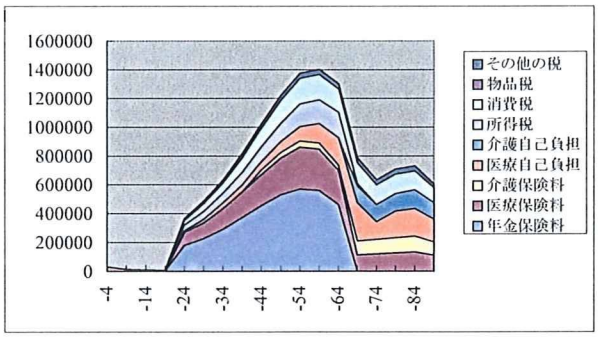
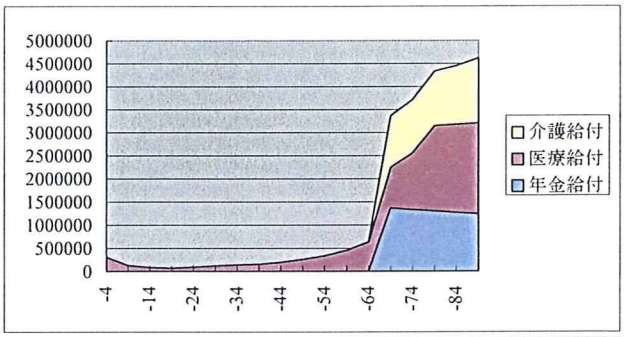
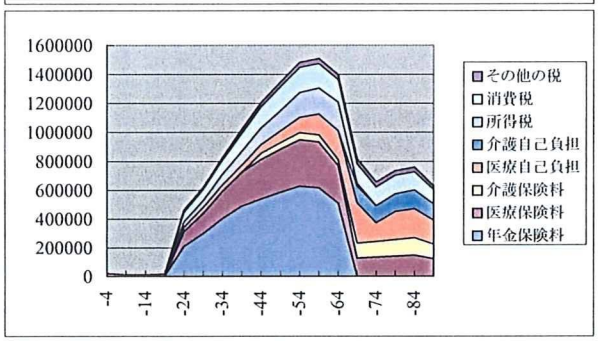
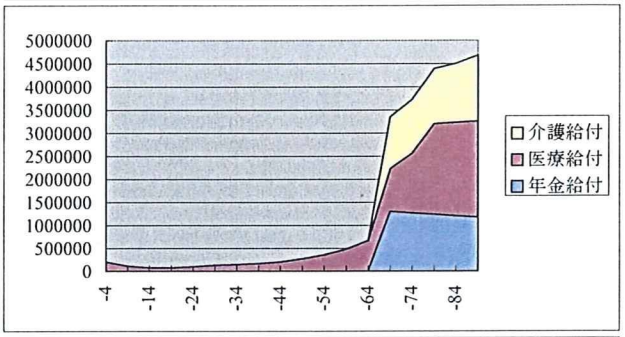
1974生



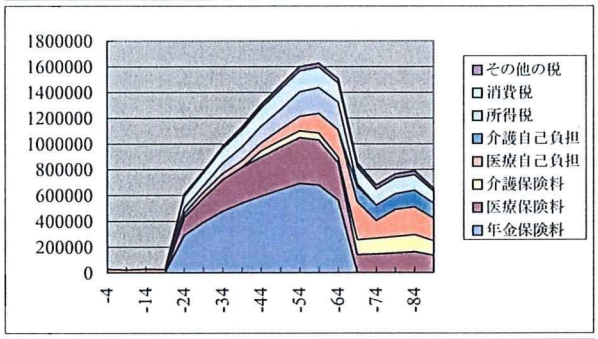


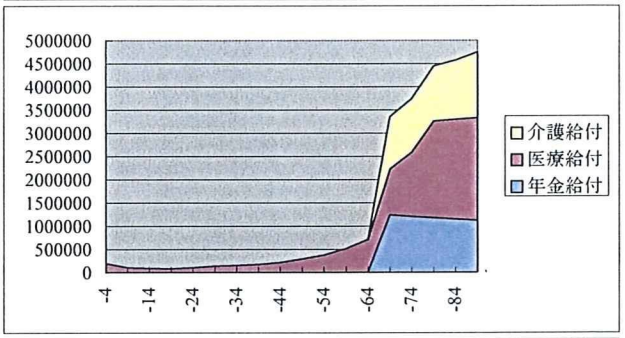
1984生



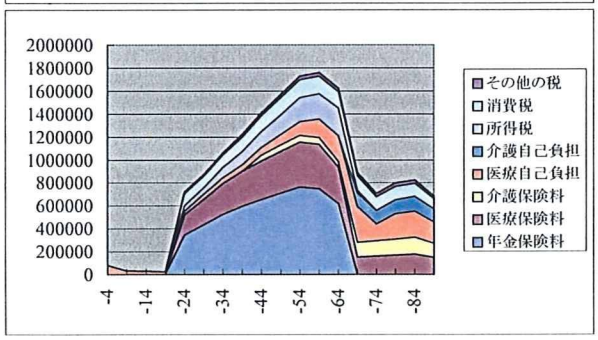


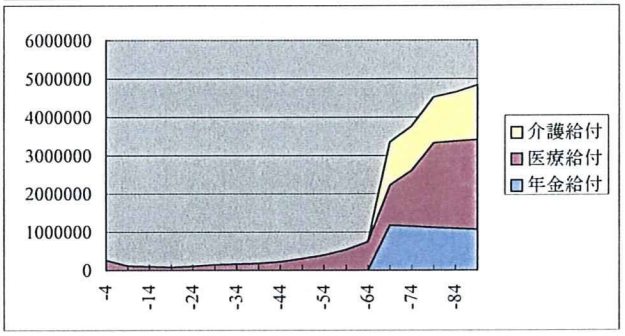
1994生





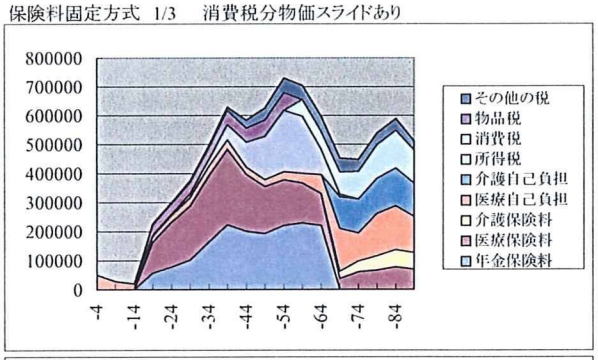
2004生

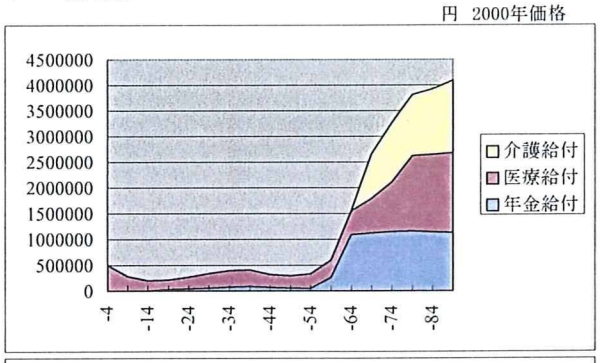




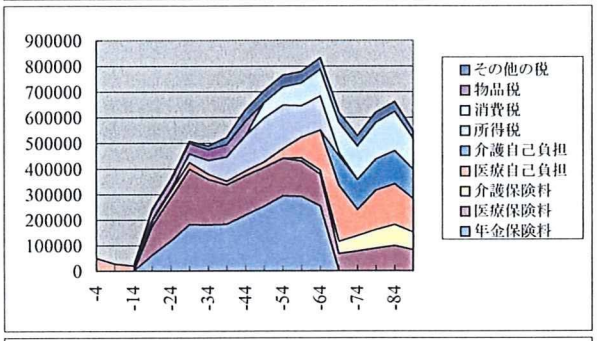
生まれ年別　1世帯あたり社会保障負担・税負担、社会保障給付　　ケース：fi1-3あり
保険料固定方式　1/3　　消費税分物価スライドあり　　円　2000年価格

