public pensions equivalent to about 40% of the employment income for the current working generation. This is significantly lower than the 50% that was pledged by the government at the time of the 2004 pension reform. Simulations like mine entail defining values for a large number of variables, so we cannot place total reliance on the figures generated. Even so, I believe we can safely draw two conclusions: that a complete switch to individual accounting would lower utility for the population as a whole; and that on the other hand, in view of the serious problem of intergenerational disparities now facing Japan, the government will be setting the bar too high if it sticks to its pledge of maintaining public pension payments at 50% of pre-retirement income.

The DB and DC systems that I have analyzed in this chapter represent two extremes on the spectrum of pension systems. Although Sweden has switched its public pension system in the direction of the DC system, the rate of return on fund operation does not vary from one subscriber to another, and in that sense it retains elements of the DB system. Thus we need to be careful not to over-simplify when we make a straight comparison between the two models. However, it is certainly the case that policy-makers will have to take great care to strike a balance between maintaining the risk-sharing function and controlling intergenerational disparities when designing public pension and social security systems from now on.

7. Analytical Appendix

Any attempt at simulation analysis must start by constructing a statistical model. For utility functions I use the following formula for expected utility, one that is used in many analyses:

$$E\left[\sum_{i=1}^{n} \beta^{i-1} P_i u\left(c_{i,t}\right)\right] \qquad \text{(Formula 5)}$$

[15] In my earlier study (Miyazato 2004), I carried out a more complex simulation, using data based on the actual population profile of Japan. That also generated an optimal income-replacement rate of around 40%.

In this formula P_i represents the survival probability of a person i years old, c_i is consumption at age i, β^i is the discount rate at age i years, t indicates point in time and i represents age in years. I also used the following CRRA to obtain relative risk avoidance levels for each utility function:

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\gamma}}{(1-\gamma)} \qquad \text{(Formula 6)}$$

Here, γ is the coefficient indicating level of risk avoidance, and may be described as showing how sensitive individuals are to fluctuations in consumption. Prior to retirement an individual receives labor income of w_i, and after retirement he receives public pension benefits of b_i. Return on assets is r, and pension contributions are represented by θ. In that case the individual's budgetary constraint equation for each period will be as follows:

$$c_{i,t} + a_{i,t} = (1+r_t)a_{i-1,t-1} + (1-\theta_t)w_{i,t} + b_{i,t} \qquad \text{(Formula 7)}$$

Here, $a_{i,t}$ represents assets held at age i years. Pension benefits $b_{i,t}$ are not receivable while still of employment age $(b_{i,t} = 0)$, but are receivable after retirement $(b_{i,t} > 0)$. Pensions are assumed to be paid under the defined benefit (DB) model, so that the amount received in pension benefits is guaranteed to be equivalent to a certain proportion of wages received prior to retirement – that proportion being the income replacement rate. If we define pre-retirement wages as $\overline{w}_t$ and the income replacement rate as κ, then the amount received in pension benefits will be obtainable by $b_{i,t} = \kappa \times \overline{w}_t$. We assume that that the pension system is financed on the tax-and-spend (*fuka*) model, meaning that contributions will be set at a level such that the sum of all pension benefits received will be equal to the sum of all pension contributions paid. Hence the pension contribution rate will be determined in accordance with formula 4 described in section 2 above. In order to take account of fluctuations in pension fund operating profit, we assume that the mean return on assets, and the distribution σ^2, fluctuate in conformity with the normal distribution $N(\mu, \sigma^2)$. Since operating profit varies between each period, in this analysis we use Dynamic Programming to solve the problem of individual optimization. We can find the solution to the problem of individual optimization by resolving the following equation:

$$V_{i,t}\left(x_{i,t}\right)=\max_{c_{i,t}}\left\{u\left(c_{i,t}\right)+\beta^{i-1}\frac{P_{i+1,t+1}}{P_{i,t}}E\left[V_{i+1,t+1}\left(x_{i+1,t+1}\right)r_t\right]\right\} \qquad \text{(Formula 8)}$$