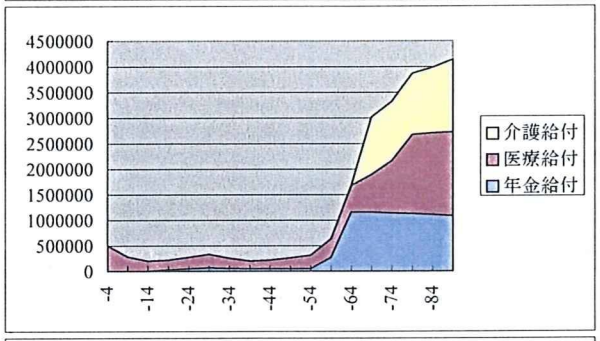
1934生



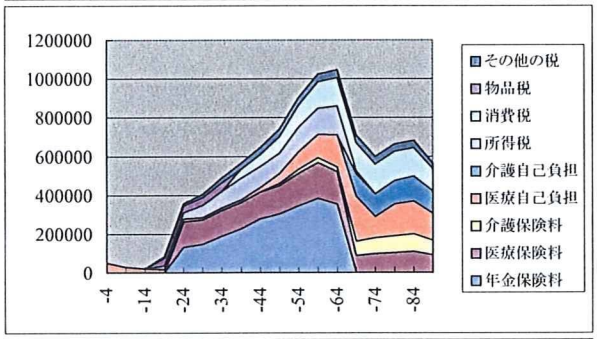


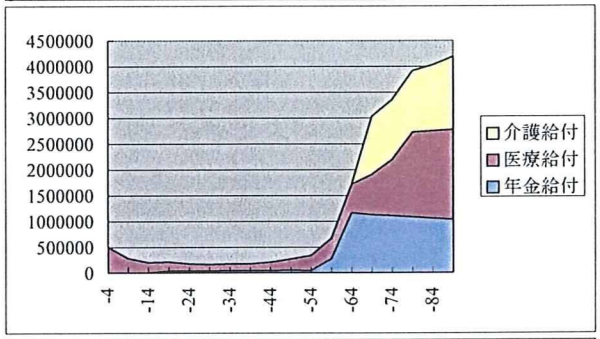
1944生



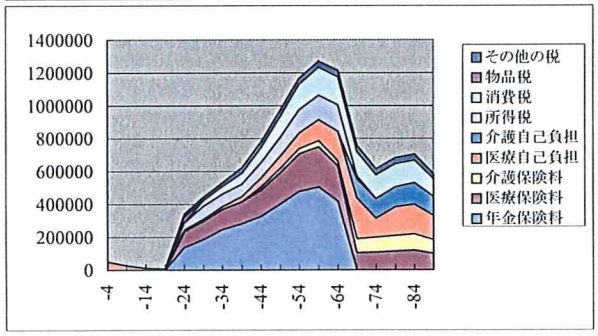


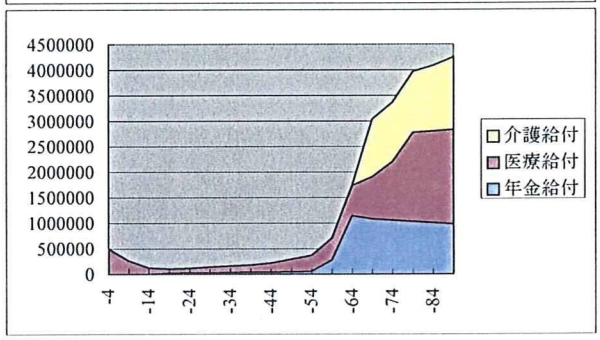
1954生



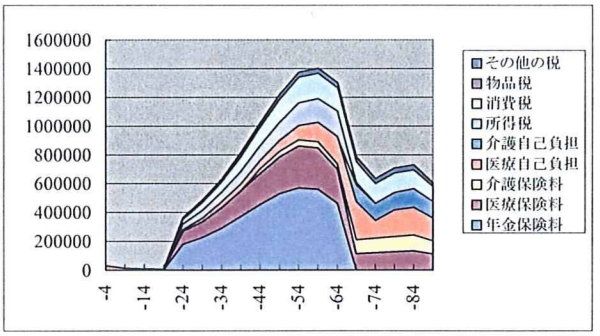


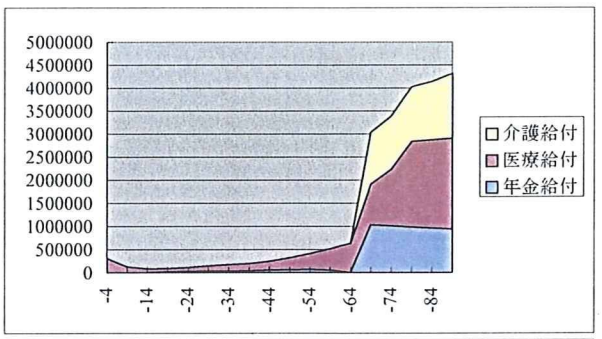
1964生



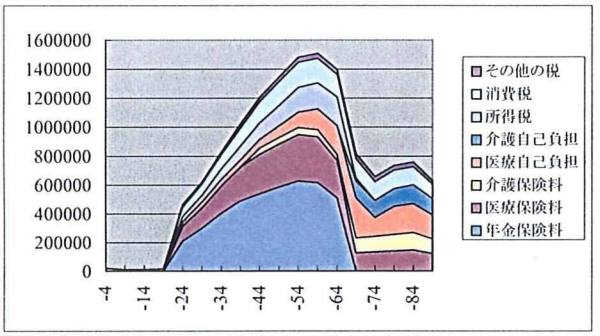


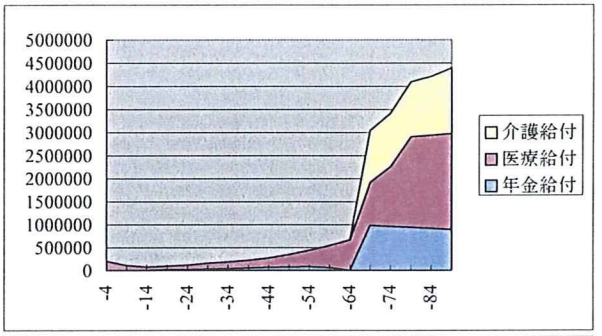
1974生



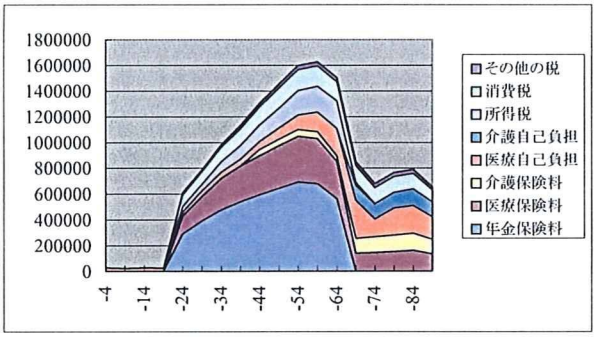


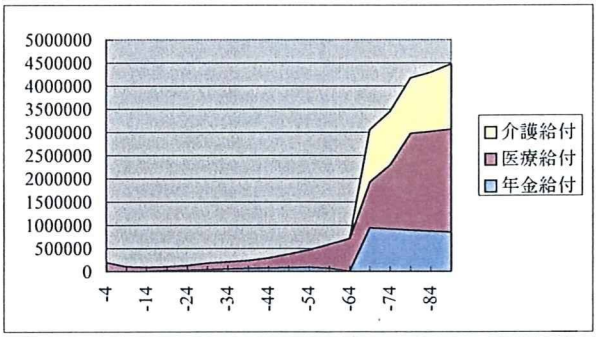
1984生



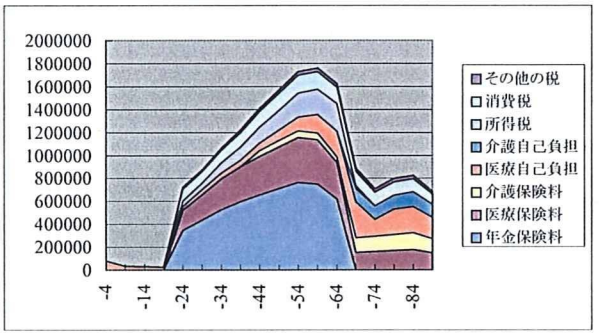


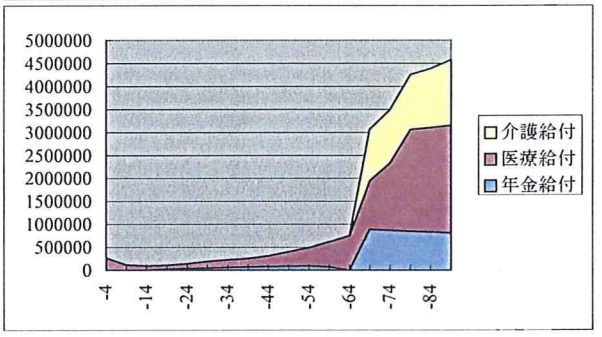
1994生





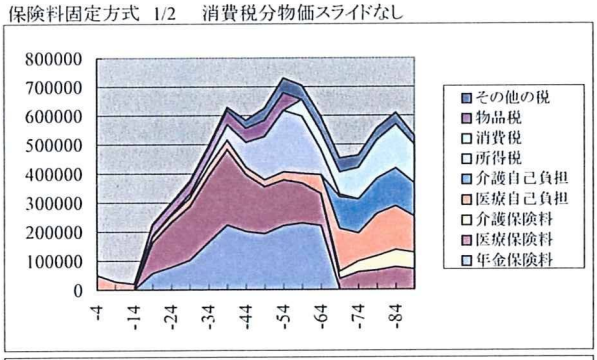
2004生

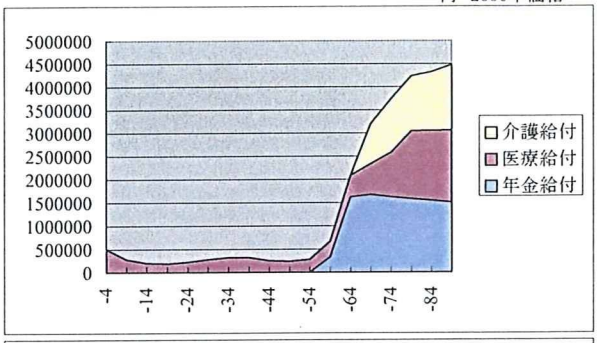




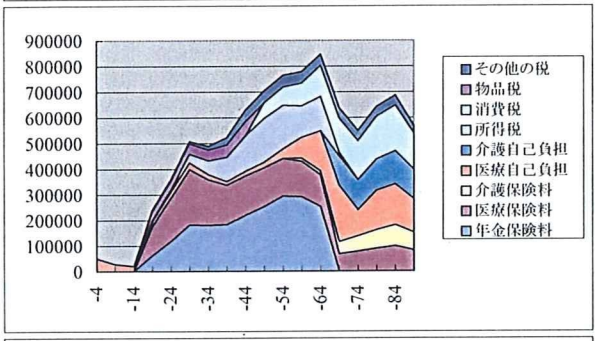
生まれ年別　1世帯あたり社会保障負担・税負担、社会保障給付　　ケース：fi1-2なし
保険料固定方式　1/2　　消費税分物価スライドなし
円　2000年価格

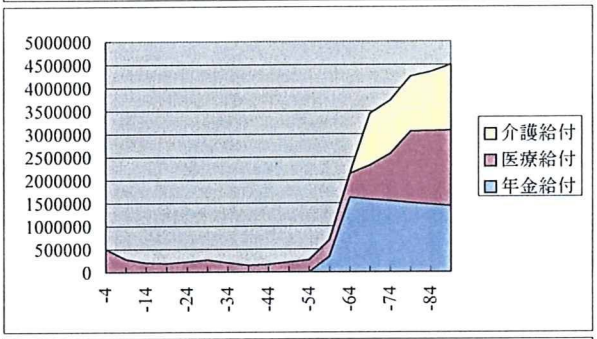
1934生



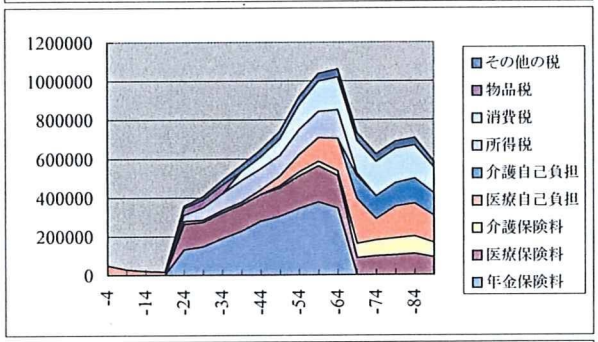


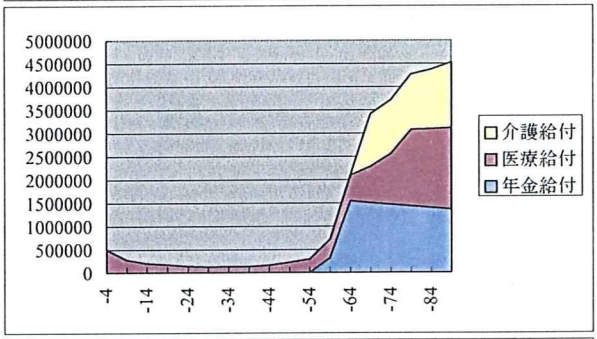
1944生



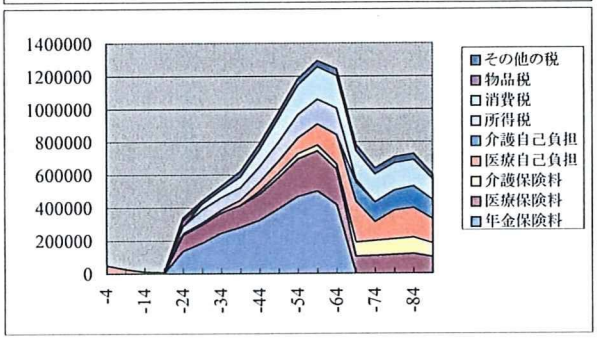


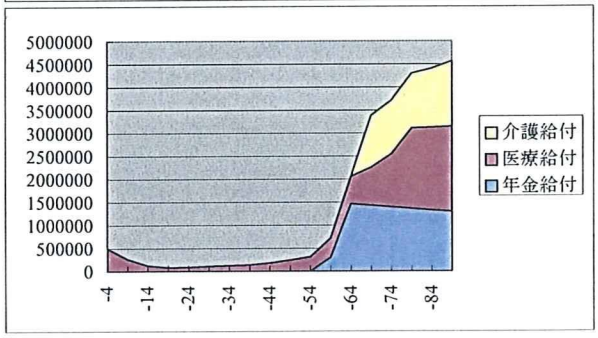
1954生



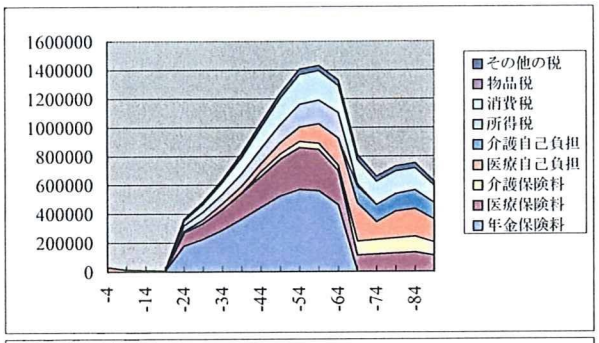


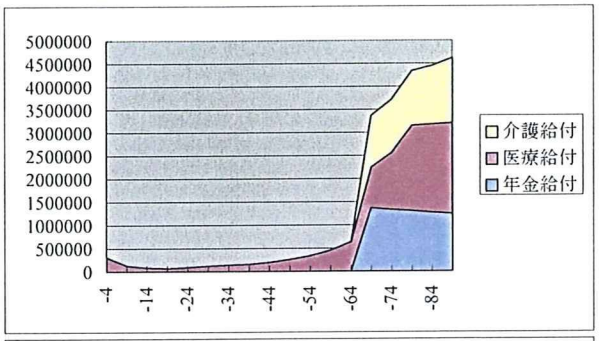
1964生



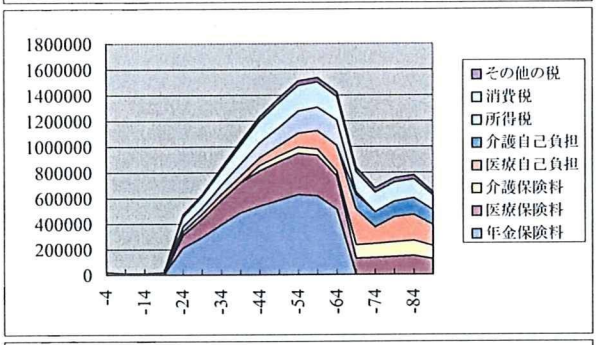


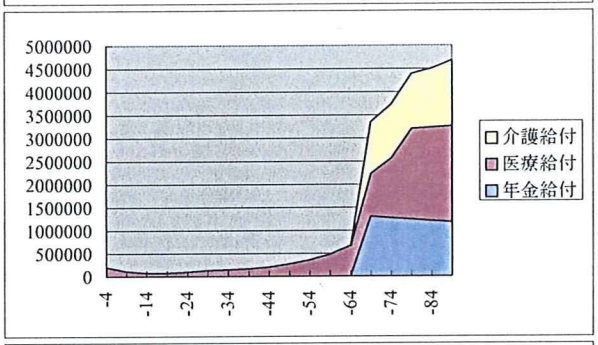
1974生



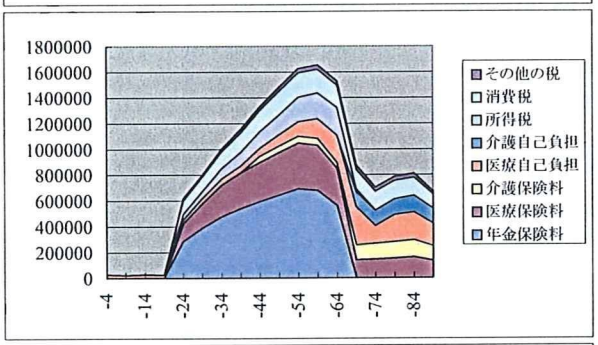


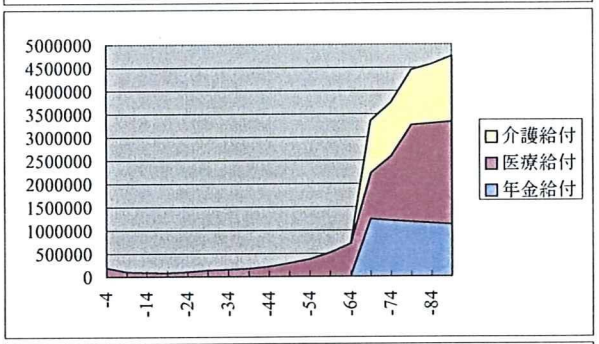
1984生



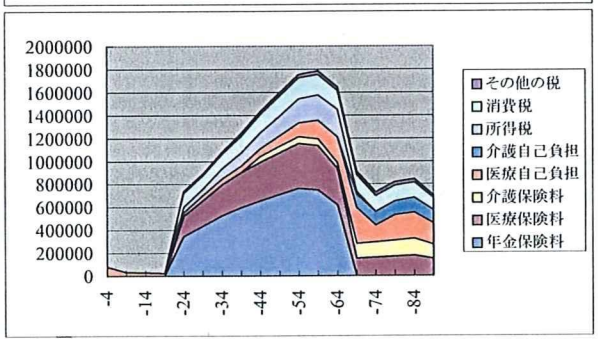


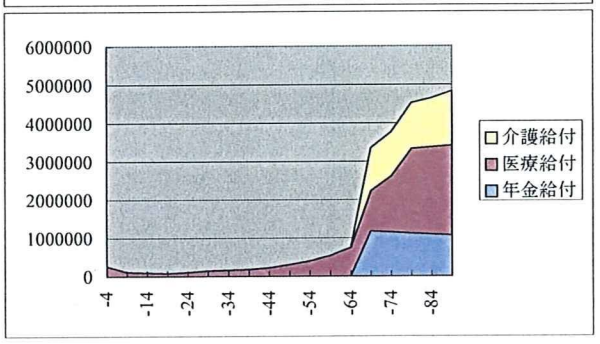
1994生





2004生



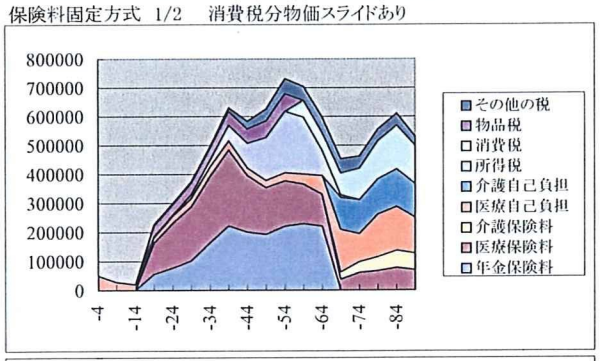


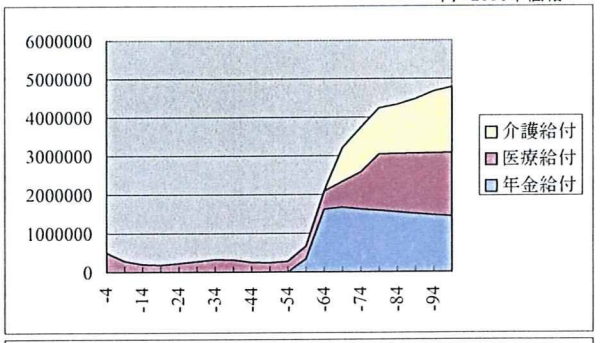
生まれ年別　1世帯あたり社会保障負担・税負担、社会保障給付
保険料固定方式　1/2　　消費税分物価スライドあり