s.t. formula 7

$V_{i,t}$ represents the status value function. Here the status variables are assets and rate of operating profit, expressed as $x_{i,t} = (a_{i-1,t-1}, r_t)$. Since it would greatly complicate the calculations if we matched the model to actual population growth rates, this analysis applies exogenous fixed population growth rates to the simulation. To further simplify calculations, the simulation assumes a model in which people's lives can be divided into a maximum of two periods. The value for each parameter is obtained by defining one period as 30 years and then raising the value of the annual rate to the power of 30. For example, if the wage growth rate is 2% then the value for one period will be $(1.02)^{30}$ = 1.811362....

Such was the model used to carry out the simulation analysis in this chapter.

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Table 1: Pros and Cons of Defined Benefit (DB) and Defined Contribution (DC) Pensions

	Merits	Demerits
Defined Benefit	*Can depend on a certain level of income after retirement * That level of income will continue even if one lives longer than expected	* Falling birthrate and aging society forces up contributions * Increase in life expectancy also forces up contributions
Defined Contribution	* No need to raise contributions even if birthrate falls, society ages, life expectancy increases	* Risks of fluctuation in operating income of system shouldered by individual * Risk of running out of assets if one lives longer than expected

Table 2: Mean return and standard deviation for safe and risky assets

	Safe investment	Risky investment
Mean return	5.22%	9.40%
Standard deviation	2.53 points	8.62 points

Note 1: 'Safe investment' is represented by purchase of newly-issued ten-year government bonds. 'Risky investment' is represented by buying of stocks on Section 1 of the Tokyo Stock Exchange and reselling them ten years later.

Note 2: Data for bond investment calculated for the period 1972-2003; data for stock investment calculated for the period 1970-1992.