ケース：fi1-2あり

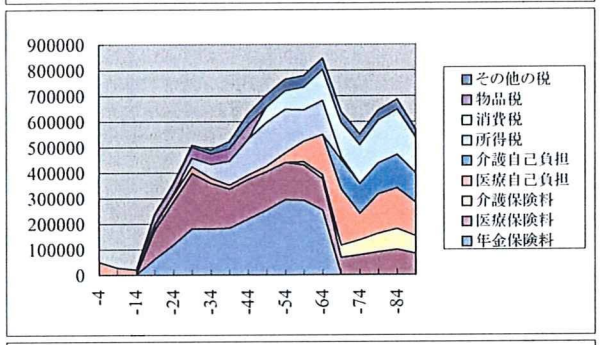
円　2000年価格

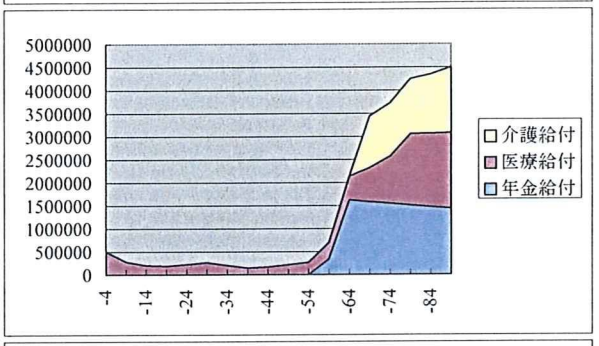
1934生



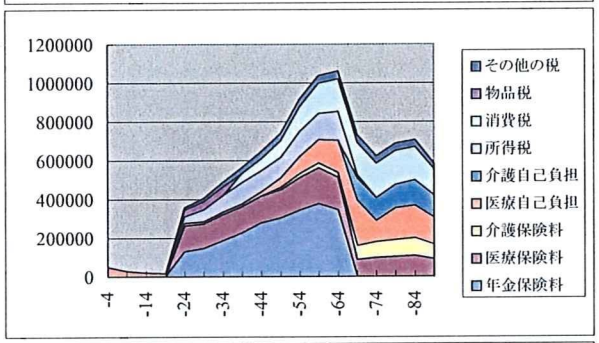


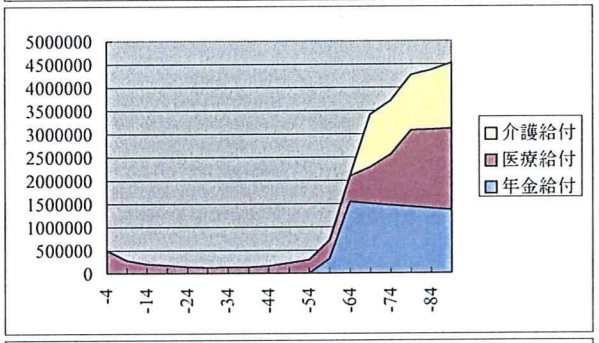
1944生



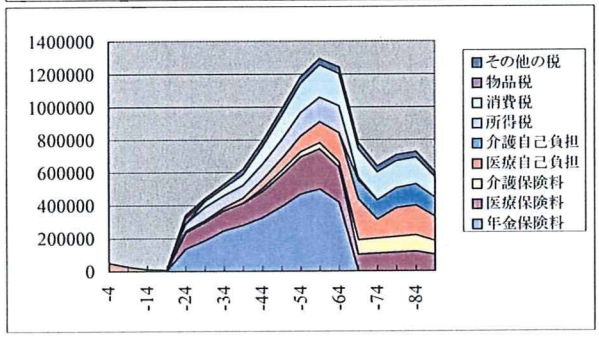


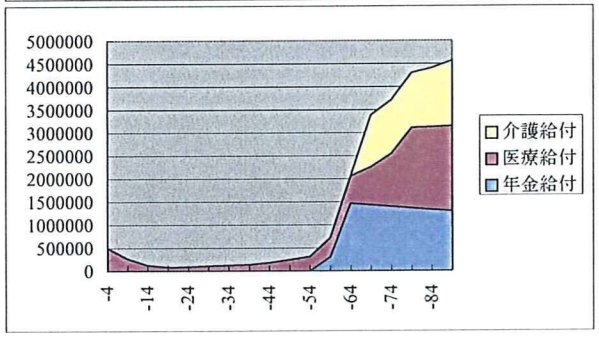
1954生



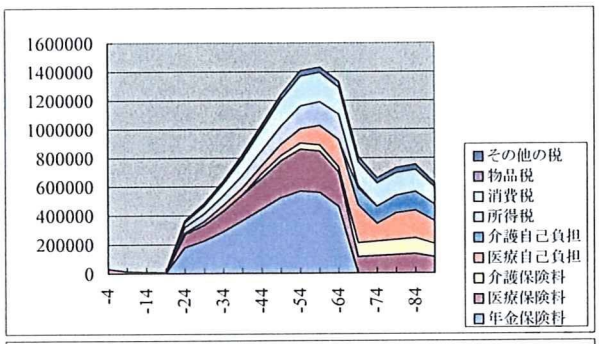


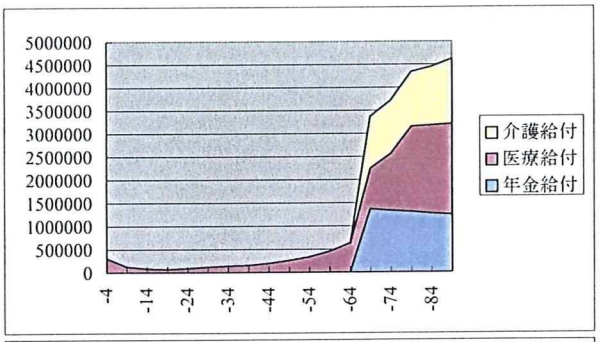
1964生



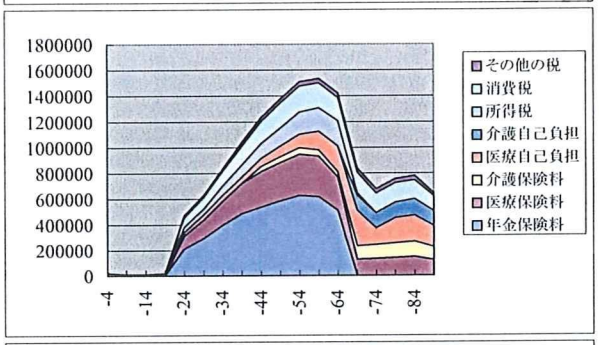


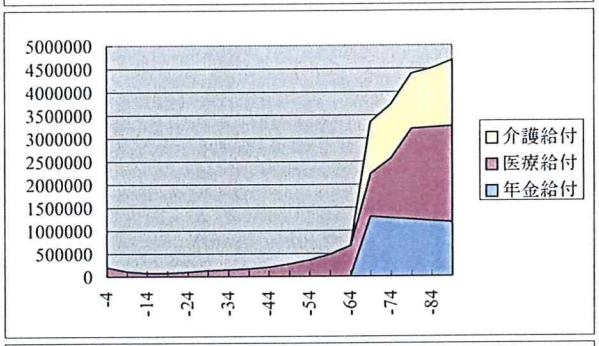
1974生



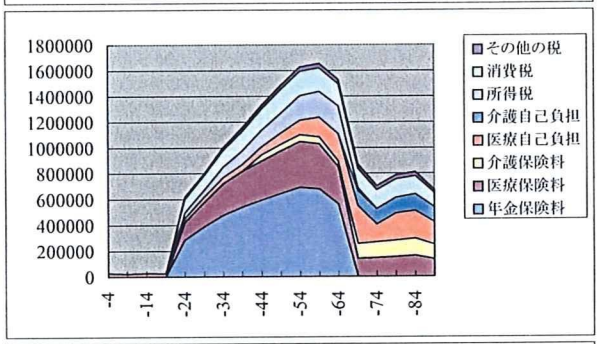


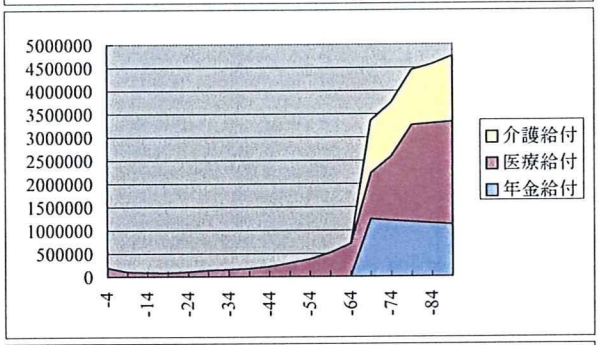
1984生



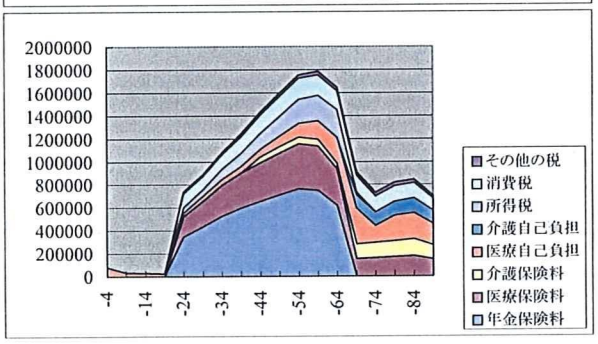


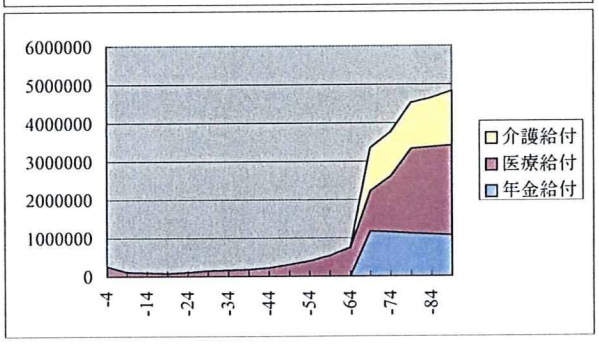
1994生





2004生





平成15年度「社会保障負担のあり方に関する研究」　世代会計を応用した年金・医療・介護の給付と負担の推計(6月
国立社会保障・人口問題研究所　金子能宏・山本克也

給付水準維持方式　国庫負担1/2　年金給付消費税スライド無し　（生涯の給付と負担の引退時点の価値（1ヶ月当たり））								
	34	44	54	64	74	84	94	04
年金保険料	1678316	2029586	2446378	2994477	3819758	4661428	5477738	6209429
医療保険料	2033172	1988647	1961958	2083396	2439475	2850522	3251775	3625047
介護保険料	241577	374454	505481	619381	714380	799545	881904	972746
医療自己負担	1101228	1315488	1467324	1596180	1702457	1829707	1981969	2164581
介護自己負担	619361	624667	624667	624667	624667	624667	624667	624667
所得税	835550	939129	928127	951624	990429	1077814	1189082	1311566
消費税	761437	1158924	1521572	1830751	2039200	2148951	2147810	2078471
物品税	369229	259274	159703	36592	0	0	0	0
その他の税	456119	468338	463243	438473	413439	397225	377980	359767
小計	8095990	9158507	10078453	11175541	12743804	14389858	15932923	17346273
	34	44	54	64	74	84	94	04
年金給付	9524886	9138501	8700277	8281069	6485978	6173462	5876004	5592879
医療給付	10018974	10084808	10265591	10568219	10773211	11280819	11948506	12742173
介護給付	5892874	6154352	6154352	6154352	6154352	6154352	6154352	6154352
小計	25436734	25377661	25120220	25003639	23413541	23608632	23978862	24489403
給付／負担（自己負担込	3.14	2.77	2.49	2.24	1.84	1.64	1.50	1.41
給付／負担（除く自己負	3.99	3.52	3.15	2.79	2.25	1.98	1.80	1.68

給付水準維持方式　国庫負担1/2　年金給付消費税スライド有り　（生涯の給付と負担の引退時点の価値（1ヶ月当たり））								
	34	44	54	64	74	84	94	04
年金保険料	1678316	2029586	2446378	2994477	3819758	4661428	5477738	6209429
医療保険料	2033172	1988647	1961958	2083396	2439475	2850522	3251775	3625047
介護保険料	241577	374454	505481	619381	714380	799545	881904	972746
医療自己負担	1101228	1315488	1467324	1596180	1702457	1829707	1981969	2164581
介護自己負担	619361	624667	624667	624667	624667	624667	624667	624667
所得税	835550	939129	928127	951624	990429	1077814	1189082	1311566
消費税	761437	1158924	1521572	1830751	2039200	2148951	2147810	2078471
物品税	333586	223631	121991	36592	0	0	0	0
その他の税	452296	464111	454947	438473	413439	397225	377980	359767
小計	8056524	9118637	10032445	11175541	12743804	14389858	15932923	17346273
	34	44	54	64	74	84	94	04
年金給付	10270487	10323909	10014655	9587924	7510693	7148803	6804350	6476494
医療給付	10018974	10084808	10265591	10568219	10773211	11280819	11948506	12742173
介護給付	5892874	6154352	6154352	6154352	6154352	6154352	6154352	6154352
小計	26182335	26563069	26434598	26310495	24438256	24583973	24907208	25373018
給付／負担（自己負担込	3.25	2.91	2.63	2.35	1.92	1.71	1.56	1.46
給付／負担（除く自己負	4.13	3.70	3.33	2.94	2.35	2.06	1.87	1.74

集計)