Figure 1: Trends in the elderly population index

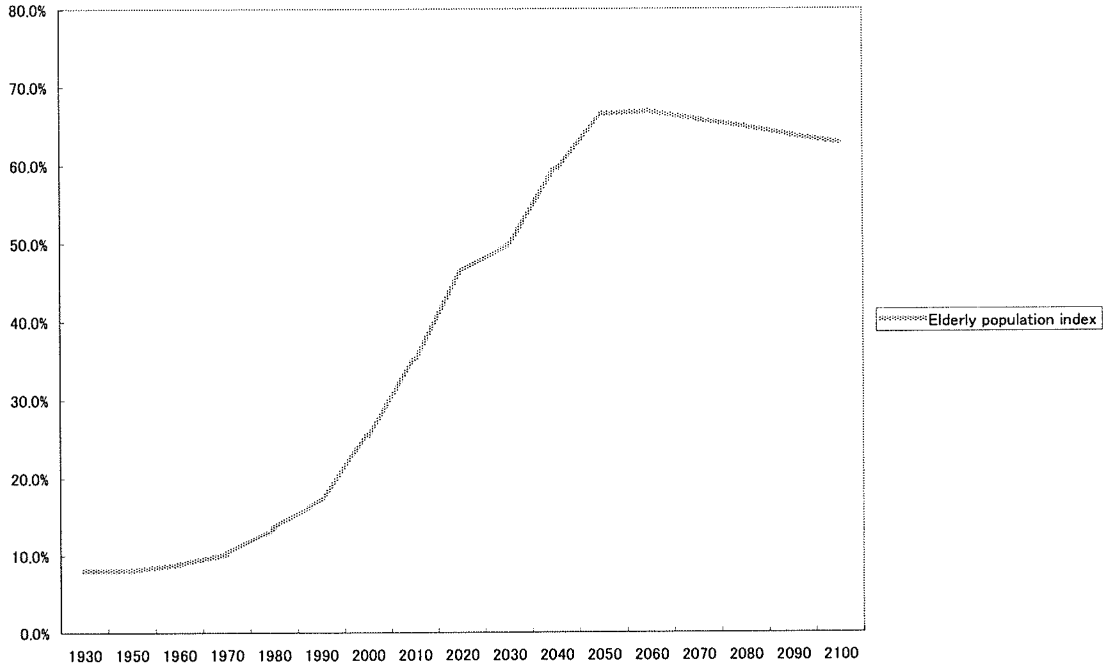

Figure 2: Trends in profitability for high-risk and low-risk investments

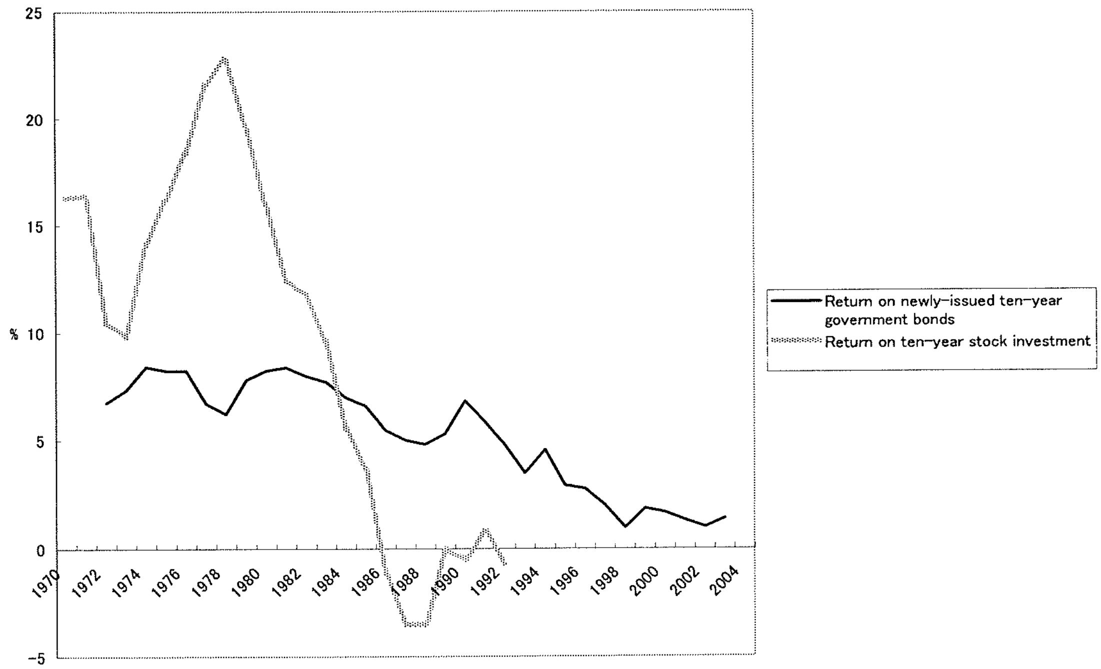

Figure 3: Trends in Japanese life expectancy

Figure 4: Survival probability by age group for Japanese people born in 2000 and 2050