給付水準維持方式　国庫負担1/3　年金給付消費税スライド無し					（生涯の給付と負担の引退時点の価値（1ヶ月当たり））			
	34	44	54	64	74	84	94	04
年金保険料	1678316	2031048	2461284	3040277	3950111	4976879	5996302	6925353
医療保険料	2033172	1988647	1961958	2083396	2439475	2850522	3251775	3625047
介護保険料	241577	374454	505481	619381	714380	799545	881904	972746
医療自己負担	1101228	1315488	1467324	1596180	1702457	1829707	1981969	2164581
介護自己負担	619361	624667	624667	624667	624667	624667	624667	624667
所得税	835550	939129	928127	951624	990429	1077814	1189082	1311566
消費税	685737	1029376	1335764	1592830	1760199	1840321	1835933	1775264
物品税	369229	259274	159703	36592	0	0	0	0
その他の税	456119	468338	463243	438473	413439	397225	377980	359767
小計	8020290	9030422	9907551	10983419	12595157	14396680	16139610	17758990
	34	44	54	64	74	84	94	04
年金給付	9524886	9138501	8700277	8281069	6485978	6173462	5876004	5592879
医療給付	10018974	10084808	10265591	10568219	10773211	11280819	11948506	12742173
介護給付	5892874	6154352	6154352	6154352	6154352	6154352	6154352	6154352
小計	25436734	25377661	25120220	25003639	23413541	23608632	23978862	24489403
給付／負担（自己負担込	3.17	2.81	2.54	2.28	1.86	1.64	1.49	1.38
給付／負担（除く自己負	4.04	3.58	3.21	2.85	2.28	1.98	1.77	1.64

給付水準維持方式　国庫負担1/3　年金給付消費税スライドあり					（生涯の給付と負担の引退時点の価値（1ヶ月当たり））			
	34	44	54	64	74	84	94	04
年金保険料	1678316	2031048	2461284	3040277	3950111	4976879	5996302	6925353
医療保険料	2033172	1988647	1961958	2083396	2439475	2850522	3251775	3625047
介護保険料	241577	374454	505481	619381	714380	799545	881904	972746
医療自己負担	1101228	1315488	1467324	1596180	1702457	1829707	1981969	2164581
介護自己負担	619361	624667	624667	624667	624667	624667	624667	624667
所得税	835550	939129	928127	951624	990429	1077814	1189082	1311566
消費税	685737	1029376	1335764	1592830	1760199	1840321	1835933	1775264
物品税	369229	259274	121991	36592	0	0	0	0
その他の税	456119	468338	454947	438473	413439	397225	377980	359767
小計	8020290	9030422	9861543	10983419	12595157	14396680	16139610	17758990
	34	44	54	64	74	84	94	04
年金給付	11115744	11059597	10611151	10108878	7917572	7536077	7172964	6827347
医療給付	10018974	10084808	10265591	10568219	10773211	11280819	11948506	12742173
介護給付	5892874	6154352	6154352	6154352	6154352	6154352	6154352	6154352
小計	27027592	27298756	27031094	26831448	24845135	24971248	25275822	25723871
給付／負担（自己負担込	3.37	3.02	2.74	2.44	1.97	1.73	1.57	1.45
給付／負担（除く自己負	4.29	3.85	3.48	3.06	2.42	2.09	1.87	1.72

平成15年度「社会保障負担のあり方に関する研究」　世代会計を応用した年金・医療・介護の給付と負担の推計(6月推
国立社会保障・人口問題研究所　金子能宏・山本克也

保険料固定方式	国庫負担1/2	年金給付消費税スライド無し			(生涯の給付と負担の引退時点の価値(1ヶ月当たり))			
	34	44	54	64	74	84	94	04
年金保険料	1678316	2029586	2446378	2953537	3607594	4246136	4853115	5389149
医療保険料	2033172	1988647	1961958	2083396	2439475	2850522	3251775	3625047
介護保険料	241577	374454	505481	619381	714380	799545	881904	972746
医療自己負担	1101228	1315488	1467324	1596180	1702457	1829707	1981969	2164581
介護自己負担	619361	624667	624667	624667	624667	624667	624667	624667
所得税	835550	939129	928127	951624	990429	1077814	1189082	1311566
消費税	756692	1148725	1505843	1809222	2012650	2118860	2115576	2046766
物品税	369229	259274	121991	36592	0	0	0	0
その他の税	456119	468338	454947	438473	413439	397225	377980	359767
小計	8091245	9148309	10016716	11113072	12505091	13944475	15276067	16494289
	34	44	54	64	74	84	94	04
年金給付	9524886	9138501	8700277	8281069	6485978	6173462	5876004	5592879
医療給付	10018974	10084808	10265591	10568219	10773211	11280819	11948506	12742173
介護給付	5892874	6154352	6154352	6154352	6154352	6154352	6154352	6154352
小計	25436734	25377661	25120220	25003639	23413541	23608632	23978862	24489403
給付／負担(自己負担込み)	3.14	2.77	2.51	2.25	1.87	1.69	1.57	1.48
給付／負担(除く自己負担)	3.99	3.52	3.17	2.81	2.30	2.05	1.89	1.79

保険料固定方式	国庫負担1/2	年金給付消費税スライド有り			(生涯の給付と負担の引退時点の価値(1ヶ月当たり))			
	34	44	54	64	74	84	94	04
年金保険料	1678316	2029586	2446378	2953537	3607594	4246136	4853115	5389149
医療保険料	2033172	1988647	1961958	2083396	2439475	2850522	3251775	3625047
介護保険料	241577	374454	505481	619381	714380	799545	881904	972746
医療自己負担	1101228	1315488	1467324	1596180	1702457	1829707	1981969	2164581
介護自己負担	619361	624667	624667	624667	624667	624667	624667	624667
所得税	835550	939129	928127	951624	990429	1077814	1189082	1311566
消費税	756692	1148725	1505843	1809222	2012650	2118860	2115576	2046766
物品税	369229	259274	121991	36592	0	0	0	0
その他の税	456119	468338	454947	438473	413439	397225	377980	359767
小計	8091245	9148309	10016716	11113072	12505091	13944475	15276067	16494289
	34	44	54	64	74	84	94	04
年金給付	9524886	9138501	8700277	8281069	6485978	6173462	5876004	5592879
医療給付	10018974	10084808	10265591	10568219	10773211	11280819	11948506	12742173
介護給付	5892874	6154352	6154352	6154352	6154352	6154352	6154352	6154352
小計	25436734	25377661	25120220	25003639	23413541	23608632	23978862	24489403
給付／負担(自己負担込み)	3.14	2.77	2.51	2.25	1.87	1.69	1.57	1.48
給付／負担(除く自己負担)	3.99	3.52	3.17	2.81	2.30	2.05	1.89	1.79

推計)

保険料固定方式	国庫負担1/3	年金給付消費税スライド無し			(生涯の給付と負担の引退時点の価値(1ヶ月当たり))			
	34	44	54	64	74	84	94	04
年金保険料	1678316	2031048	2461284	2975607	3628674	4264291	4862626	5389149
医療保険料	2033172	1988647	1961958	2083396	2439475	2850522	3251775	3625047
介護保険料	241577	374454	505481	619381	714380	799545	881904	972746
医療自己負担	1101228	1315488	1467324	1596180	1702457	1829707	1981969	2164581
介護自己負担	619361	624667	624667	624667	624667	624667	624667	624667
所得税	835550	939129	928127	951624	990429	1077814	1189082	1311566
消費税	682339	1022389	1325094	1578373	1742507	1820350	1814745	1754417
物品税	369229	259274	159703	36592	0	0	0	0
その他の税	456119	468338	463243	438473	413439	397225	377980	359767
小計	8016892	9023435	9896880	10904293	12256027	13664120	14984747	16201939
	34	44	54	64	74	84	94	04
年金給付	9524886	9138501	8700277	8281069	6485978	6173462	5876004	5592879
医療給付	10018974	10084808	10265591	10568219	10773211	11280819	11948506	12742173
介護給付	5892874	6154352	6154352	6154352	6154352	6154352	6154352	6154352
小計	25436734	25377661	25120220	25003639	23413541	23608632	23978862	24489403
給付／負担(自己負担込	3.17	2.81	2.54	2.29	1.91	1.73	1.60	1.51
給付／負担(除く自己負	4.04	3.58	3.22	2.88	2.36	2.11	1.94	1.83

保険料固定方式	国庫負担1/3	年金給付消費税スライドあり			(生涯の給付と負担の引退時点の価値(1ヶ月当たり))			
	34	44	54	64	74	84	94	04
年金保険料	1678316	2031048	2461284	2975607	3628674	4264291	4862626	5389149
医療保険料	2033172	1988647	1961958	2083396	2439475	2850522	3251775	3625047
介護保険料	241577	374454	505481	619381	714380	799545	881904	972746
医療自己負担	1101228	1315488	1467324	1596180	1702457	1829707	1981969	2164581
介護自己負担	619361	624667	624667	624667	624667	624667	624667	624667
所得税	835550	939129	928127	951624	990429	1077814	1189082	1311566
消費税	682339	1022389	1325094	1578373	1742507	1820350	1814745	1754417
物品税	369229	259274	159703	36592	0	0	0	0
その他の税	456119	468338	463243	438473	413439	397225	377980	359767
小計	8016892	9023435	9896880	10904293	12256027	13664120	14984747	16201939
	34	44	54	64	74	84	94	04
年金給付	7237392	7167224	6892033	6602342	5311182	5143621	4995044	4854912
医療給付	10018974	10084808	10265591	10568219	10773211	11280819	11948506	12742173
介護給付	5892874	6154352	6154352	6154352	6154352	6154352	6154352	6154352
小計	23149240	23406384	23311976	23324913	22238745	22578791	23097902	23751437
給付／負担(自己負担込	2.89	2.59	2.36	2.14	1.81	1.65	1.54	1.47
給付／負担(除く自己負	3.68	3.30	2.99	2.69	2.24	2.01	1.87	1.77

4. Public Pension Reform under Uncertainty: The Risk of Return and Increasing Longevity[1]

Naomi Miyazato[2]

National Institute of Population and Social Security Research

Abstruct:

The situation of the aging population in Japan is more severe than in most developed countries. Recently, the Ministry of Health, Labor and Welfare announced the 2004 public pension reform plans, recognizing above facts. The 2004 public pension reform plans are beneficial because the intergenerational inequality on tax burdens is corrected due to the fixed tax rate (or the pension premium rate) the adjusted benefit level with the current working labor force. However, we need to consider further whether the replacement rate ratio of 50% level is optimal or not. The purpose of this paper is to investigate the optimal level of replacement rate ratio in Japan by considering not only the risk of fluctuation of return but also of increasing longevity.

1 This paper was written while I stayed the Wharton School, University of Pennsylvania as a visiting research fellow of the research grant from the Imperial Gift Foundation(Boshi Aiiku Kai) that supports this research project for the Science of Health, Labor and Welfare. I would like to appreciate for useful comments and help from Prof. Olivia Michel during my stay in the Wharton School. Finally, I want to note that the results of this paper do not represent any views and opinions of the institute that I belong to.

2 宮里尚三（国立社会保障・人口問題研究所社会保障応用分析研究部研究員）

1. Introduction

The situation of the aging population in Japan is more severe than in most developed countries. The percentage of people over 65 was 17.4% in 2000. It will be 22.5% in 2010, 28.7% in 2025. The present Japanese public pension is a Defined Benefit pension system. It is based on a Pay-As-You-Go system. A Pay-As-You-Go system is vulnerable due to the increasing aging population, and almost all people in Japan have doubts about the sustainability of the preset public pension system.

Recently, the Ministry of Health, Labor and Welfare announced the 2004 public pension reform plans, recognizing above facts. Characteristics of the reform plans are as follows: 1) Pension premiums (or taxes) will increase gradually by 2022, and will be fixed at 18.0% after 2022. 2) Benefits levels will be adjusted in conjunction with the two factors: the size of the work force and the wage level of the work force. 3) Replacement rate ratio will be secured at the 50% level. The 2004 public pension reform plans are beneficial because the intergenerational inequality on tax burdens is corrected due to the fixed tax rate (or the pension premium rate) the adjusted benefit level with the current working labor force. However, we need to consider further whether the replacement rate ratio of 50% level is optimal or not.

There are two main pension systems. One is the Defined Benefit pension system, the other is the Defined Contribution pension system. Each system has an advantage and a disadvantage. The advantage of Defined Benefit pension is that this system guarantees some amount, in other word replacement ratio, after retirement. Therefore, the volatility of assets or income after retirement is low. Moreover, Defined Benefit pension reduces the risk of increasing longevity on the benefit aspect because people can receive some amount before death. The disadvantage of this system is that the more the aging population increases, the more the working generation has to pay taxes (or premiums). Therefore, the rising longevity combined with low birth rate has a negative impact on the pension tax burden aspect. On the other hand, the advantage of Defined Contribution pension system is that this system is neutral to aging or a change of demographic structure. In other words, aging does not have a negative impact on the burden aspect under Defined Contribution pension system. The disadvantage of this system is that the volatility of assets or income after retirement is high. Moreover, Defined Contribution pension system is vulnerable to the risk of increasing longevity from the benefit aspect because people can not receive a benefit beyond their accumulated amount.

The purpose of this paper is to investigate the optimal level of replacement rate ratio in Japan by considering not only the risk of fluctuation of return but also of increasing

longevity.

2. Model description

We use the overlapping generation model to analyze the optimal level of replacement ratio. An individual lives a maximum of N periods and faces a possibility of death at each period. As we use the population projection published by the National Institute of Population and Social Security Research as demographic patterns[2]. An advantage of using those projected population is that simulation results are more realistic than in the case of assuming a population growth rate is constant. We assume than individual does not take into consideration his or her child. Thus there is no bequest motive. In addition, we assume that individual does not have any assets before he or she enters the economy.

2.1. Utility function

We consider a representative individual with the following utility function:

$$E\left[\sum_{j=1}^{N}\beta^{j-1}\pi_{j,t+j-1}u\left(c_{j,t+j-1}\right)\right] \qquad (1)$$

where E is the expectation operator, $\beta^{j-1}$is the time discount factor at age j, $c_{j,t+j-1}$ is the consumption at age j in year $t+j-1$, and t is generation. $\pi_{j,t+j-1}$ is a probability that individual is alive at age j conditional on being alive at age 1. Survival rates of an individual differ at each generation. According to our assumption of population projection for Japan, the later individual enters the economy, the higher will be his or her survival rates at each time[3].

The period utility function $u(c)$ is of the constant relative risk aversion class

$$u(c)=\begin{cases} c^{1-\gamma}/(1-\gamma) & \gamma > 1 \\ log(c) & \gamma = 1 \end{cases}$$

where γ is the coefficient of relative risk aversion. The more the coefficient of relative risk aversion is large, the more individual is risk averse. If γ equals 1, the period

---

[2] In The Population Projection for Japan (2002), it is assumed that a population growth rate is constant after 2100.

[3] However, it is presumed that survival rates after 2050 is same as that of 2050 in Population Projection for Japan (2002).

utility function is *log(c)*.

2.2. Budget constraints

An individual earns wage income during working periods and receives pension benefits after retirement. The individual consumes at each period and saves the difference between accumulated assets and consumption. We assume that individual consumes all accumulated assets at age N if he or she lives to the maximum period N. Thus the individual does not save at age N. Wage income is zero after retirement and pension benefits is zero before retirement. Pension premiums (or taxes) are proportionally imposed on wage income. Individual budget constraint is as follows:

$$c_{j,t+j-1} + s_{j,t+j-1} = (1 + r_{t+j-1})s_{j-1,(t+j-1)-1} + (1 - \tau_{t+j-1})w_{j,t+j-1} + b_{j,t+j-1} \qquad (2)$$

where $s_{j,t+j-1}$ is savings, $w_{j,t+j-1}$ is wage income, and $b_{j,t+j-1}$ is pension benefits at age j in year $t+j-1$. τ_{t+j-1} is pension premiums in year $t+j-1$ and r_{t+1-1} is return of assets at initial of period $t+j-1$. The equation (2) becomes $c_{j,t+j-1} + s_{j,t+j-1} = (1+r_{t+j-1})s_{j-1,(t+j-1)-1} + (1-\pi_{t+j-1})w_{j,t+j-1}$ if individual is in working periods. As we assume assets of individual are zero before appearance in economy, it is $c_{1,t} + s_{1,t} = (1-\pi_t)w_{1,t}$ at age 1. On the other hand, it becomes $c_{j,t+j-1} + s_{j,t+j-1} = (1+r_{t+j-1})s_{j-1,(t+j-1)-1} + b_{j,t+j-1}$ if individual has retired. It is $c_{N,t+N-1} = (1+r_{t+N-1})s_{N-1,(t+N-1)-1} + b_{N,t+N-1}$ at age N if individual lives up to the maximum period N.

In this model, a public pension system is presumed as the Defined Benefit pension system based on pay-as-you-go. Under this pension system, pension benefits are defined by the replacement ratio multiplied by the average wage of working person. Therefore, pension benefits are described below:

$$b_{j,j+t-1} = \kappa \overline{w}_{j+t-1}, \qquad R+1 = j = N \qquad (3)$$

where κ is replacement rate ratio, $R+1$ is retirement age, $\overline{w}_{j+t-1}$ is average wage of workers in year $j+t-1$. If individual is in working periods ($1 = j = R$), $b_{j,j+t-1}$ equals zero. We assume that the retirement age is fixed for all generations. Therefore, the later individual enters the economy, the more he or she takes the risk of increasing longevity because his or her mortality rate is low. For simplicity, the wage growth rate is assumed to be constant at g. Thus the wage at next period is $w_{j+1,j+t} = (1+g)w_{j,j+t-1}$. If the

individual has retired($R+1 = i = N$), $w_{j,j+t-1}$ equals zero. In addition, labor supply is assumed to be exogenous. As we emphasize an advantage of the Defined Benefits pension, we assume there is no risk of a wage fluctuation.