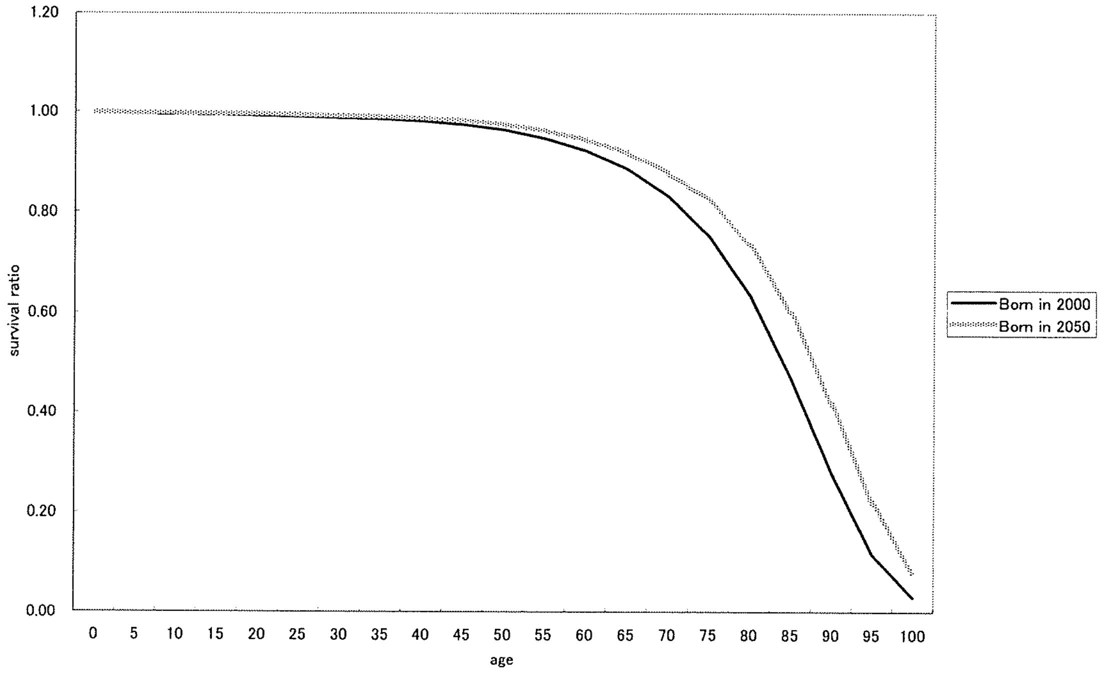

Figure 5: Influence on Optimal Income-replacement Rate of Declining Population Growth

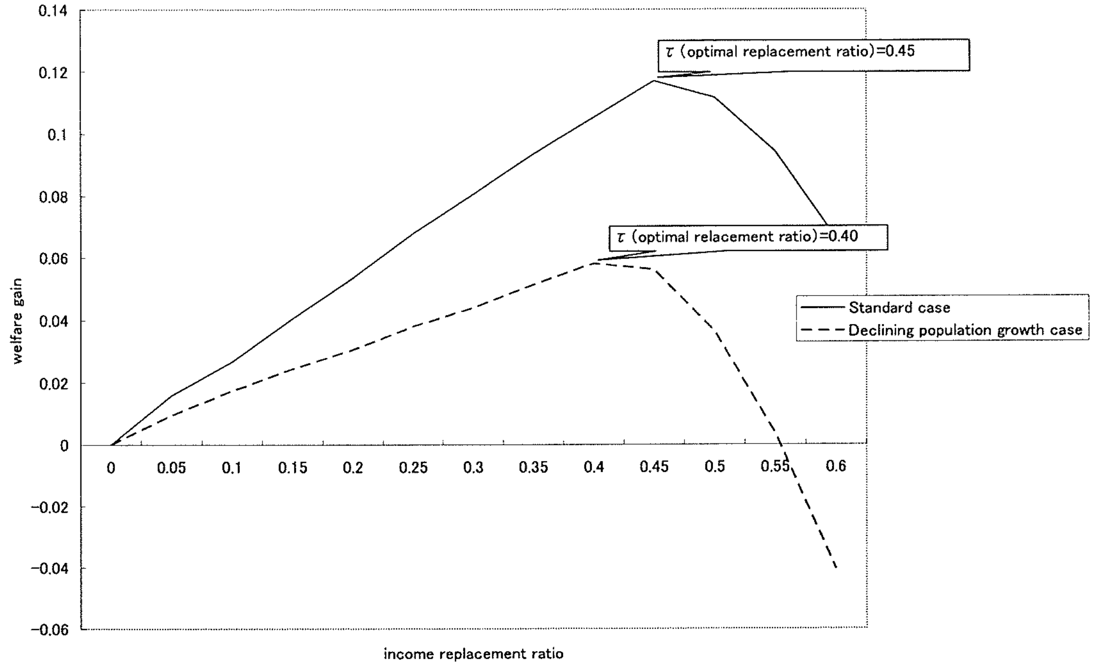

Figure 6: Influence on Optimal Income-replacement Rate of Increasing Survival Probability