Another risk rises from the volatility of return of assets. We assume that return of assets is described by the following equations:

$$r_i = \mu + z_i \tag{4}$$

$$z_i = \rho z_{i-1} + \varepsilon \tag{5}$$

$$\varepsilon \sim N(0, \sigma^2) \tag{6}$$

where μ is the constant term and z_i is the permanent error term. For simplicity, this permanent error is assumed to follow AR(1) process that is either a unit-root or close to a unit-root to capture the persistence of return of assets over time. The term ρ controls the degree of persistence of previous error term. In addition, the transient error term ε is assumed to be normally distributed with zero mean and variance of σ^2.

2.3. Individual decision problem

The decision problem of individual can be described as the dynamic programming problem. In this model, the state variable of generation t at age is expressed as $x_{j,j+t-1} = (s_{j,(j+t-1)-1}, z_{j+t-1})$ and the control variable is consumption $c_{j,j+t-1}$ or savings $s_{j,j+t-1}$. Let $V_{j,t+j-1}(x_{j,t+j-1})$ be the maximized value of the objective function of generation t at age j with the state variable $x_{j,j+t-1}$. $V_{j,t+j-1}(x_{j,t+j-1})$ is given as the solution to the following dynamic program.

$$V_{j,t+j-1}(x_{j,t+j-1}) = \max_{c_{j,t+j-1}} \left\{ u(c_{j,t+j-1}) + \beta^{j-1} \frac{\pi_{j+1,t+j}}{\pi_{j,t+j-1}} E\left[V_{j+1,t+j}(x_{j+1,t+j}) \middle| z_{t+j-1} \right] \right\}$$

$$(i = 1, \ldots, N) \tag{7}$$

subject to equation (2) and $V_{j+1,t+j}(x_{j+1,t+j}) = 0$

The value function at age $N+1$ is identically zero because death is certain beyond age N ($\pi_{N+1,t+N} = 0$). The decision rules and the value functions for each age $j = 1, 2, \ldots, N$

can be found by working the backward recursion from the last period of life[4].

2.4. Government

A public pension system in this paper is described as the Defined Benefit pension system based on a pay-as-you-go. A characteristic of this system is that total benefits of retired persons and total taxes (or pension premium) of working persons are equal. Since total benefits is pension benefits multiplied by number of retire persons and total taxes is pension taxes multiplied by number of working persons, the budget balance is described as follows:

$$\sum_{j=R+1}^{N} b_{j,i} \times L_{j,i} = \sum_{j=1}^{R} \tau_i \times w_{j,i} \times L_{j,i} \qquad (8)$$

where $L_{j,I}$ is population of age j agent in year i. Using equation (3) to substitute $b_{j,i}$ in equation (8), it exchange as follows:

$$\sum_{j=R+1}^{N} \kappa \overline{w}_i \times L_{j,i} = \sum_{j=1}^{R} \tau_i \times w_{j,i} \times L_{j,i} \qquad (9)$$

The equation (9) implies that τ_i is determined if a replacement rate ratio κ settles. The degree of κ means the size of public pension because the degree of pension benefits and taxes (or premium) depend on κ.

The purpose of this paper is to find out what level of replacement ratio leads to the highest welfare of the individual. We can consider the optimal public pension size by finding that replacement ratio.

3. Simulation results

3.1. Parameters setting

We explain how to select the parameters of the model. The parameters used by a benchmark simulation are listed in Table 1.

We regard one period of the model as 5 years. The time discount factor is set as 0.98. This value is the same as using in Feldstein and Ranguelova (2001). The coefficient of relative risk aversion is set as 2 in a benchmark case because it is plausible that this

[4] Using the budget constraint (2) to substitute for $c_{j,t+j-1}$ in Bellman's equation (7), the problem reduces to choosing the decision control variable s_{jj+t-1}.

value would be less than 3 and probably 2 in Feldstein and Ranguelova (2001).

[Table 1]

The demographic patterns follow the dates in the Population Projection for Japan (2002). Since one period of the model is assumed to be 5 years, we make an population projection in 5-year increments. Every 5 year represents an additional period. We use those dates taking the average of male and female population. In this model, we assume that individuals enter the economy at age 20 (model period 1) and die at age 100 (model period 17). Thus we ignore the population dates of age below 20 although the Population Projection for Japan shows these dates. In addition, retirement age is assumed to be age 65 (model period 9) and retirement age does not differ for all generations.

The survival rates follow the life table in the Population Projection for Japan. We also use the average dates of male and female survival rates. In Table 2, we find that the later individuals enter the economy, the higher the survival rates will be at every age. Thus, the later generations that enter the economy take more the risk of increasing longevity than generations that enter the economy at early age because retirement age does not differ for all generations. This means that the later generations gain more advantage of the Defined Benefits pension from the benefits aspect.

We assume that the wage growth rate g is 2 % and the expected return of assets μ is 2%. The coefficient of auto-regression ρ is assumed to be 1, and σ^2 is assumed to be 0.1.

[Table 2]

3.2. Results

First of all, we show welfare levels of the generation who enter the economy in 2000 under every replacement rate ratio. We assume that the coefficient of relative risk aversion γ is 2 in Figure 1. We find that the welfare level in the case where a replacement ratio κ is over zero is higher than the case where κ equals zero. However the welfare level decreases if κ is too high. This reason is as follows. In the range of low replacement rate ratio, the individual gains from the advantage of the Defined Benefits pension which covers the risk of assets fluctuation and this individual lives longer after retirement. However the disadvantage of the Defined Benefits pension, in which the expected return of this pension is lower than the expected interest rate,

exceeds the advantage of this pension. The optimal replacement rate ratio of the generation 2000 is 0.45

[Figure 1]

Next, we show the welfare levels of the generation 2000 in the case where the coefficient of relative risk aversion γ equals 1 and 3. Figure 2 shows the case where γ is 1. We find that the optimal replacement rate ratio is 0.32 in this case. This result is consistent with the fact that the individual is less (more) sensitive to the fluctuation of the consumption if the coefficient of relative risk aversion is low (high). Therefore, the degree of the advantage of the Defined Benefits pension becomes small (large). In the case where γ is 3, the optimal replacement rate ratio is 5.0.

[Figure 2]
[Figure 3]

Next, table 3 shows welfare levels of all generations under every replacement rate ratio. We find that the optimal replacement rate ratio of all generations is below 5.0 in the case where γ is 1 and 2. In the case where γ equal to 3, the optimal level is also below 5.0 except for those who enter the economy in 2000.

[Table 3]
[Table 4]
[Table 5]

4. Conclusion

We simply conclude as follows. If γ =1, or γ =2, reducing the replacement rate ratio below 50% increases the welfare of all future generations. If γ =3, reducing the replacement rate ratio below 50% increases the welfare of future generations except for a generation in 2000.

Appendix 1

In this paper, we assumed that the wage growth rate is constant. Although this

assumption is strong, we can derive the constant wage growth rate to specify the production function using the following equations.

$$f(k_i) = a_i + b_i k_i \qquad (A.1)$$

$$a_i = (1+g)a_{i-1} \qquad (A.2)$$

$$b_i = \mu + z_i \qquad (A.3)$$

where $f(k_i)$ is the output per worker, k_i is the capital stock per worker in year i. The term of a_i and b_i follow the equations (A.2) and (A.3) respectively. g and μ are constant values. In this production function, the wage rate w_i and the return of assets (or capital) r_i is given by

$$r_i = f'(k_i) = b_i \qquad (A.4)$$

$$w_i = f(k_i) - r_i k_i = a_i \qquad (A.5)$$

where f'(k_i) is differentiated from k_i. Thus, we can derive the following the wage rate and the return of assets.

$$r_i = \mu + z_i$$

$$w_t = (1+g)w_{t-1}$$

Appendix 2

We describe how to calculate the optimal decision rules of the individual. We use the grid method to solve the individual decision rules. First of all, we put a grid on the state variable $x_{k,l} = (s_{k,l}, z_l)$. We start in the last period of N of an individual's life and solve for the control variable $s_{j,i}$ for grid point in the state variable $x_{j+1,i+1} = (s_{j+1,i+1}, z_{i+1})$, setting $V_{j+1,i+1}(x_{j+1,i+1}) = 0$. Given the maximizing control variable $s_{j,i}$ on gridpoints, we can determine values for the value function $V_{j,i}(x_{j,i})$ on gridpoints. We repeat this procedure to solve for value functions and decision rules for all earlier periods.

In this paper, we assigned a value of 500 points and 5 points $(s_{k,l}, z_l)$ on the grid. The spacing between points on the asset grid increases with asset levels. More specifically, asset gridpoints are placed according to $s^1 = 0$, $s^m = d \times m^{2.35}$, m = 2,...,500, where $d = \bar{s}/500^{2.35}$ and $\bar{s}$ is an upper bound imposed on the asset grid.